Ordu Üniv. Bil. Tek. Derg., Cilt:1, Sayı:1,2011,1-14 Ordu Univ. J. Sci. Tech., Vol:1, No:1, 2011,1-14

ON DUAL SPACELIKE MANNHEIM PARTNER CURVES IN ID_1^3

Özcan BEKTAŞ, Süleyman ŞENYURT

Department of Mathematics, Arts and Science Faculty, Ordu University, Ordu-Turkey

Abstract
The first aim of this paper is to define the dual spacelike Mannheim partner curves in Dual Lorentzian Space ID_1^3, the second aim of this paper is to obtain the relationships between the curvatures and the torsions of the dual spacelike Mannheim partner curves with respect to each other and the final aim of this paper is to get the necessary and sufficient conditions for the dual spacelike Mannheim partner curves in ID_1^3.

Key words: Mannheim curves, dual Lorentzian Space, curvature, torsion.

AMS Mathematics Subject Classification(2000): 53B30,51M30,53A35,53A04

ID_1^3 ' DE DUAL SPACELİKE MANNHEİM EĞRİ ÇİFTLERİ ÜZERİNE

Özet
Bu çalışmanın amacı: ilk olarak dual Lorentz uzayında dual spacelike Mannheim eğri çiftini tanımlamak, ikinci olarak dual spacelike Mannheim eğri çiftinin birbirlerine göre eğrilik ve burulmaları arasındaki bağıntıları vermek ve son olarak da ID_1^3 dual Lorentz uzayında verilen bir eğri çiftinin dual spacelike eğri olması için gerek ve yeter şartları elde etmektir.

Anahtar kelimeler: Manheim eğri, Dual Lorentzian uzay, eğrilme, burulma

1.INTRODUCTION

As is well-known, a surface is said to be "ruled" if it is generated by moving a straight line continuously in Euclidean space (O'Neill, 1997). Ruled surfaces are one of the simplest objects in geometric modeling. One important fact about ruled surfaces is that they can be generated by straight lines. A practical application of this type surfaces is that they are used in civil engineering and physics (Guan et al., 1997).

Since building materials such as wood are straight, they can be considered as straight lines. The results is that if engineers are planning to construct something with curvature, they can use a ruled surface since all the lines are straight (Orbay et al., 2009).

In the differential geometry of a regular curve in the Euclidean 3 - space IE^3, it is well-known that one of the important problem is the characterization of a regular curve. The curvature functions k_1 and k_2 of a reguler curve play an important role to determine the shape and size of the curve (Kuhnel, 1999; Do Carmo, 1976). For example, If $k_1 = k_2 = 0$, the curve is geodesic. If $k_1 \neq 0(constant)$ and $k_2 = 0$, then the curve is a circle with radius $1/k_1$. If $k_1 \neq 0(constant)$ and $k_2 \neq 0$ $(constant)$, then the curve is a helix in the space.

Another way to classification and characterization of curves is the relationship between the Frenet vectors of the curves. For example Saint Venant proposed the question whether upon the surfaces generated by the principal normal of a curve, a second curve can exist which has for its principal normal the principal normal of the given curve. This question was answered by Bertrand in 1850; he showed that a necessary and sufficient condition for the existence of such a second curve is that a linear relationship with constant coefficients exists between the first and second curvatures of the given original curve. The pairs of curves of this kind have been called Conjugate Bertrand curves, or more commonly Bertrand Curves. There are many works related with Bertrand curves in the Euclidean space and Minkowski space. Another kind of associated curves are called Mannheim curve and Mannheim partner curve. If there exists a corresponding relationship between the space curves α and β such that, at the corresponding points of the curves, principal normal lines of α coincides with the binormal lines of β, then α is called a Mannheim curve, and β Mannheim partner curve of α.

In recent studies, Liu and Wang (2007, 2008) are curious about the Mannheim curves in both Euclidean and Minkowski 3- space and they obtained the necessary and sufficient conditions between the curvature and the torsion for a curve to be the Mannheim partner curves. Meanwhile, the detailed discussion concerned with the Mannheim curves can be found in literature (Wang and Liu, 2007; Liu and Wang, 2008; Orbay et al., 2009; Özkaldı et al., 2009; Azak, 2009) and references therein.

Dual numbers had been introduced by W.K. Clifford (1849 - 1879) as a tool for his geometrical investigations. After him E. Study used dual numbers and dual vectors in his research on line geometry and kinematics. He devoted special attention to the representation of oriented lines by dual unit vectors and defined the famous mapping: The set of oriented lines in an Euclidean three– dimension space IE^3 is one to one correspondence with the points of a dual space ID^3 of triples of dual numbers.

In this paper, we study the dual spacelike Mannheim partner curves in dual Lorentzian space ID_1^3.

