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The Electromagnetic Three-Body Problem With Radiation Terms - Derivation of Equations of Motion (I)

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Abstract

The primary purpose of the present paper is to continue the previous investigations of the author and apply the technique from the two-body problem of classical electrodynamics to the three-body problem. We derive equations of motion with radiation terms which are neutral type nonlinear differential equations with state-dependent delays. The derivation approach is analogous to that of the two body-problem, which allows a unified consideration of the problem for any number of bodies. In the next paper, we prove the existence of periodic solution of the three-body problem and in such a way the Bohr-Sommerfeld postulate for stationary states is confirmed.

Keywords: Classical electrodynamics; Three-body problem, Radiation terms.
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1. Introduction

The main purpose of the present paper is to derive equations of motion describing three charged particles in the framework of the classical electrodynamics. Our considerations based on the previous author's results concerning a general formulation of the N-body problem of classical electrodynamics exposed in [1], [2]. The formulation is based on the principle of independent action of the electromagnetic fields generated by moving particles. The core of this principle is that each particle is under the influence of the rest ones, as the interaction between them is disregarded. From mathematical point of view this means: for every fixed particle the Lorentz force (in the right-hand sides of the equations of motion) can be calculated by vector

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summation of the retarded fields produced by all the other N-1 particles. This leads to a nonlinear system of $4N$ equations of motion (cf. [1], [2]):

$$m_k \frac{d\lambda_r^{(k)}}{ds_k} = \frac{e_k}{c^2}\left(F_{rl}^{(k,1)}\lambda_l^{(k)} + F_{rl}^{(k,2)}\lambda_l^{(k)} + \ldots + F_{rl}^{(k,k-1)}\lambda_l^{(k)} + F_{rl}^{(k,k+1)}\lambda_l^{(k)} + \ldots + F_{rl}^{(k,N)}\lambda_l^{(k)}\right) \quad (1.1)$$

$(r = 1,2,3,4; \quad k = 1,2,3,...,N)$, where the Einstein summation convention is valid, that is, in our case the summation in repeating l. The right-hand sides $F_{rl}^{(k,n)} = \frac{\partial\Phi_l^{(n)}}{\partial x_r^{(k)}} - \frac{\partial\Phi_r^{(n)}}{\partial x_l^{(k)}}$ can be calculated by the retarded Lienard-Wiechert potentials $\Phi_r^{(n)} = -\frac{e_n\,\lambda_r^{(n)}}{\left\langle\lambda^{(n)},\xi^{(kn)}\right\rangle_4}$ (cf. [17, 19]). A formulation and investigation of the two-body problem of classical electrodynamics in series of author's papers are given [19, 13, 14, 3, 4, 5, 6]. The equations of motion for the two-body problem are neutral type nonlinear differential equations with retarded arguments depending on the unknown trajectories (cf. [15, 16]). The same type are the equations of motion for the three-body problem. Here we follow the line of investigations from [7, 8, 9], where we have proved the existence of Bohr-Sommerfeld stationary states [20]. It turns out that they are implied by the classical electrodynamics unlike previous claims that they contradict it (cf. for instance [18]).

The paper consists of six sections and references. Section 1 is an Introduction. In Section 2, a derivation of the explicit form of the equations of motion for three-body problem is made. Every vector equation contains in the right-hand side the Lorentz force and a radiation term that shows the self-interaction of every moving particle. The Lorentz force is a sum of two terms each of which shows the influence of the other two particles on the first one. In Section 3, the formalism of the transition to the Euclidean coordinates is described. In Section 4, we reduce the system of equations of motion in a simplified form using suitable denotations. In Section 5, a transformation of the radiation terms under the Dirac assumption is made. Section 6 is a conclusion.

Before we begin our exposition, we note another approach contained in the papers [12, 11].

Here we apply the results from [1] to the particular case of the three-body problem and investigate in the next papers the He-atom. Namely, the equations of motion (1.1) for 3-body problem become

$$\begin{aligned}
m_1 \tfrac{d\lambda_r^{(1)}}{ds_1} &= \tfrac{e_1}{c^2}\left(F_{rl}^{(12)}\lambda_l^{(1)} + F_{rl}^{(13)}\lambda_l^{(1)}\right),\\
m_2 \tfrac{d\lambda_r^{(2)}}{ds_2} &= \tfrac{e_2}{c^2}\left(F_{rl}^{(21)}\lambda_l^{(2)} + F_{rl}^{(23)}\lambda_l^{(2)}\right),\\
m_3 \tfrac{d\lambda_r^{(3)}}{ds_3} &= \tfrac{e_3}{c^2}\left(F_{rl}^{(31)}\lambda_l^{(3)} + F_{rl}^{(32)}\lambda_l^{(3)}\right).
\end{aligned}$$

