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Certain Geometric Properties and Matrix Transformations on a Newly Introduced Banach Space

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Article Info

Abstract

The main purpose of this study is to characterize some matrix classes from classical sequence spaces into a newly introduced space and find the norm of some special matrix operators. Also, we give certain geometric properties of this space.

1. Introduction

The matrix transformations in sequence spaces have been studied by many authors over years. Since the most general linear operators from a sequence space to another one can be given by an infinite matrix, the theory of matrix transformations has been of great importance in the study of sequence spaces. For the relevant literature consult to [1]-[6].

In the recent times, the interest in investigating geometric properties of sequence spaces with topological properties have increased. Over years several papers on the geometric properties of various spaces have appeared. For instance, Mursaleen et al. [7] examined the geometric properties of Euler sequence space. More information about the relevant literature can be found in [8]-[14].

The main purpose of this work is to characterize some matrix classes on a newly introduced sequence space and find the norm of certain bounded linear matrix operators. Also, we prove that the resulting space is of type p Banach-Saks and it has the weak fixed point property. Finally, we investigate the strictly convexity and uniformly convexity of this space.

2. Preliminaries and notations

A sequence space is a linear subspace of the space of all real valued sequences ω. ℓ_∞, c, c_0 and ℓ_p $(1 \leq p < \infty)$ are the sequence spaces of all bounded, convergent, null sequences and absolutely p-summable sequences, respectively.

Given any sequence spaces X and Y and an infinite matrix $T = (t_{ij})$, T is called a matrix mapping from X into Y if for every sequence $x = (x_j) \in X$, $Tx = (T_i(x))$ with

$$T_i(x) = \sum_{j=1}^{\infty} t_{ij} x_j$$

is in Y and the series is convergent for each $i \in \mathbb{N} = \{1, 2, ...\}$. Then, Tx is called the T-transform of x.

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The set

$$X_T = \{x = (x_j) \in \omega : Tx \in X\}$$

is called the matrix domain of T in the space X and it is also a sequence space.

Let $\varphi : \mathbb{N} \to \mathbb{N}$ be the Euler function defined as

$$\varphi(i) = \sum_{j=1,(j,i)=1}^{i} 1,$$

where (j,i) is the greatest common divisor of j and i. That is, $\varphi(i)$ gives the number of positive integers less than i which are coprime with j.

The Euler function φ satisfies the following properties:

1. $i = \sum_{j|i} \varphi(j)$ holds for every $i \in \mathbb{N}$.
2. $\varphi(i) = i \prod_{p|i}(1 - \frac{1}{p})$, where p is the prime divisor of i.
3. $\varphi(ij) = \varphi(i)\varphi(j)$ holds for $(i,j) = 1$.

Let $i = p_1^{\alpha_1} p_2^{\alpha_2} ... p_l^{\alpha_l}$. The Möbius function $\mu : \mathbb{N} \to \{-1,0,1\}$ is defined as

$$\mu(i) = (-1)^l \text{ if } \alpha_1 = \alpha_2 = ... = \alpha_l = 1$$

$$\mu(i) = 0 \text{ if } \alpha_k \neq 1 \text{ for at least one } k \in \{1,2,...,l\},$$

where $p_1, p_2, ..., p_l$ are non-equivalent prime numbers and $p_1^{\alpha_1} p_2^{\alpha_2} ... p_l^{\alpha_l}$ is the prime factorization of $i > 1$. Also,

$$\mu(1) = 1$$

and for $i \neq 1$

$$\sum_{p|i} \mu(p) = 0$$

holds.

Φ-summability was introduced by Schoenberg [15] in order to study the Riemann integrability of a generalized Dirichlet function in $[0,1]$. It is said that a sequence $x = (x_j)$ is φ-convergent to l if

$$\lim_{i \to \infty} \frac{1}{i} \sum_{j|i} \varphi(j) x_j = l.$$

Let $\Phi = (\phi_{ij})$ be the matrix defined as

$$\phi_{ij} = \begin{cases} \frac{\varphi(j)}{i} & , & \text{if } j \mid i, \\ 0 & , & \text{if } j \nmid i. \end{cases}$$

The regularity of this special matrix is also observed by Schoenberg [15]. This means that the matrix Φ maps c into c and the limit is preserved.