2. PRELIMINARY

By a dual number A, we mean an ordered pair of the form (a, a^*) for all $a, a^* \in IR$. Let the set $IR \times IR$ be denoted as ID. Two inner operations and an equality on $ID = \{(a, a^*) | a, a^* \in IR\}$ are defined as follows:

$(i) \oplus : ID \times ID \to ID$ for $A = (a, a^*)$, $B = (b, b^*)$ defined as

$$A \oplus B = (a, a^*) \oplus (b, b^*) = (a + b, a^* + b^*)$$

is called the addition in ID.

$(ii) \odot : ID \times ID \to ID$ for $A = (a, a^*)$, $B = (b, b^*)$ defined as

$$A \odot B = (a, a^*) \odot (b, b^*) = (ab, ab^* + a^*b)$$

is called the multiplication in ID.

(iii) If $a=b$, $a^*=b^*$ $A=(a,a^*)$, $B=(b,b^*)$, A and B are equal, and it is indicated as $A=B$.

If the operations of addition, multiplication and equality on $ID=IR\times IR$ with set of real numbers IR are defined as above, the set ID is called the dual numbers system and the element (a,a^*) of ID is called a dual number. In a dual number $A=(a,a^*)\in ID$, the real number a is called the real part of A and the real number a^* is called the dual part of A. The dual number 1=(1,0) is called the unit element of multiplication operation ID with respect to multiplication and denoted by ε. In accordance with the definition of the operation of multiplication, it can be easily seen that $\varepsilon^2=0$. Also, the dual number $A=(a,a^*)\in ID$ can be written as $A=a+\varepsilon a^*$.

The set $ID=\left\{A=a+\varepsilon a^* \,\middle|\, a,a^*\in IR\right\}$ of dual numbers is a commutative ring according to the operations,

$(i)\left(a+\varepsilon a^*\right)+\left(b+\varepsilon b^*\right)=(a+b)+\varepsilon\left(a^*+b^*\right)$

$(ii)\left(a+\varepsilon a^*\right)\left(b+\varepsilon b^*\right)=ab+\varepsilon\left(ab^*+ba^*\right).$

The dual number $A=a+\varepsilon a^*$ divided by the dual number $B=b+\varepsilon b^*$ provided $b\neq 0$ can be defined as

$$\frac{A}{B}=\frac{a+\varepsilon a^*}{b+\varepsilon b^*}=\frac{a}{b}+\varepsilon\frac{a^*b-ab^*}{b^2}.$$

Now let us consider the differentiable dual function. If the dual function f expansions the Taylor series then we have

$$f\left(a+\varepsilon a^*\right)=f(a)+\varepsilon a^* f'(a)$$

where $f'(a)$ is the derivation of f. Thus we can obtain

$$\sin\left(a+\varepsilon a^*\right)=\sin a+\varepsilon a^*\cos a$$

$$\cos\left(a+\varepsilon a^*\right)=\cos a-\varepsilon a^*\sin a.$$

The set of

$$ID^3=\left\{\vec{A} \;\middle|\; \vec{A}=\vec{a}+\varepsilon\vec{a^*},\ \vec{a},\vec{a^*}\in IR^3\right\}$$

is a module on the ring ID. For any $\vec{A}=\vec{a}+\varepsilon\vec{a^*}$, $\vec{B}=\vec{b}+\varepsilon\vec{b^*}\in ID^3$, the scalar or inner product and the vector product of $\vec{A}$ and $\vec{B}$ are defined by, respectively,

$$\left\langle\vec{A},\vec{B}\right\rangle=\left\langle\vec{a},\vec{b}\right\rangle+\varepsilon\left(\left\langle\vec{a},\vec{b^*}\right\rangle+\left\langle\vec{a^*},\vec{b}\right\rangle\right)$$

$$\vec{A}\wedge\vec{B}=\vec{a}\wedge\vec{b}+\varepsilon\left(\vec{a}\wedge\vec{b}^*+\vec{a}^*\wedge\vec{b}\right).$$

If $\vec{a} \neq 0$, the norm $\|\vec{A}\|$ of $\vec{A} = \vec{a} + \varepsilon \overrightarrow{a^*}$ is defined by

$$\|\vec{A}\| = \sqrt{\langle \vec{A}, \vec{A} \rangle} = \|\vec{a}\| + \varepsilon \frac{\langle \vec{a}, \overrightarrow{a^*} \rangle}{\|\vec{a}\|} \ , \ \|\vec{a}\| \neq 0 .$$

A dual vector $\vec{A}$ with norm 1 is called a dual unit vector. The set

$$S^2 = \left\{ \vec{A} = \vec{a} + \varepsilon \overrightarrow{a^*} \in ID^3 \middle| \ \|\vec{A}\| = (1,0); \vec{a}, \overrightarrow{a^*} \in IR^3 \right\}$$

is called the dual unit sphere with the center $\vec{O}$ in ID^3.