Including the radiation terms we obtain

$$\begin{aligned}
m_1 \frac{d\lambda_r^{(1)}}{ds_1} &= \frac{e_1}{c^2}\left(F_{rl}^{(12)}\lambda_l^{(1)} + F_{rl}^{(13)}\lambda_l^{(1)} + F_{rl}^{(1)rad}\lambda_l^{(1)}\right),\ (r = 1,2,3,4)\\
m_2 \frac{d\lambda_r^{(2)}}{ds_2} &= \frac{e_2}{c^2}\left(F_{rl}^{(21)}\lambda_l^{(2)} + F_{rl}^{(23)}\lambda_l^{(2)} + F_{rl}^{(2)rad}\lambda_l^{(2)}\right),\ (r = 1,2,3,4) \quad (1.2)\\
m_3 \frac{d\lambda_r^{(3)}}{ds_3} &= \frac{e_3}{c^2}\left(F_{rl}^{(31)}\lambda_l^{(3)} + F_{rl}^{(32)}\lambda_l^{(3)} + F_{rl}^{(3)rad}\lambda_l^{(3)}\right),\ (r = 1,2,3,4).
\end{aligned}$$

Let us note the number of equations in (1.2) is 12, while the unknown functions (trajectories) are 12. In [4, 1] it is proved that every 4-th equation is a consequence of the previous three ones. In a similar way one can prove that the independent equations in (1.2) are 9 in number. So we have to solve a system containing 9 equations for 9 unknown functions – components of the velocities of the three moving particles.

2. Derivations of an Explicit Form of the Lorentz Force in Equations of Motion and Introducing the Radiation Terms

We follow the procedure introduced in [1]. By $\left(x_1^{(k)}(t),\, x_2^{(k)}(t),\, x_3^{(k)}(t), x_4^{(k)} = ict\right), (k = 1,2,3)$ we denote the space-time coordinates of the charged particles. As usually Latin indices run from 1 to 4, while Greek – from 1 to 3. The scalar product in the Minkowski space is $\langle a,b\rangle_4 = a_r b_r = \sum_{r=1}^{4} a_r b_r$, while in the 3-dimensional subspace — $\langle a,b\rangle = a_\alpha b_\alpha = \sum_{\alpha=1}^{3} a_\alpha b_\alpha$; c is the vacuum speed of light; $m_k\,(k = 1,2,3)$ are the proper masses of the particles; $e_k\,(k = 1,2,3)$ are their charges. The elements of proper times are $ds_k\,(k = 1,2,3)$ and the unit tangent vectors to the world lines are

$$\lambda^{(k)} = \left(\lambda_1^{(k)}, \lambda_2^{(k)}, \lambda_3^{(k)}, \lambda_4^{(k)}\right) = \left(\frac{u_1^{(1)}(t)}{\Delta_k}, \frac{u_2^{(1)}(t)}{\Delta_k}, \frac{u_3^{(1)}(t)}{\Delta_k}, \frac{ic}{\Delta_k}\right),$$

where $\vec{u}^{(k)} = \left(u_1^{(k)}(t), u_2^{(k)}(t), u_3^{(k)}(t)\right) = \left(\dot{x}_1^{(k)}(t), \dot{x}_2^{(k)}(t), \dot{x}_3^{(k)}(t)\right)$, $\Delta_k = \sqrt{c^2 - \langle \vec{u}^{(k)}(t), \vec{u}^{(k)}(t)\rangle}$.

In order to compare denotations with originally accepted ones we recall [19]

$$\gamma_k = 1/\sqrt{1 - \frac{\langle \vec{u}^{(k)}, \vec{u}^{(k)}\rangle}{c^2}};\ \frac{d}{ds_k} = \frac{\gamma_k}{c}\frac{d}{dt};\ \lambda_\alpha^{(k)} = \frac{\gamma_k\, u_\alpha^{(k)}(t)}{c} = \frac{u_\alpha^{(k)}(t)}{\Delta_k}\,(\alpha = 1,2,3);\ \lambda_4^{(k)} = i\gamma_k = \frac{ic}{\Delta_k}.$$

The components of the accelerations are

$$\frac{d\lambda^{(k)}}{ds_k} = \left(\frac{\gamma_k}{c}\frac{d}{dt}\frac{\gamma_k \lambda_\alpha^{(k)}}{c},\ i\frac{\gamma_k}{c}\frac{d\gamma_k}{dt}\right) = \left(\frac{1}{\Delta_k}\frac{d}{dt}\frac{u_\alpha^{(k)}(t)}{\Delta_k},\ \frac{ic}{\Delta_k}\frac{d}{dt}\frac{1}{\Delta_k}\right),\ (\alpha = 1,2,3), (k = 1,2,3).$$

The isotropic vectors $\xi^{(kn)}(k = 1,2,3\,;\, n \neq k)$ are obtained as in [1] (cf. also [19]): fix any event on the world line of the k-th particle and draw the light cone into the past. This cone intersects the world line of the n-th particle in any other (past) event. Then

$$\begin{aligned}\xi^{(kn)} &= \left(\xi_1{}^{(kn)}, \xi_2{}^{(kn)}, \xi_3{}^{(kn)}, \xi_4{}^{(kn)}\right)\\ &= \left(x_1^{(k)}(t) - x_1^{(n)}(t - \tau_{kn}), x_1^{(k)}(t) - x_1^{(n)}(t - \tau_{kn}),\, x_1^{(k)}(t) - x_1^{(n)}(t - \tau_{kn}),\, ic\tau_{kn}\right).\end{aligned}$$

Since $\langle \xi^{(kn)}, \xi^{(kn)}\rangle_4 = 0$, the retarded functions $\tau_{kn}(t)$ should satisfy the following functional equations

$$\tau_{kn}(t) = \frac{1}{c}\sqrt{\langle \xi^{(kn)}, \xi^{(kn)}\rangle_4} \equiv \frac{1}{c}\sqrt{\sum_{\alpha=1}^{3}\left[x_\alpha^{(k)}(t) - x_\alpha^{(n)}(t - \tau_{kn}(t)\right]^2}.$$

The above equations are $3.2 = 6$ in number $(k = 1,2,3\,;\, n \neq k)$.