In [16], by using this matrix, the sequence spaces

$$\ell_p(\Phi) = \left\{ x = (x_i) \in \omega : \sum_i \left| \frac{1}{i} \sum_{j|i} \varphi(j) x_j \right|^p < \infty \right\} \quad (1 \leq p < \infty)$$

and

$$\ell_\infty(\Phi) = \left\{ x = (x_i) \in \omega : \sup_i \left| \frac{1}{i} \sum_{j|i} \varphi(j) x_j \right| < \infty \right\}$$

are introduced and proved that these spaces are Banach spaces with the norms

$$\|x\|_{\ell_p(\Phi)} = \left(\sum_i \left| \frac{1}{i} \sum_{j|i} \varphi(j) x_j \right|^p \right)^{1/p} \quad (1 \leq p < \infty)$$

and

$$\|x\|_{\ell_\infty(\Phi)} = \sup_i \left| \frac{1}{i} \sum_{j|i} \varphi(j) x_j \right|,$$

respectively.

Unless otherwise stated, $\tilde{x} = (\tilde{x}_i)$ will be the Φ-transform of a sequence $x = (x_i)$, that is,

$$\tilde{x}_i = \Phi_i(x) = \frac{1}{i} \sum_{j|i} \varphi(j) x_j \tag{2.1}$$

for all $i \in \mathbb{N}$.

3. Some matrix transformations and norms of matrix operators

In this part of the study, we firstly give the characterization of matrix classes $(X, \ell_p(\Phi))$, where $X \in \{\ell_\infty, c, c_0, \ell_1\}$ and $1 \leq p \leq \infty$. For this aim, we give the following results, where $\mathscr{F}$ denotes the collection of all finite subsets of $\mathbb{N}$. q is conjugate of p; that is $p^{-1} + q^{-1} = 1$ with $1 < p, q < \infty$.

Lemma 3.1. *[17] Let $1 \leq p < \infty$.*

(a) $T = (t_{ij}) \in (\ell_\infty, \ell_p) = (c, \ell_p) = (c_0, \ell_p)$ *if and only if*

$$\sup_{K \in \mathscr{F}} \sum_i \left| \sum_{j \in K} t_{ij} \right|^p < \infty.$$

(b) $T = (t_{ij}) \in (\ell_1, \ell_p)$ *if and only if*

$$\sup_j \sum_i \left| t_{ij} \right|^p < \infty.$$

(c) $T = (t_{ij}) \in (\ell_\infty, \ell_\infty) = (c, \ell_\infty) = (c_0, \ell_\infty)$ *if and only if*

$$\sup_i \sum_j \left| t_{ij} \right| < \infty.$$

(d) $T = (t_{ij}) \in (\ell_1, \ell_\infty)$ *if and only if*

$$\sup_{i,j} \left| t_{ij} \right| < \infty.$$

Theorem 3.2. *Let $1 \leq p < \infty$.*

(a) $T = (t_{ij}) \in (\ell_\infty, \ell_p(\Phi)) = (c, \ell_p(\Phi)) = (c_0, \ell_p(\Phi))$ *if and only if*

$$\sup_{K \in \mathscr{F}} \sum_i \left| \sum_{j \in K} \sum_{l|i} \frac{\varphi(l)}{i} t_{lj} \right|^p < \infty.$$

(b) $T = (t_{ij}) \in (\ell_1, \ell_p(\Phi))$ *if and only if*

$$\sup_j \sum_i \left| \sum_{l|i} \frac{\varphi(l)}{i} t_{lj} \right|^p < \infty.$$

(c) $T = (t_{ij}) \in (\ell_\infty, \ell_\infty(\Phi)) = (c, \ell_\infty(\Phi)) = (c_0, \ell_\infty(\Phi))$ *if and only if*

$$\sup_i \sum_j \left| \sum_{l|i} \frac{\varphi(l)}{i} t_{lj} \right| < \infty.$$