Let $\alpha(t) = (\alpha_1(t), \alpha_2(t), \alpha_3(t))$ and $\beta(t) = (\beta_1(t), \beta_2(t), \beta_3(t))$ be real valued curves in IE^3. Then $\tilde{\alpha}(t) = \alpha(t) + \varepsilon \alpha^*(t)$ is a curve in ID^3 and it is called dual space curve. If the real valued functions $\alpha_i(t)$ and $\alpha_i^*(t)$ are differentiable then the dual space curve $\tilde{\alpha}(t)$ is differentiable in ID^3. The real part $\alpha(t)$ of the dual space curve $\tilde{\alpha} = \tilde{\alpha}(t)$ is called indicatrix. The dual arc-length of the dual space curve $\tilde{\alpha}(t)$ from t_1 to t is defined by

$$\tilde{s} = \int_{t_1}^{t} \|\overrightarrow{\tilde{\alpha}'}(t)\| dt = \int_{t_1}^{t} \|\overrightarrow{\alpha'}(t)\| dt + \varepsilon = \int_{t_1}^{t} \left\langle \vec{t}, \left(\overrightarrow{\alpha^*}(t)\right)' \right\rangle dt = s + \varepsilon s^*$$

where $\vec{t}$ is unit tangent vector of the indicatrix $\alpha(t)$ which is a real space curve in IE^3. From now on we will take the arc length s of $\vec{\alpha}(t)$ as the parameter instead of t.

The Lorentzian inner product of dual vectors $\vec{A}, \vec{B} \in ID^3$ is defined by

$$\langle \vec{A}, \vec{B} \rangle = \langle \vec{a}, \vec{b} \rangle + \varepsilon \left(\langle \vec{a}, \vec{b}^* \rangle + \langle \vec{a}^*, \vec{b} \rangle \right)$$

with the Lorentzian inner product $\vec{a} = (a_1, a_2, a_3)$ and $\vec{b} = (b_1, b_2, b_3) \in IR^3$

$$\langle \vec{a}, \vec{b} \rangle = -a_1 b_1 + a_2 b_2 + a_3 b_3 .$$

Thus, $(ID^3, \langle , \rangle)$ is called the dual Lorentzian space and denoted by ID_1^3. We call the elements of ID_1^3 as the dual vectors. For $\vec{A} \neq \vec{0}$, the norm $\|\vec{A}\|$ of $\vec{A}$ is defined by $\|\vec{A}\| = \sqrt{|\langle \vec{A}, \vec{A} \rangle|}$. The dual vector $\vec{A} = \vec{a} + \varepsilon \overrightarrow{a^*}$ is called dual spacelike vector if $\langle \vec{A}, \vec{A} \rangle > 0$ or $\vec{A} = 0$, dual timelike vector if $\langle \vec{A}, \vec{A} \rangle < 0$, dual lightlike vector if $\langle \vec{A}, \vec{A} \rangle = 0$ for $\vec{A} \neq 0$. The dual Lorentzian cross-product of $\vec{A}, \vec{B} \in ID^3$ is defined by

$$\vec{A} \wedge \vec{B} = \vec{a} \wedge \vec{b} + \varepsilon\left(\vec{a} \wedge \vec{b}^* + \vec{a}^* \wedge \vec{b}\right)$$

where $\vec{a} \wedge \vec{b} = (a_3b_2 - a_2b_3, a_1b_3 - a_3b_1, a_1b_2 - a_2b_1)$ $\vec{a}, \vec{b} \in IR^3$ i s the Lorentzian cross product.

Dual number $\Phi = \varphi + \varepsilon\varphi^*$ is called dual angle between $\vec{A}$ ve $\vec{B}$ unit dual vectors. Then we was

$$\sinh(\varphi + \varepsilon\varphi^*) = \sinh\varphi + \varepsilon\varphi^* \cosh\varphi \quad \text{and}$$

$$\cosh(\varphi + \varepsilon\varphi^*) = \cosh\varphi + \varepsilon\varphi^* \sinh\varphi \ .$$

Let $\{T(s), N(s), B(s)\}$ be the moving Frenet frame along the curve $\widetilde{\alpha}(s)$. Then $T(s), N(s)$ and $B(s)$ are dual tangent, the dual principal normal and the dual binormal vector of the curve $\widetilde{\alpha}(s)$, respectively. Depending on the casual character of the curve $\widetilde{\alpha}$, we have the following dual Frenet –Serret formulas:

If $\widetilde{\alpha}$ is a dual spacelike curve with a dual timelike binormal B;

$$\begin{pmatrix} T' \\ N' \\ B' \end{pmatrix} = \begin{pmatrix} 0 & \kappa & 0 \\ -\kappa & 0 & \tau \\ 0 & \tau & 0 \end{pmatrix} \begin{pmatrix} T \\ N \\ B \end{pmatrix} \tag{2.1}$$

where $\langle T,T\rangle = \langle N,N\rangle = 1, \langle B,B\rangle = -1, \langle T,N\rangle = \langle N,B\rangle = \langle T,B\rangle = 0.$

We denote by $\{V_1(s), V_2(s), V_3(s)\}$ the moving Frenet frame along the curve $\widetilde{\beta}(s)$. Then $V_1(s), V_2(s)$ and $V_3(s)$ are dual tangent, the dual principal normal and the dual binormal vector of the curve $\widetilde{\beta}(s)$, respectively. Depending on the casual character of the curve $\widetilde{\beta}$, we have the following dual Frenet – Serret formulas:

If $\widetilde{\beta}$ is a dual spacelike curve with a dual spacelike binormal V_3;

$$\begin{pmatrix} V_1' \\ V_2' \\ V_3' \end{pmatrix} = \begin{pmatrix} 0 & P & 0 \\ P & 0 & Q \\ 0 & Q & 0 \end{pmatrix} \begin{pmatrix} V_1 \\ V_2 \\ V_2 \end{pmatrix} \tag{2.2}$$

where $\langle T,T\rangle = \langle B,B\rangle = 1, \langle N,N\rangle = -1, \langle T,N\rangle = \langle N,B\rangle = \langle T,B\rangle = 0.$

If the curves are unit speed curve, then curvature and torsion calculated by,

$$\kappa = \sqrt{\langle T',T'\rangle} \ , \tau = \frac{\det(T,T',T'')}{\langle T',T'\rangle}$$

or

$$P=\sqrt{\left\langle V_1',V_1'\right\rangle}\ , \qquad Q=\frac{\det\left(V_1,V_1',V_1''\right)}{\left\langle V_1',V_1'\right\rangle}.$$

If the curves are not unit speed curve, then curvature and torsion calculated by,

$$\kappa=\frac{\left\|\tilde{\alpha}'\wedge\tilde{\alpha}''\right\|}{\left\|\tilde{\alpha}'\right\|^3},\ \tau=\frac{\det\left(\alpha,\tilde{\alpha}',\tilde{\alpha}''\right)}{\left\|\tilde{\alpha}'\wedge\tilde{\alpha}''\right\|^2}$$

or

$$P=\frac{\left\|\tilde{\beta}'\wedge\tilde{\beta}''\right\|}{\left\|\tilde{\beta}'\right\|^3},\quad Q=\frac{\det\left(\tilde{\beta},\tilde{\beta}',\tilde{\beta}''\right)}{\left\|\tilde{\beta}'\wedge\tilde{\beta}''\right\|^2}.$$

Definition 2.1. a) Dual Hyperbolic angle: Let $\vec{A}$ and $\vec{B}$ be dual timelike vectors in ID_1^3. Then the dual angle between $\vec{A}$ and $\vec{B}$ is defined by $\left\langle\vec{A},\vec{B}\right\rangle=-\left\|\vec{A}\right\|\left\|\vec{B}\right\|\cosh\Phi$. The dual number $\Phi=\theta+\varepsilon\theta^*$ is called the dual hyberbolic angle.

b) Dual Central angle: Let $\vec{A}$ and $\vec{B}$ be spacelike vectors in ID_1^3 that span a dual timelike vector subspace. Then the dual angle between $\vec{A}$ and $\vec{B}$ is defined by $\left\langle\vec{A},\vec{B}\right\rangle=\left\|\vec{A}\right\|\left\|\vec{B}\right\|\cosh\Phi$. The dual number $\Phi=\theta+\varepsilon\theta^*$ is called the dual central angle.

c) Dual Spacelike angle: Let $\vec{A}$ and $\vec{B}$ be dual spacelike vectors in ID_1^3 that span a dual spacelike vector subspace. Then the dual angle between $\vec{A}$ and $\vec{B}$ is defined by $\left\langle\vec{A},\vec{B}\right\rangle=\left\|\vec{A}\right\|\left\|\vec{B}\right\|\cos\Phi$. The dual number $\Phi=\theta+\varepsilon\theta^*$ is called the dual spacelike angle.

a) Dual Lorentzian timelike angle: Let $\vec{A}$ be a dual spacelike vector and $\vec{B}$ be a dual timelike vector in ID_1^3. Then the dual angle between $\vec{A}$ and $\vec{B}$ is defined by $\left\langle\vec{A},\vec{B}\right\rangle=\left\|\vec{A}\right\|\left\|\vec{B}\right\|\sinh\Phi$. The dual number $\Phi=\theta+\varepsilon\theta^*$ is called the dual Lorentzian timelike angle.

3. DUAL SPACELIKE MANNHEIM PARTNER CURVE IN ID_1^3

In this section, we define dual spacelike Mannheim partner curves in ID_1^3 and we give some characterization for dual spacelike Mannheim partner curves in the same space. Using these relationships, we will comment again Shell's and Mannheim's theorems.

Definition 3.1. Let $\tilde{\alpha}: I \to ID_1^3$, $\tilde{\alpha}(s)=\alpha(s)+\varepsilon\alpha^*(s)$ and $\tilde{\beta}: I \to ID_1^3, \tilde{\beta}(s)=\beta(s)+\varepsilon\beta^*(s)$ be dual spacelike curves. If there exists a corresponding relationship between the dual spacelike curves with dual timelike binormal $\tilde{\alpha}$ and the dual spacelike curves with dual spacelike binormal $\tilde{\beta}$ such that, at the corresponding points of the dual spacelike curves, the dual binormal lines of $\tilde{\alpha}$ coincides with the dual principal normal lines of $\tilde{\beta}$, then $\tilde{\alpha}$ is called a dual spcelike Mannheim curve, and $\tilde{\beta}$ is called a dual Mannheim partner curve of $\tilde{\alpha}$. The pair $\{\tilde{\alpha},\tilde{\beta}\}$ is said to be dual spacelike Mannheim pair.