In what follows we obtain an explicit type of the radiation terms, following [10] (cf. also [7]- [9]). Let us consider a charge $e_k(k = 1,2,3)$ describing any curve $L_k(k = 1,2,3)$ in the space-time. Let $A_k\left(x_1^{(k)}(t), x_2^{(k)}(t), x_3^{(k)}(t), ict\right)$ be any event and let $A_k^{ret}\left(x_1^{(k)}(\breve{t}_k), x_2^{(k)}(\breve{t}_k), x_3^{(k)}(\breve{t}_k), ic\breve{t}_k\right)$, $\breve{t}_k < t$ be the intersection of $L_k(k = 1,2,3)$ with the null-cone drawn into the past from $A_k(k = 1,2,3)$, and $A_k^{adv}\left(x_1^{(k)}(\widehat{t}_k), x_2^{(k)}(\widehat{t}_k), x_3^{(k)}(\widehat{t}_k), ic\,\widehat{t}_k\right)$, $\widehat{t}_k > t$ be the intersection of $L_k\,(k = 1,2,3)$ with the null-cone drawn into the future from $A_k(k = 1,2,3)$. The components of the velocity vector to the world-line $L_k(k = 1,2,3)$ at A_k^{ret} are

$$\lambda^{(k)ret} = \left(\lambda_1^{(k)ret}, \lambda_2^{(k)ret}, \lambda_3^{(k)ret}, \lambda_4^{(k)ret}\right) = \left(\frac{u_1^{(k)}(\breve{t}_k)}{\Delta_{(k)ret}}, \frac{u_2^{(k)}(\breve{t}_k)}{\Delta_{(k)ret}}, \frac{u_3^{(k)}(\breve{t}_k)}{\Delta_{(k)ret}}, \frac{ic}{\Delta_{(k)ret}}\right) = \left(\frac{\vec{u}^{(k)}(\breve{t}_k)}{\Delta_{(k)ret}}, \frac{ic}{\Delta_{(k)ret}}\right),$$

where $\Delta_{(k)ret} = \sqrt{c^2 - \langle \vec{u}^{(k)}(\breve{t}_k), \vec{u}^{(k)}(\breve{t}_k)\rangle}$.

Let $A_k^{ret}A_k$ be the isotropic vector $\xi^{(k)ret}=\left(\xi_1^{(k)ret},\xi_2^{(k)ret},\xi_3^{(k)ret},\xi_4^{(k)ret}\right)$.

Following the idea of P. A. M. Dirac [10] we set $\tau_k^{ret}(t)=t-\breve{t}_k \Rightarrow \breve{t}_k=t-\tau_k^{ret}(t)$. In a similar way we introduce the velocity vector to the world-line $L_k(k=1,2,3)$ at A_k^{adv}:

$$\lambda^{(k)adv}=\left(\lambda_1^{(k)adv},\lambda_2^{(k)adv},\lambda_3^{(k)adv},\lambda_4^{(k)adv}\right)=\left(\frac{u_1^{(k)}(\widehat{t}_k)}{\Delta_{(k)adv}},\frac{u_2^{(k)}(\widehat{t}_k)}{\Delta_{(k)adv}},\frac{u_3^{(k)}(\widehat{t}_k)}{\Delta_{(k)adv}},\frac{ic}{\Delta_{(k)adv}}\right)=\left(\frac{\vec{u}^{(k)}(\widehat{t}_k)}{\Delta_{(k)adv}},\frac{ic}{\Delta_{(k)adv}}\right),$$

where $\Delta_{(k)adv}=\sqrt{c^2-\left\langle\vec{u}^{(k)}(\widehat{t}_k),\vec{u}^{(k)}(\widehat{t}_k)\right\rangle}$.