(d) $T = (t_{ij}) \in (\ell_1, \ell_\infty(\Phi))$ *if and only if*

$$\sup_{i,j} \left| \sum_{l|i} \frac{\varphi(l)}{i} t_{lj} \right| < \infty.$$

Proof. Given any infinite matrix $T = (t_{ij}) \in (\ell_\infty, \ell_p(\Phi))$, define a new matrix $\hat{T} = (\hat{t}_{ij})$ by

$$\hat{t}_{ij} = \sum_{l|i} \frac{\varphi(l)}{i} t_{lj}$$

for all $i, j \in \mathbb{N}$. Then, for any $x = (x_j) \in \ell_\infty$, the equality

$$\sum_j \hat{t}_{ij} x_j = \sum_{l|i} \frac{\varphi(l)}{i} \sum_j t_{lj} x_j$$

means that $\hat{T}_i(x) = \Phi_i(Tx)$ for all $i \in \mathbb{N}$. This implies that $Tx \in \ell_p(\Phi)$ for $x = (x_j) \in \ell_\infty$ if and only if $\hat{T}x \in \ell_p$ for $x = (x_j) \in \ell_\infty$. Hence, we conclude from Lemma 3.1 (a) that

$$\sup_{K \in \mathscr{F}} \sum_i \left| \sum_{j \in K} \sum_{l|i} \frac{\varphi(l)}{i} t_{lj} \right|^p < \infty.$$

The other results follow with the same technique by using Lemma 3.1 (b), (c) and (d). $\qquad\square$

Now, we investigate the norm of the bounded linear matrix operators from $\ell_p(\Phi)$ into $\ell_1(\Phi)$ and $\ell_\infty(\Phi)$ for $1 \le p \le \infty$. Firstly, we have a lemma which is essential for our investigation.

Lemma 3.3. *Given any infinite matrix $T = (t_{ij})$, the following statements hold:*

(a) The norm of $T \in B(\ell_p, \ell_\infty)$ is defined by

$$\|T\|_{(\ell_1, \ell_\infty)} = \sup_{i,j} |t_{ij}|$$

and

$$\|T\|_{(\ell_p, \ell_\infty)} = \sup_i \sum_j |t_{ij}|^q \quad (1 < p \le \infty).$$

(b) The norm of $T \in B(\ell_p, \ell_1)$ is defined by

$$\|T\|_{(\ell_1, \ell_1)} = \sup_j \sum_i |t_{ij}|$$

and

$$\|T\|_{(\ell_p, \ell_1)} = \sup_{K \in \mathscr{F}} \sum_j \left| \sum_{i \in K} t_{ij} \right|^q \quad (1 < p \le \infty).$$

Theorem 3.4. *Let $T = (t_{ij})$ be an infinite matrix.*

(a) If $T \in B(\ell_1(\Phi), \ell_\infty(\Phi))$, then

$$A_1^\infty = \sup_{i,j} \left| \sum_{j|l} \frac{\mu(\frac{l}{j})}{\varphi(l)} j \sum_{k|i} \frac{\varphi(k)}{i} t_{kl} \right|$$

is finite. In this case, $\|T\|_{(\ell_1(\Phi), \ell_\infty(\Phi))} = A_1^\infty$.

(b) Let $1 < p \le \infty$. If $T \in B(\ell_p(\Phi), \ell_\infty(\Phi))$, then

$$A_p^\infty = \sup_i \sum_j \left| \sum_{j|l} \frac{\mu(\frac{l}{j})}{\varphi(l)} j \sum_{k|i} \frac{\varphi(k)}{i} t_{kl} \right|^q$$

is finite. In this case, $\|T\|_{(\ell_p(\Phi), \ell_\infty(\Phi))} = A_p^\infty$.

(c) If $T \in B(\ell_1(\Phi), \ell_1(\Phi))$, then

$$A_1^1 = \sup_j \sum_i \left| \sum_{j|l} \frac{\mu(\frac{l}{j})}{\varphi(l)} j \sum_{k|i} \frac{\varphi(k)}{i} t_{kl} \right|$$

is finite. In this case, $\|T\|_{(\ell_1(\Phi), \ell_1(\Phi))} = A_1^1$.