Let $\{T,N,B\}$ be the dual Frenet frame field along $\tilde{\alpha}=\tilde{\alpha}(s)$ and let $\{V_1,V_2,V_3\}$ be the Frenet frame field along $\tilde{\beta}=\tilde{\beta}(s)$. On the way $\Phi=\theta+\varepsilon\theta^*$ is dual angle between T and V_1, there is an following equations between the Frenet vectors;

$$\begin{pmatrix} V_1 \\ V_2 \\ V_3 \end{pmatrix}=\begin{pmatrix} \cos\Phi & \sin\Phi & 0 \\ 0 & 0 & 1 \\ \sin\Phi & -\cos\Phi & 0 \end{pmatrix}\begin{pmatrix} T \\ N \\ B \end{pmatrix} \tag{3.1}$$

Theorem 2.1. The distance between corresponding dual points of the dual spacelike Mannheim partner curves in ID_1^3 is constant.

Proof: From the definition of dual spacelike Mannheim curve, we can write

$$\tilde{\beta}(s)=\tilde{\alpha}(s)+\lambda(s)B(s) \tag{3.2}$$

By taking the derivate of this equation with respect to s and applying the Frenet formulas, we get

$$V_1\frac{ds^*}{ds}=T+\lambda\tau N+\lambda' B \tag{3.3}$$

where the superscript $(')$ denotes the derivate with respect to the arc length parameters of the dual curve $\tilde{\alpha}(s)$. Since the dual vectors B and V_2 are linearly, we get

$$\left\langle V_1\frac{ds^*}{ds},B\right\rangle=0,\ \ \langle T+\lambda\tau N+\lambda' B,B\rangle=0,\ \lambda'=0.$$

If we take $\lambda=\lambda_1+\varepsilon\lambda_1^*$, we get $\lambda_1'=0$ ve $\lambda_1^{*\prime}=0$. From here, we can write

$$\lambda_1=c_1 \text{ and } \lambda_1^*=c_2\ ,\ c_1,c_2=\text{constant}.$$

Then we get $\lambda=c_1+\varepsilon c_2$. On the other hand, from the definition of distance function between $\tilde{\alpha}(s)$ and $\tilde{\beta}(s)$ we can write

$$d\left(\tilde{\alpha}(s),\tilde{\beta}(s)\right)=\left\|\tilde{\beta}(s)-\tilde{\alpha}(s)\right\|=\left\|\lambda(s)B(s)\right\|=\left\|\lambda_1 b+\varepsilon\left(\lambda_1^* b+\lambda_1 b^*\right)\right\|$$
$$=\left\|\lambda_1 b\right\|+\varepsilon\frac{\left\langle\lambda_1 b,\lambda_1^* b+\lambda_1 b^*\right\rangle}{\left\|\lambda_1 b\right\|}=\left|\lambda_1\right|\mp\varepsilon\lambda_1^*=\left|c_1\right|\mp\varepsilon c_2.$$

This is completed the proof.

Theorem 2.2. For a dual spacelike curve $\tilde{\alpha}$ in ID_1^3, there is a dual spacelike curve $\tilde{\beta}$ so that $\{\tilde{\alpha},\tilde{\beta}\}$ is a dual spacelike Mannheim pair.

Proof: Since the dual vectors V_2 and B are linearly dependent, the equation (3.2) can be written as

$$\tilde{\alpha}=\tilde{\beta}-\lambda V_2 \tag{3.4}$$

Since λ is a nonzero constant, there is a dual spacelike curve $\tilde{\beta}$ for all values of λ.

Now, we can give the following theorem related to curvature and torsion of the dual spacelike Mannheim partner curves.

Theorem 2.3. Let $\{\tilde{\alpha},\tilde{\beta}\}$ be a dual spacelike Mannheim pair in ID_1^3**.** If τ is dual torsion of $\tilde{\alpha}$ and P is dual curvature and Q is dual torsion of $\tilde{\beta}$, then

$$\tau=-\frac{P}{\lambda Q} \tag{3.5}$$

Proof: By taking the derivate of equation (3.3) with respect to s and applying the Frenet formulas, we obtain

$$V_1\frac{ds^*}{ds}=T+\lambda\tau N \tag{3.6}$$

Let $\Phi=\varphi+\varepsilon\varphi^*$ be dual angle between the dual tangent vectors T and V_1, we can write

$$\begin{cases}V_1=\cos\Phi\,T+\sin\Phi\,N,\\ V_3=\sin\Phi\,T-\cos\Phi\,N.\end{cases} \tag{3.7}$$