Let $A_k^{adv}A_k$ be the isotropic vector $\xi^{(k)adv}=\left(\xi_1^{(k)adv},\xi_2^{(k)adv},\xi_3^{(k)adv},\xi_4^{(k)adv}\right)$, where

$$\xi_\alpha^{(k)adv}=x_\alpha^{(k)}(\widehat{t}_k)-x_\alpha^{(k)}(t)\ ;\ \xi_4^{(k)ret}=ic(\widehat{t}_k-t),\ \ t<\widehat{t}_k\ .$$

Now, we put $\tau_k^{adv}(t)=\widehat{t}_k-t\Rightarrow \widehat{t}_k=t+\tau_k^{adv}(t)$. Following again [10] we define the radiation term as a half of the difference between both retarded and advanced potentials:

$$F_{mn}^{(k)rad}=\frac{1}{2}\left[\left(\frac{\partial A_n^{(k)ret}}{\partial x_m^{(k)ret}}-\frac{\partial A_m^{(n)ret}}{\partial x_n^{(k)ret}}\right)-\left(\frac{\partial A_n^{(k)adv}}{\partial x_m^{(k)adv}}-\frac{\partial A_m^{(n)adv}}{\partial x_n^{(k)adv}}\right)\right],$$

where $A_n^{(k)ret}=-\frac{e_k\lambda_n^{(k)ret}}{\left\langle\lambda^{(k)ret},\xi^{(k)ret}\right\rangle_4}$ and $A_n^{(k)adv}=-\frac{e_k\lambda_n^{(k)adv}}{\left\langle\lambda^{(k)adv},\xi^{(k)adv}\right\rangle_4}$. Then the equations of motion become $(r=1,2,3,4)$

$$m_1\frac{d\lambda_r^{(1)}}{ds_1}=\frac{e_1}{c^2}\left(F_{rl}^{(12)}\lambda_l^{(1)}+F_{rl}^{(13)}\lambda_l^{(1)}+\frac{1}{2}\left[\frac{\partial A_l^{(1)ret}}{\partial x_r^{(1)ret}}-\frac{\partial A_r^{(1)ret}}{\partial x_l^{(1)ret}}-\left(\frac{\partial A_l^{(1)adv}}{\partial x_r^{(1)adv}}-\frac{\partial A_r^{(1)adv}}{\partial x_l^{(1)adv}}\right)\right]\lambda_l^{(1)}\right),$$

$$m_2\frac{d\lambda_r^{(2)}}{ds_2}=\frac{e_2}{c^2}\left(F_{rl}^{(21)}\lambda_l^{(2)}+F_{rl}^{(23)}\lambda_l^{(2)}+\frac{1}{2}\left[\frac{\partial A_l^{(2)ret}}{\partial x_r^{(2)ret}}-\frac{\partial A_r^{(2)ret}}{\partial x_l^{(2)ret}}-\left(\frac{\partial A_l^{(2)adv}}{\partial x_r^{(2)adv}}-\frac{\partial A_r^{(2)adv}}{\partial x_l^{(2)adv}}\right)\right]\lambda_l^{(2)}\right),$$

$$m_3\frac{d\lambda_r^{(3)}}{ds_3}=\frac{e_3}{c^2}\left(F_{rl}^{(31)}\lambda_l^{(3)}+F_{rl}^{(32)}\lambda_l^{(3)}+\frac{1}{2}\left[\frac{\partial A_l^{(3)ret}}{\partial x_r^{(3)ret}}-\frac{\partial A_r^{(3)ret}}{\partial x_l^{(3)ret}}-\left(\frac{\partial A_l^{(3)adv}}{\partial x_r^{(3)adv}}-\frac{\partial A_r^{(3)adv}}{\partial x_l^{(3)adv}}\right)\right]\lambda_l^{(3)}\right).$$

The formalism from [7] leads to $(k=1,2,3)$:

$$\begin{aligned}
\frac{d\lambda_\alpha^{(k)}}{ds_k} &= \sum_{n=1,n\neq k}^{3}\frac{e_ke_n}{m_kc^2}\left\{\frac{\xi_\alpha^{(kn)}\left\langle\lambda^{(k)},\lambda^{(n)}\right\rangle_4-\lambda_\alpha^{(n)}\left\langle\lambda^{(k)},\xi^{(kn)}\right\rangle_4}{\left\langle\lambda^{(n)},\xi^{(kn)}\right\rangle_4^3}\left(1+\left\langle\frac{d\lambda^{(n)}}{ds_n},\xi^{(kn)}\right\rangle_4\right)\right.\\
&\left.+\frac{1}{\left\langle\lambda^{(n)},\xi^{(kn)}\right\rangle_4^2}\left[\frac{d\lambda_\alpha^{(n)}}{ds_n}\left\langle\lambda^{(k)},\xi^{(kn)}\right\rangle_4-\xi_\alpha^{(kn)}\left\langle\lambda^{(k)},\frac{d\lambda^{(n)}}{ds_n}\right\rangle_4\right]\right\}\\
&+\frac{5_k^2}{2m_kc^2}\left[\frac{\xi_\alpha^{(k)ret}\langle\lambda^{(k)},\lambda^{(k)ret}\rangle_4-\lambda_\alpha^{(k)ret}\langle\xi^{(k)ret},\lambda^{(k)}\rangle_4}{\langle\lambda^{(k)ret},\xi^{(k)ret}\rangle_4^3}\left(1+\left\langle\frac{d\lambda^{(k)ret}}{ds_{ret}},\xi^{(k)ret}\right\rangle_4\right)\right.\\
&\left.+\frac{1}{\langle\lambda^{(k)ret},\xi^{(k)ret}\rangle_4^2}\left(\frac{d\lambda_\alpha^{(k)ret}}{ds_{ret}}\langle\xi^{(k)ret},\lambda^{(k)}\rangle_4-\xi_\alpha^{(k)ret}\left\langle\lambda^{(k)},\frac{d\lambda^{(k)ret}}{ds_{ret}}\right\rangle_4\right)\right]\\
&-\frac{5_k^2}{2m_kc^2}\left[\frac{\xi_\alpha^{(k)adv}\langle\lambda^{(k)},\lambda^{(k)adv}\rangle_4-\lambda_\alpha^{(k)adv}\langle\xi^{(k)adv},\lambda^{(k)}\rangle_4}{\langle\lambda^{(k)adv},\xi^{(k)adv}\rangle_4^3}\left(1+\left\langle\frac{d\lambda^{(k)adv}}{ds_{adv}},\xi^{(k)adv}\right\rangle_4\right)\right.\\
&\left.+\frac{1}{\langle\lambda^{(k)adv},\xi^{(k)adv}\rangle_4^2}\left(\frac{d\lambda_\alpha^{(k)adv}}{ds_{adv}}\langle\xi^{(k)adv},\lambda^{(k)}\rangle_4-\xi_\alpha^{(k)adv}\left\langle\lambda^{(k)},\frac{d\lambda^{(k)adv}}{ds_{adv}}\right\rangle_4\right)\right]
\end{aligned}\tag{2.1}$$