(d) Let $1 < p \le \infty$. If $T \in B(\ell_p(\Phi), \ell_1(\Phi))$, then

$$A_p^1 = \sup_{K \in \mathscr{F}} \sum_j \left| \sum_{i \in K} \sum_{j|l} \frac{\mu(\frac{l}{j})}{\varphi(l)} j \sum_{k|i} \frac{\varphi(k)}{i} t_{kl} \right|^q$$

is finite. In this case, $\|T\|_{(\ell_p(\Phi), \ell_1(\Phi))} = A_p^1$.

Proof. Let $\tilde{T} = \Phi T \Phi^{-1}$. From Theorem 3 in [16], it is known that the spaces $\ell_p(\Phi)$ and ℓ_p are linearly isomorphic, where $1 \leq p \leq \infty$. Hence, we deduce from the following diagram

$$
\begin{array}{ccc}
\ell_p(\Phi) & \xrightarrow{\;\;T\;\;} & X(\Phi) \\[4pt]
{\scriptstyle \Phi^{-1}}\big\uparrow & & \big\downarrow{\scriptstyle \Phi} \\[4pt]
\ell_p & \xrightarrow[\tilde{T}=\Phi T \Phi^{-1}]{} & X
\end{array}
$$

that $\|T\|_{(\ell_p(\phi),X(\Phi))} = \|\tilde{T}\|_{(\ell_p,X)}$, where $X \in \{\ell_\infty, \ell_1\}$ and $1 \leq p \leq \infty$. Thus, the desired results follows from Lemma 3.3.   □

4. Certain geometric properties of $\ell_p(\Phi)$

In this part of the study, some geometric properties of the space $\ell_p(\Phi)$ for $1 < p < \infty$ is given. $\mathscr{B}_X$ denotes the unit ball in a normed space $(X, \|.\|)$.

It is said that a Banach space X satisfies the *Banach-Saks property* if every sequence (u_n) in $X \cap \ell_\infty$ has a subsequence (t_n) such that the sequence $(a_k(t))$ is convergent, where

$$
a_k(t) = \frac{1}{k+1}(t_0 + t_1 + ... + t_k); \;\; (k \in \mathbb{N}).
$$

It is said that a Banach space X satisfies the *weak Banach-Saks property* if there exists a subsequence (t_n) of a given weakly null sequence (u_n) in X such that the sequence $(a_k(t))$ is strongly convergent to zero.

It is said that a Banach space satisfies the property *Banach-Saks type p* if every weakly null sequence (u_k) has a subsequence (u_{k_j}) such that for some $C > 0$,

$$
\left\| \sum_{j=1}^{n} u_{k_j} \right\| < Cn^{1/p}
$$

for all $n \in \mathbb{N}$. Note that $n^{1/\infty} = 1$ for all $n \in \mathbb{N}$ ([18]).

Theorem 4.1. *The space $\ell_p(\Phi)$ is of type p Banach-Saks for $1 < p < \infty$.*

Proof. Let (δ_n) be a sequence such that $\delta_n > 0$ for all $n \in \mathbb{N}$ and $\sum_n \delta_n \leq 1/2$. Choose a weakly null sequence (u_n) in $\mathscr{B}_{\ell_p(\Phi)}$. Put $t_1 = u_{n_1} = u_1$. There exists $m_1 \in \mathbb{N}$ such that

$$
\left\| \sum_{i=m_1+1}^{\infty} t_1^i \varepsilon^i \right\|_{\ell_p(\Phi)} < \delta_1.
$$

Since (u_n) is weakly null sequence implies $u_n \to 0$ coordinatewise, there is an $n_2 \in \mathbb{N}$ such that

$$
\left\| \sum_{i=1}^{m_1} u_n^i \varepsilon^i \right\|_{\ell_p(\Phi)} < \delta_1,
$$

for all $n \geq n_2$. Put $t_2 = u_{n_2}$. Then, there exists an $m_2 > m_1$ such that

$$
\left\| \sum_{i=m_2+1}^{\infty} t_2^i \varepsilon^i \right\|_{\ell_p(\Phi)} < \delta_2.
$$

Again using the fact that $u_n \to 0$ coordinatewise, there exists an $n_3 > n_2$ such that

$$
\left\| \sum_{i=1}^{m_2} u_n^i \varepsilon^i \right\|_{\ell_p(\Phi)} < \delta_2,
$$

for all $n \geq n_3$.