From (3.6) and (3.7) , we get

$$\frac{ds^*}{ds}=\frac{1}{\cos\Phi},\quad \lambda\tau=\sin\Phi\frac{ds^*}{ds} \tag{3.8}$$

By taking the derivate of equation (3.4) with respect to s and applying the Frenet formulas, we obtain

$$T=\left(1-\lambda P\right)V_1\frac{ds^*}{ds}-\lambda Q V_3\frac{ds^*}{ds} \tag{3.9}$$

From equation (3.7) we can write

$$\begin{cases} T = \cos\Phi\, V_1 + \sin\Phi\, V_3, \\ N = \sin\Phi\, V_1 - \cos\Phi\, V_3. \end{cases} \quad (3.10)$$

where Φ is the dual angle between T and V_1 at the corresponding points of the dual curves of $\tilde{\alpha}$ and $\tilde{\beta}$. By taking into consideration equations (3.9) and (3.10), we get

$$\begin{cases} \cos\Phi = \left(1-\lambda P\right)\dfrac{ds^*}{ds}, \\ \sin\Phi = -\lambda Q\dfrac{ds^*}{ds}. \end{cases} \quad (3.11)$$

Substituting $\dfrac{ds^*}{ds}$ into (3.11) , we get

$$\begin{cases} \cos^2\Phi = \left(1-\lambda P\right), \\ \sin^2\Phi = -\lambda^2 \tau Q. \end{cases} \quad (3.12)$$

From the last equation, we can write

$$\tau = -\frac{P}{\lambda Q}.$$

If the last equation is seperated into the dual and real parts, we can obtain

$$\begin{cases} k_2 = -\dfrac{p}{c_1 q}, \\ k_2^* = \dfrac{c_1\left(pq^* - p^* q\right) + c_2 qp}{\left(c_1 q\right)^2}. \end{cases} \quad (3.13)$$

Corollary 3.1. Let $\{\tilde{\alpha}, \tilde{\beta}\}$ be a dual spacelike Mannheim pair in ID_1^3**.** Then, the dual product of torsions τ and Q at the corresponding points of the dual spacelike Mannheim partner curves is not constant.

Namely, Schell's theorem is invalid for the dual spacelike Mannheim curves. By considering Theorem 2.3 we can give the following results.

Corollary 3.2. Let $\{\tilde{\alpha}, \tilde{\beta}\}$ be a dual spacelike Mannheim pair in ID_1^3**.** Then, torsions τ and Q has a negative sign.

Theorem 3.4. Let $\{\tilde{\alpha}, \tilde{\beta}\}$ be a dual spacelike Mannheim pair in ID_1^3**.** Between the curvature and the torsion of the dual spacelike curve $\widetilde{\beta}$, there is the relationship

$$\mu Q + \lambda P = 1 \quad (3.14)$$

where μ and λ are nonzero dual numbers.

Proof: From equation (3.11) , we obtain

$$\frac{\cos\Phi}{1-\lambda P}=\frac{\sin\Phi}{-\lambda Q},$$

arranging this equation, we get

$$\cot\Phi=\frac{1-\lambda P}{-\lambda Q},$$

and if we choose $\mu=-\lambda\cot\Phi$ for brevity, we see that

$$\mu Q+\lambda P=1.$$

Theorem 3.5. Let $\{\tilde{\alpha},\tilde{\beta}\}$ be a dual spacelike Mannheim pair in ID_1^3. There are the following equations fort he curvatures and the torsions of the curves $\tilde{\alpha}$ ve $\tilde{\beta}$

$$i)\kappa=-\frac{d\Phi}{ds}, \qquad ii)\ \tau=P\sin\Phi\frac{ds^*}{ds}-Q\cos\Phi\frac{ds^*}{ds},,$$

$$iii)\ P=\tau\sin\Phi\frac{ds}{ds^*}, \qquad iv)\ Q=-\tau\cos\Phi\frac{ds}{ds^*}.$$

Proof: ***i)*** By considering equation (3.7), we can easily that $\langle T,V_1\rangle=\cos\Phi$. Differentiating of this equality with respect to s by considering equation (3.1) , we have

$$\langle T',V_1\rangle+\left\langle T,V_1'\frac{ds^*}{ds}\right\rangle=-\sin\Phi\frac{d\Phi}{ds},$$

from equations (3.1) and (3.2), we can write

$$\langle \kappa N,V_1\rangle+\left\langle T,PV_2\frac{ds^*}{ds}\right\rangle=-\sin\Phi\frac{d\Phi}{ds},$$

from equations (3.10), we get

$$\kappa=-\frac{d\Phi}{ds}.$$

If the last equation is seperated into the dual and real part, we can obtain

$$\begin{cases} k_1=-\dfrac{d\varphi}{ds}, \\ k_1^*=-\dfrac{d\varphi^*}{ds}. \end{cases}$$

ii) By considering equation (3.7), we can easily that $\langle N,V_2\rangle=0$. Differentiating of this equality with respect to *s and* by considering equation (3.1) , we have

$$\langle N',V_2\rangle+\left\langle N,V_2'\frac{ds^*}{ds}\right\rangle=0,$$

From equations (3.1) and (3.2), we can write

$$\left\langle -\kappa T+\tau B,V_2\right\rangle+\left\langle \sin\Phi V_1-\cos\Phi V_3,\left(PV_1+QV_3\right)\frac{ds^*}{ds}\right\rangle=0,$$