$$
\begin{aligned}
\frac{d\lambda_4^{(k)}}{ds_k} &= \sum_{n=1,n\neq k}^{3} \frac{Q^{(kn)}}{c^2}\left\{ \frac{\xi_4^{(kn)}\left\langle\lambda^{(k)},\lambda^{(n)}\right\rangle_4 - \lambda_4^{(n)}\left\langle\lambda^{(k)},\xi^{(kn)}\right\rangle_4}{\left\langle\lambda^{(n)},\xi^{(kn)}\right\rangle_4^3}\left(1+\left\langle\frac{d\lambda^{(n)}}{ds_n},\xi^{(kn)}\right\rangle_4\right)\right.\\
&\left. + \frac{1}{\left\langle\lambda^{(n)},\xi^{(kn)}\right\rangle_4^2}\left[\frac{d\lambda_4^{(n)}}{ds_n}\left\langle\lambda^{(k)},\xi^{(kn)}\right\rangle_4 - \xi_4^{(kn)}\left\langle\lambda^{(k)},\frac{d\lambda^{(n)}}{ds_n}\right\rangle_4\right]\right\}\\
&+ \frac{5_k^2}{2m_k c^2}\left[\frac{\xi_4^{(k)ret}\langle\lambda^{(k)},\lambda^{(k)ret}\rangle_4 - \lambda_4^{(k)ret}\langle\xi^{(k)ret},\lambda^{(k)}\rangle_4}{\langle\lambda^{(k)ret},\xi^{(k)ret}\rangle_4^3}\left(1+\left\langle\xi^{(k)ret},\frac{d\lambda^{(k)ret}}{ds_{ret}}\right\rangle_4\right)\right.\\
&\left. + \frac{1}{\langle\lambda^{(k)ret},\xi^{(k)ret}\rangle_4^2}\left(\langle\xi^{(k)ret},\lambda^{(k)}\rangle_4\frac{d\lambda_4^{(k)ret}}{ds_{ret}} - \left\langle\lambda^{(k)},\frac{d\lambda^{(k)ret}}{ds_{ret}}\right\rangle_4\xi_4^{(k)ret}\right)\right]\\
&- \frac{5_k^2}{2m_k c^2}\left[\frac{\xi_\alpha^{(k)adv}\langle\lambda^{(k)},\lambda^{(k)adv}\rangle_4 - \lambda_\alpha^{(k)adv}\langle\xi^{(k)adv},\lambda^{(k)}\rangle_4}{\langle\lambda^{(k)adv},\xi^{(k)adv}\rangle_4^3}\left(1+\left\langle\xi^{(k)adv},\frac{d\lambda^{(k)adv}}{ds_{adv}}\right\rangle_4\right)\right.\\
&\left. + \frac{1}{\langle\lambda^{(k)adv},\xi^{(k)adv}\rangle_4^2}\left(\langle\xi^{(k)adv},\lambda^{(k)}\rangle_4\frac{d\lambda_4^{(k)adv}}{ds_{adv}} - \left\langle\lambda^{(k)},\frac{d\lambda^{(k)adv}}{ds_{adv}}\right\rangle_4\xi_4^{(k)adv}\right)\right].
\end{aligned}
\tag{2.2}
$$