By continuing this process, we obtain two sequences (m_i) with $m_1 < m_2 < ... < m_i < ...$ and (n_i) with $n_1 < n_2 < ... < n_i < ...$ such that

$$
\left\| \sum_{i=1}^{m_j} u_n^i \varepsilon^i \right\|_{\ell_p(\Phi)} < \delta_j,
$$

for all $n \geq n_{j+1}$ and

$$\left\| \sum_{i=m_j+1}^{\infty} t_j^i \varepsilon^i \right\|_{\ell_p(\Phi)} < \delta_j,$$

where $t_j = u_{n_j}$. It follows that

$$\left\| \sum_{j=1}^{n} t_j \right\|_{\ell_p(\Phi)} = \left\| \sum_{j=1}^{n} \left(\sum_{i=1}^{m_{j-1}} t_j^i \varepsilon^i + \sum_{i=m_{j-1}+1}^{m_j} t_j^i \varepsilon^i + \sum_{i=m_j+1}^{\infty} t_j^i \varepsilon^i \right) \right\|_{\ell_p(\Phi)}$$

$$\leq \left\| \sum_{j=1}^{n} \left(\sum_{i=m_{j-1}+1}^{m_j} t_j^i \varepsilon^i \right) \right\|_{\ell_p(\Phi)} + 2 \sum_{j=0}^{n} \delta_j.$$

Also, given any $u \in \mathscr{B}_{\ell_p(\Phi)}$, we have $\|u\|_{\ell_p(\Phi)}^p = \sum_{i=1}^{\infty} \left| \frac{1}{i} \sum_{k|i} \varphi(k) u_k \right|^p < 1$. Therefore, we have that

$$\left\| \sum_{j=1}^{n} \left(\sum_{i=m_{j-1}+1}^{m_j} t_j^i \varepsilon^i \right) \right\|_{\ell_p(\Phi)}^p = \sum_{j=1}^{n} \sum_{i=m_{j-1}+1}^{m_j} \left| \frac{1}{i} \sum_{k|i} \varphi(k) t_j^k \right|^p$$

$$\leq \sum_{j=1}^{n} \sum_{i=1}^{\infty} \left| \frac{1}{i} \sum_{k|i} \varphi(k) t_j^k \right|^p \leq n.$$

Hence, we obtain

$$\left\| \sum_{j=1}^{n} \left(\sum_{i=m_{j-1}+1}^{m_j} t_j^i \varepsilon^i \right) \right\|_{\ell_p(\Phi)} \leq n^{1/p}.$$

Since $n^{1/p} \geq 1$ holds for all $n \in \mathbb{N}$ and $1 < p < \infty$, we have

$$\left\| \sum_{j=1}^{n} t_j \right\|_{\ell_p(\Phi)} \leq n^{1/p} + 1 \leq 2n^{1/p}.$$

Hence, we conclude that $\ell_p(\Phi)$ is of type p Banach-Saks for $1 < p < \infty$. $\qquad \square$

García-Falset [19] introduce the following coefficient:

$$R(X) = \sup \left\{ \liminf_{n \to \infty} \|u_n - L\| : (u_n) \text{ is a sequence in } \mathscr{B}_X, \ u_n \xrightarrow{w} 0, L \in \mathscr{B}_X \right\}.$$

Here $u_n \xrightarrow{w} 0$ means that (u_n) is weakly convergent to zero. A Banach space X with $R(X) < 2$ has the *weak fixed point property* ([20]).

Remark 4.2. $R(\ell_p(\Phi)) = R(\ell_p) = 2^{1/p}$ since $\ell_p(\Phi)$ is linearly isomorphic to ℓ_p.