From equations (3.10), we get

$$\tau=P\sin\Phi\frac{ds^*}{ds}-Q\cos\Phi\frac{ds^*}{ds}.$$

iii) By considering equation (3.7), we can easily that $\left\langle B,V_1\right\rangle=0$. Differentiating of this equality with respect to *s and* by considering equation (3.1) , we have

$$\left\langle B',V_1\right\rangle+\left\langle B,V_1'\frac{ds^*}{ds}\right\rangle=0,$$

From equations (3.1), (3.2) and (3.10) we can write

$$\left\langle \tau\left(\sin\Phi V_1-\cos\Phi V_3\right),V_1\right\rangle+\left\langle B,PV_2\frac{ds^*}{ds}\right\rangle=0,\qquad P=\tau\sin\Phi\frac{ds}{ds^*}.$$

iv) By considering equation (3.7), we can easily that $\left\langle B,V_3\right\rangle=0$. Differentiating of this equality with respect to *s* by considering equation (3.1) , we have

$$\left\langle B',V_3\right\rangle+\left\langle B,V_3'\frac{ds^*}{ds}\right\rangle=0,$$

From equations (3.1), (3.2) and (3.10) we can write

$$\left\langle \tau\left(\sin\Phi V_1-\cos\Phi V_3\right),V_3\right\rangle+\left\langle B,QV_2\frac{ds^*}{ds}\right\rangle=0,\quad Q=-\tau\cos\Phi\frac{ds}{ds^*}.$$

Corollary 3.3. Let $\{\tilde{\alpha},\tilde{\beta}\}$ be a dual spacelike Mannheim pair in ID_1^3. If the statements of Theorem 3.5 is seperated into the dual and real part, we can obtain

$$i)\begin{cases} k_2=p\sin\theta\dfrac{ds^*}{ds}-q\cos\theta\dfrac{ds^*}{ds},\\ k_2^*=\left(p^*\sin\theta+p\theta^*\cos\theta\right)\dfrac{ds^*}{ds}-\left(q^*\cos\theta-q\theta^*\sin\theta\right)\dfrac{ds^*}{ds}.\end{cases}$$

$$ii)\begin{cases} p=k_2\sin\theta\dfrac{ds}{ds^*},\\ p^*=\left(k_2^*\sin\theta+k_2\theta^*\cos\theta\right)\dfrac{ds}{ds^*},\end{cases}$$

$$iii)\begin{cases} q = -k_2 \cos\theta \dfrac{ds}{ds^*}, \\ q^* = -\left(k_2^* \cos\theta - k_2\theta^* \sin\theta\right)\dfrac{ds}{ds^*}. \end{cases}$$

By considering the statements iii and iv) of Theorem 3.5 we can give the following results.

Corollary 3.4. Let $\{\tilde{\alpha}, \tilde{\beta}\}$ be a dual spacelike Mannheim pair in ID_1^3**.** Then there exist the following relation between curvature and torsion of $\tilde{\beta}$ and torsion of $\tilde{\alpha}$;

$$Q^2 + P^2 = \tau^2 \left(\frac{ds}{ds^*}\right)^2 \tag{3.15}$$

Theorem 3.6. A dual spacelike space curve in ID_1^3 is a dual spacelike Mannheim curve if and only if its curvature P and torsion Q satisfy the formula

$$-\lambda\left(P^2 + Q^2\right) = P \tag{3.16}$$

where λ is never pure dual constant.

Proof: By taking the derivate of the statement $\tilde{\alpha} = \tilde{\beta} - \lambda V_2$ with respect to s and applying the Frenet formulas we obtain

$$T\frac{ds}{ds^*} = V_1 + \lambda\left(PV_1 + QV_3\right),$$

$$\kappa N\left(\frac{ds}{ds^*}\right)^2 + T\frac{d^2s}{ds^{*2}} = PV_2 + \lambda\left(P'V_1 + Q'V_3 + \left(P^2 + Q^2\right)V_2\right).$$

Taking the inner product the last equation with B , we get

$$-\lambda\left(P^2 + Q^2\right) = P.$$

If the last equation is seperated into the dual and real part, we can obtain

$$\begin{cases} p = -c_1\left(p^2 + q^2\right), \\ p^* = -2c_1\left(pp^* + qq^*\right) - c_2\left(p^2 + q^2\right) \end{cases} \tag{3.17}$$

where $\lambda = c_1 + \varepsilon c_2$.