3. Transition to the Euclidean Coordinates Introducing Suitable Denotations

Using the formalism from [19], [3] and [4] we are able to move to the Euclidean space: for the isotropic vectors $\left\langle\xi^{(kn)},\xi^{(kn)}\right\rangle_4=0$, $\left\langle\xi^{(k)ret},\xi^{(k)ret}\right\rangle_4=0$, $\left\langle\xi^{(k)adv},\xi^{(k)adv}\right\rangle_4=0$ we have

$$
\begin{aligned}
\xi^{(kn)} &= \left(\xi_1{}^{(kn)},\xi_2{}^{(kn)},\xi_3{}^{(kn)},ic\tau_{kn}\right) = \left(\vec{\xi}^{(kn)},ic\tau_{kn}\right)\\
&= \left(x_1^{(k)}(t)-x_1^{(n)}(t-\tau_{kn}),x_2^{(k)}(t)-x_2^{(n)}(t-\tau_{kn}),x_3^{(k)}(t)-x_3^{(n)}(t-\tau_{kn}),ic\tau_{kn}\right),\\
\xi^{(k)ret} &= \left(\xi_1^{(k)ret},\xi_2^{(k)ret},\xi_3^{(k)ret},ic\tau_k^{ret}\right) = \left(\vec{\xi}^{(k)ret},ic\tau_k^{ret}\right)\\
&= \left(x_1^{(k)}(t)-x_1^{(k)}(t-\tau_k^{ret}),x_2^{(k)}(t)-x_2^{(k)}(t-\tau_k^{ret}),x_3^{(k)}(t)-x_3^{(k)}(t-\tau_k^{ret}),ic\tau_k^{ret}\right),\\
\xi^{(k)adv} &= \left(\xi_1{}^{(k)adv},\xi_2{}^{(k)adv},\xi_3{}^{(k)adv},ic\tau_k^{adv}\right) = \left(\vec{\xi}^{(k)adv},ic\tau_k^{adv}\right)\\
&= \left(x_1^{(k)}(t)-x_1^{(k)}(t+\tau_k^{adv}),x_2^{(k)}(t)-x_2^{(k)}(t+\tau_k^{adv}),x_3^{(k)}(t)-x_3^{(k)}(t+\tau_k^{adv}),ic\tau_k^{adv}\right)
\end{aligned}
$$

which implies $\tau_{kn}=\frac{1}{c}\sqrt{\left\langle\vec{\xi}^{(kn)},\vec{\xi}^{(kn)}\right\rangle}$, $\tau_k^{ret}=\frac{1}{c}\sqrt{\left\langle\vec{\xi}^{(k)ret},\vec{\xi}^{(k)ret}\right\rangle}$, $\tau_k^{adv}=\frac{1}{c}\sqrt{\left\langle\vec{\xi}^{(k)adv},\vec{\xi}^{(k)adv}\right\rangle}$.

For the velocity vectors we have

$$
\lambda^{(n)} = \left(\frac{u_1^{(n)}(t-\tau_{kn})}{\Delta_{kn}},\ \frac{u_2^{(n)}(t-\tau_{kn})}{\Delta_{kn}},\ \frac{u_3^{(n)}(t-\tau_{kn})}{\Delta_{kn}},\ \frac{ic}{\Delta_{kn}}\right) = \left(\frac{\vec{u}^{(n)}(t-\tau_{kn})}{\Delta_{kn}},\ \frac{ic}{\Delta_{kn}}\right),
$$

where

$$
\Delta_{kn} = \sqrt{c^2-\left\langle\vec{u}^{(n)}(t-\tau_{kn}),\vec{u}^{(n)}(t-\tau_{kn})\right\rangle}
$$

and

$$
\frac{d}{ds_k} = \frac{1}{\Delta_k}\frac{d}{dt}\ ;\quad \frac{d}{ds_n} = \frac{1}{\Delta_{kn}}\frac{d}{dt_{kn}} = \frac{1}{\Delta_{kn}}\frac{dt}{dt_{kn}}\frac{d}{dt}\ ,\quad \frac{d}{dt_{kn}} = D_{kn};
$$

$$\frac{dt}{dt_{kn}}=\frac{c\sqrt{\left\langle\vec{\xi}^{(kn)},\vec{\xi}^{(kn)}\right\rangle}-\left\langle\vec{\xi}^{(kn)},\vec{u}^{(n)}\right\rangle}{c\sqrt{\left\langle\vec{\xi}^{(kn)},\vec{\xi}^{(kn)}\right\rangle}-\left\langle\vec{\xi}^{(kn)},\vec{u}^{(k)}\right\rangle}=\frac{c^2\tau_{kn}-\left\langle\vec{\xi}^{(kn)},\vec{u}^{(n)}\right\rangle}{c^2\tau_{kn}-\left\langle\vec{\xi}^{(kn)},\vec{u}^{(k)}\right\rangle}=D_{kn}.$$