Hence, we have the following result.

Theorem 4.3. *The space $\ell_p(\Phi)$ has the weak fixed point property for $1 < p < \infty$.*

Let $\mathscr{S}_X = \{u \in X : \|u\| = 1\}$. The *Gurarii's modulus of convexity* is

$$\beta_X(\delta) = \inf \left\{ 1 - \inf_{0 \leq \lambda \leq 1} \|\lambda u + (1-\lambda) v\| : u, v \in \mathscr{S}_X, \|u - v\| = \delta \right\},$$

where $0 \leq \delta \leq 2$ ([21]).

Theorem 4.4. *The inequality $\beta_{\ell_p(\Phi)}(\delta) \leq 1 - [1 - (\frac{\delta}{2})^p]^{1/p}$ holds, where $0 \leq \delta \leq 2$.*

Proof. Let $0 \leq \delta \leq 2$. Consider the sequences

$$\tilde{u} = \left(\left(1 - \left(\frac{\delta}{2} \right)^p \right)^{1/p}, \frac{\delta}{2}, 0, 0, 0, \ldots \right)$$

and

$$\tilde{v} = \left(\left(1 - \left(\frac{\delta}{2} \right)^p \right)^{1/p}, -\frac{\delta}{2}, 0, 0, 0, \dots \right).$$

Set $u = \Phi^{-1}\tilde{u}$ and $v = \Phi^{-1}\tilde{v}$. By using the relation (2.1), we obtain that

$$\|u\|_{\ell_p(\Phi)}^p = \|\Phi u\|_{\ell_p}^p = \|\tilde{u}\|_{\ell_p}^p = \left| \left(1 - \left(\frac{\delta}{2} \right)^p \right)^{1/p} \right|^p + \left| \frac{\delta}{2} \right|^p = 1$$

and

$$\|v\|_{\ell_p(\Phi)}^p = \|\Phi v\|_{\ell_p}^p = \|\tilde{v}\|_{\ell_p}^p = \left| \left(1 - \left(\frac{\delta}{2} \right)^p \right)^{1/p} \right|^p + \left| -\frac{\delta}{2} \right|^p = 1.$$

Also, we have

$$\|u - v\|_{\ell_p(\Phi)}^p = \|\tilde{u} - \tilde{v}\|_{\ell_p}^p = (|\delta|^p)^{1/p} = \delta.$$

Hence, we conclude that

$$
\begin{aligned}
\beta_{\ell_p(\Phi)}(\delta) \quad &\leq \quad 1 - \inf_{0 \leq \lambda \leq 1} \|\lambda u + (1-\lambda)v\|_{\ell_p(\Phi)} \\
&\leq \quad 1 - \inf_{0 \leq \lambda \leq 1} \|\lambda \tilde{u} + (1-\lambda)\tilde{v}\|_{\ell_p} \\
&\leq \quad 1 - \inf_{0 \leq \lambda \leq 1} \left[\left| \lambda \left(1 - \left(\frac{\delta}{2} \right)^p \right)^{1/p} + (1-\lambda) \left(1 - \left(\frac{\delta}{2} \right)^p \right)^{1/p} \right|^p + \left| \lambda \frac{\delta}{2} - (1-\lambda)\frac{\delta}{2} \right|^p \right]^{1/p} \\
&\leq \quad 1 - \inf_{0 \leq \lambda \leq 1} \left[1 - \left(\frac{\delta}{2} \right)^p + |2\lambda - 1|^p \frac{\delta^p}{2} \right]^{1/p} \\
&\leq \quad 1 - \left[1 - \left(\frac{\delta}{2} \right)^p \right]^{1/p}.
\end{aligned}
$$

$\square$

Corollary 4.5. *If $\beta_{\ell_p(\Phi)}(\delta) = 1$, then $\ell_p(\Phi)$ is strictly convex.*

Corollary 4.6. *If $0 < \beta_{\ell_p(\Phi)}(\delta) \leq 1$, then $\ell_p(\Phi)$ is uniformly convex.*

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