Theorem 3.7. Let $\{\tilde{\alpha}, \tilde{\beta}\}$ be a dual spacelike Mannheim partner curves in ID_1^3**.** Moreover, the dual points $\tilde{\alpha}(s)$, $\tilde{\beta}(s)$ be two corresponding dual points of $\{\tilde{\alpha}, \tilde{\beta}\}$ and M ve M^* be the curvature centers at these points, respectively. Then, the ratio

$$\frac{\left\|\widetilde{\beta}(s)M\right\|}{\left\|\widetilde{\alpha}(s)M\right\|}:\frac{\left\|\widetilde{\beta}(s)M^*\right\|}{\left\|\widetilde{\alpha}(s)M^*\right\|}=(1+\lambda P)\sqrt{1-\lambda^2\kappa^2}\neq \text{constant.} \tag{3.18}$$

is not constant.

Proof: A circle that lies in the dual osculating plane of the point $\widetilde{\alpha}(s)$ on the dual spacelike curve $\widetilde{\alpha}$ and that has the centre $M=\widetilde{\alpha}(s)+\frac{1}{\kappa}N$ lying on the dual principal normal N of the point $\widetilde{\alpha}(s)$ and the radius $\frac{1}{\kappa}$ far from $\widetilde{\alpha}(s)$, is called dual osculating circle of the dual curve $\widetilde{\alpha}$ in the point $\widetilde{\alpha}(s)$. Similar definition can be given fort he dual curve $\widetilde{\beta}$ too.

Then, we can write

$$\left\|\widetilde{\alpha}(s)M\right\|=\left\|\frac{1}{\kappa}N\right\|=\frac{1}{\kappa},\qquad \left\|\widetilde{\alpha}(s)M^*\right\|=\left\|\lambda B+\frac{1}{P}V_2\right\|=\frac{1}{P}+\lambda,$$

$$\left\|\widetilde{\beta}(s)M^*\right\|=\left\|\frac{1}{P}V_2\right\|=\frac{1}{P},\qquad \left\|\widetilde{\beta}(s)M\right\|=\left\|-\lambda B+\frac{1}{\kappa}N\right\|=\frac{\sqrt{1-\lambda^2\kappa^2}}{\kappa}.$$

Therefore, we obtain

$$\frac{\left\|\widetilde{\beta}(s)M\right\|}{\left\|\widetilde{\alpha}(s)M\right\|}:\frac{\left\|\widetilde{\beta}(s)M^*\right\|}{\left\|\widetilde{\alpha}(s)M^*\right\|}=(1+\lambda P)\sqrt{1-\lambda^2\kappa^2}\neq \text{constant.}$$

Thus, we can give the following

Corollary 3.5. Mannheim's Theorem is invalid fort he dual spacelike Mannheim partner curve $\{\tilde{\alpha},\tilde{\beta}\}$ in ID_1^3.

REFERENCES

Azak, A. Z. 2009, Üç Boyutlu Lorentz Uzayı L^3 de Timelike Mannheim Eğri Çifti Üzerine, Sakarya University Faculty of Arts and Science The Journal of Arts and Science, Vol. 11(2), 35-45.

Blum, R. 1966, A Remarkable Class of Mannheim-Curves, Canad. Math. Bull., Vol. 9(2), 223-228.

Do Carmo, M.P 1976, Differential Geometry of Curves and Surfaces, Pearson Education., New York: Academic Press.

Guan, Z., Ling, J.,Ping X., and Rongxi T. 1997, Study and Application of Physics-Based Deformable Curves and surfaces, Computers and Graphics 21: 305-313.

Gungor, M. A. and Tosun M. 2010, A study on dual Mannheim partner curves." Int. Math. Forum 5, no. 45-48 2319–2330.

Kazaz, M. and Önder, M., Mannheim Offsets of Timelike Ruled Surfaces in Minkowski 3-space IR_1^3 , eprint/arXiv:0906.2077v3. 3 1 R .

Kuhnel, W. 1999, Differential Geometry: Curves-Surfaces-Manifolds, Braunschweig, Wiesbaden.

Liu, H. and Wang F. 2007, Mannheim Partner Curves in 3-space, Procedings of the Eleventh International Workshop on Diff. Geom.11,25-31.

Liu, H. and Wang F. 2008, Mannheim Partner Curves in 3-space, J. Geom., Vol. 88(1-2), 120-126.

O'Neill, B. 1997, Elemantary Differential Geometry, 2nd ed., Academic Press, New York.

Orbay, K., Kasap, E. and Aydemir, İ. 2009, Mannheim Offsets of Ruled Surfaces, Mathematical Problems in Engineering, Article Number:160917.

Orbay, K., Kasap, E. 2009, On Mannheim Partner Curves in E^3 , International Journal of Physical Sciences, Vol.4 (5), 261-264.

Özkaldi, S., İlarslan, K. and Yayli,Y 2009, On Mannheim Partner Curves In Dual Space, Analele Stiintifice ale Universitatii Ovidius Constanta, Seria Matematica, vol XVII, fasc. 2.