For the acceleration vectors and scalar products in the Minkowski space we have

$$\begin{aligned}
\frac{d\lambda_\alpha^{(k)}}{ds_k}&=\frac{1}{\Delta_k}\frac{d}{dt}\left(\frac{u_\alpha^{(k)}(t)}{\Delta_k}\right)=\frac{\dot{u}_\alpha^{(k)}(t)}{\Delta_k^2}+\frac{u_\alpha^{(k)}(t)\left\langle\vec{u}^{(k)}(t),\dot{\vec{u}}^{(k)}(t)\right\rangle}{\Delta_k^4};\\
\frac{d\lambda_4^{(k)}}{ds_k}&=\frac{ic}{\Delta_k}\frac{d}{dt}\left(\frac{1}{\Delta_k}\right)=\frac{ic\left\langle\vec{u}^{(k)}(t),\dot{\vec{u}}^{(k)}(t)\right\rangle}{\Delta_k^4};\\
\frac{d\lambda_\alpha^{(n)}}{ds_n}&=D_{kn}\left(\frac{\dot{u}_\alpha^{(n)}(t-\tau_{kn})}{\Delta_{kn}^2}+\frac{u_\alpha^{(n)}(t-\tau_{kn})\left\langle\vec{u}^{(n)}(t-\tau_{kn}),\dot{\vec{u}}^{(n)}(t-\tau_{kn})\right\rangle}{\Delta_{kn}^4}\right);\\
\frac{d\lambda_4^{(n)}}{ds_n}&=\frac{icD_{kn}\left\langle\vec{u}^{(n)}(t-\tau_{kn}),\dot{\vec{u}}^{(n)}(t-\tau_{kn})\right\rangle}{\Delta_{kn}^4};\\
\left\langle\lambda^{(k)},\lambda^{(n)}\right\rangle_4&=\frac{\left\langle\vec{u}^{(k)}(t),\vec{u}^{(n)}(t-\tau_{kn})\right\rangle-c^2}{\Delta_k\Delta_{kn}};\\
\left\langle\lambda^{(k)},\xi^{(kn)}\right\rangle_4&=\frac{\left\langle\vec{u}^{(k)}(t),\vec{\xi}^{(kn)}\right\rangle-c^2\tau_{kn}}{\Delta_k};\\
\left\langle\lambda^{(n)},\xi^{(kn)}\right\rangle_4&=\frac{\left\langle\vec{u}^{(n)}(t-\tau_{kn}),\vec{\xi}^{(kn)}\right\rangle-c^2\tau_{kn}}{\Delta_{kn}};\\
\left\langle\xi^{(kn)},\frac{d\lambda^{(n)}}{ds_n}\right\rangle_4&=D_{kn}\left(\frac{\left\langle\vec{\xi}^{(kn)},\dot{\vec{u}}^{(n)}(t-\tau_{kn})\right\rangle}{\Delta_{kn}^2}+\frac{\left\langle\vec{\xi}^{(kn)},\vec{u}^{(n)}(t-\tau_{kn})\right\rangle-c^2\tau_{kn}}{\Delta_{kn}^4}\right.\\
&\qquad\left.\left\langle\vec{u}^{(n)}(t-\tau_{kn}),\dot{\vec{u}}^{(n)}(t-\tau_{kn})\right\rangle\right);\\
\left\langle\lambda^{(k)},\frac{d\lambda^{(n)}}{ds_n}\right\rangle_4&=\frac{D_{kn}}{\Delta_k}\left(\frac{\left\langle\vec{u}^{(k)}(t),\dot{\vec{u}}^{(n)}(t-\tau_{kn})\right\rangle}{\Delta_{kn}^2}+\frac{\left\langle\vec{u}^{(k)}(t),\vec{u}^{(n)}(t-\tau_{kn})\right\rangle-c^2\tau_{kn}}{\Delta_{kn}^4}\right.\\
&\qquad\left.\left\langle\vec{u}^{(n)}(t-\tau_{kn}),\dot{\vec{u}}^{(n)}(t-\tau_{kn})\right\rangle\right).
\end{aligned}$$

For the retarded part of radiation term in a similar way differentiating

$$t-\breve{t}_k=\frac{1}{c}\sqrt{\sum_{\gamma=1}^{3}\left[x_\gamma^{(k)}(t)-x_\gamma^{(k)}(\breve{t}_k)\right]^2}$$

with respect to $\breve{t}_k$ $(t=t(\breve{t}_k))$, we obtain $\frac{dt}{d\breve{t}_k}-1=\frac{\sum_{\gamma=1}^{3}\left[x_\gamma^{(k)}(t)-x_\gamma^{(k)}(\breve{t}_k)\right]\left[u_\gamma^{(k)}(t)\frac{dt}{d\breve{t}_k}-u_\gamma^{(k)}(\breve{t}_k)\right]}{c\sqrt{\sum_{\gamma=1}^{3}\left[x_\gamma^{(k)}(t)-x_\gamma^{(k)}(\breve{t}_k)\right]^2}}$. Hence

$$D_k^{ret}\equiv\frac{dt}{d\breve{t}_k}=\frac{c\sqrt{\left\langle\vec{\xi}^{(k)ret},\vec{\xi}^{(k)ret}\right\rangle}-\left\langle\vec{\xi}^{(k)ret},\vec{u}^{(k)ret}\right\rangle}{c\sqrt{\left\langle\vec{\xi}^{(k)ret},\vec{\xi}^{(k)ret}\right\rangle}-\left\langle\vec{\xi}^{(k)ret},\vec{u}^{(k)}\right\rangle}=\frac{c\tau_k^{ret}-\left\langle\vec{\xi}^{(k)ret},\vec{u}^{(k)}(t-\tau_k^{ret})\right\rangle}{c\tau_k^{ret}-\left\langle\vec{\xi}^{(k)ret},\vec{u}^{(k)}(t)\right\rangle}.$$

Analogously for advanced term we have

$$D_k^{adv}\equiv\frac{dt}{d\,\widehat{t}_k}=\frac{c\sqrt{\left\langle\vec{\xi}^{(k)adv},\vec{\xi}^{(k)adv}\right\rangle}-\left\langle\vec{\xi}^{(k)adv},\vec{u}^{(k)adv}\right\rangle}{c\sqrt{\left\langle\vec{\xi}^{(k)adv},\vec{\xi}^{(k)adv}\right\rangle}-\left\langle\vec{\xi}^{(k)adv},\vec{u}^{(k)}\right\rangle}=\frac{c\tau_k^{adv}-\left\langle\vec{\xi}^{(k)adv},\vec{u}^{(k)}(t+\tau_k^{adv})\right\rangle}{c\tau_k^{adv}-\left\langle\vec{\xi}^{(k)adv},\vec{u}^{(k)}(t)\right\rangle}.$$

Further

$$\left\langle\lambda^{(k)},\lambda^{(k)ret}\right\rangle_4=\frac{\left\langle\vec{u}^{(k)}(t),\vec{u}^{(k)}(t-\tau_k^{ret}\right\rangle-c^2}{\Delta_k\Delta_{(k)ret}};\left\langle\lambda^{(k)},\lambda^{(k)adv}\right\rangle_4=\frac{\left\langle\vec{u}^{(k)}(t),\vec{u}^{(k)}(t+\tau_k^{adv}\right\rangle-c^2}{\Delta_k\Delta_{(k)adv}};$$

$$\left\langle \xi^{(k)ret}, \lambda^{(k)} \right\rangle_4 = \frac{\left\langle \vec{u}^{(k)}(t), \vec{\xi}^{(k)ret} \right\rangle - c^2 \tau_k^{ret}}{\Delta_k}; \left\langle \xi^{(k)adv}, \lambda^{(k)} \right\rangle_4 = \frac{\left\langle \vec{u}^{(k)}(t), \vec{\xi}^{(k)adv} \right\rangle - c^2 \tau_k^{adv}}{\Delta_k};$$

$$\left\langle \xi^{(k)ret}, \lambda^{(k)ret} \right\rangle_4 = \frac{\left\langle \vec{u}^{(k)ret}(t - \tau_k^{ret}), \vec{\xi}^{(k)ret} \right\rangle - c^2 \tau_k^{ret}}{\Delta_{(k)ret}};$$

$$\left\langle \xi^{(k)adv}, \lambda^{(k)adv} \right\rangle_4 = \frac{\left\langle \vec{u}^{(k)adv}(t + \tau_k^{adv}), \vec{\xi}^{(k)adv} \right\rangle - c^2 \tau_k^{adv}}{\Delta_{(k)adv}};$$

$$\frac{d}{ds_{ret}} = \frac{1}{\Delta_{(k)ret}} \frac{d}{d\breve{t}} = \frac{1}{\Delta_{(k)ret}} \frac{dt}{d\breve{t}_k} \frac{d}{dt} = \frac{1}{\Delta_{(k)ret}} D_k^{ret} \frac{d}{dt};$$

$$\frac{d}{ds_{adv}} = \frac{1}{\Delta_{(k)adv}} \frac{d}{d\breve{t}} = \frac{1}{\Delta_{(k)adv}} \frac{dt}{d\breve{t}_k} \frac{d}{dt} = \frac{1}{\Delta_{(k)adv}} D_k^{adv} \frac{d}{dt};$$

$$\frac{d\lambda_\alpha^{(k)ret}}{ds_{ret}} = D_k^{ret} \left(\frac{\dot{u}_\alpha^{(k)}(t-\tau_k^{ret})}{\Delta_{(k)ret}^2} + \frac{u_\alpha^{(k)}(t-\tau_k^{ret}) \left\langle \vec{u}^{(k)}(t-\tau_k^{ret}), \dot{\vec{u}}^{(k)}(t-\tau_k^{ret}) \right\rangle}{\Delta_{(k)ret}^4} \right);$$

$$\frac{d\lambda_4^{(k)ret}}{ds_{ret}} = \frac{icD_k^{ret} \left\langle \vec{u}^{(k)}(t-\tau_k^{ret}), \dot{\vec{u}}^{(k)}(t-\tau_k^{ret}) \right\rangle}{\Delta_{(k)ret}^4};$$

$$\frac{d\lambda_\alpha^{(k)adv}}{ds_{adv}} = D_k^{adv} \left(\frac{\dot{u}_\alpha^{(k)}(t+\tau_k^{adv})}{\Delta_{(k)adv}^2} + \frac{u_\alpha^{(k)}(t+\tau_k^{adv}) \left\langle \vec{u}^{(k)}(t+\tau_k^{adv}), \dot{\vec{u}}^{(k)}(t+\tau_k^{adv}) \right\rangle}{\Delta_{(k)adv}^4} \right);$$

$$\frac{d\lambda_4^{(k)adv}}{ds_{adv}} = \frac{icD_k^{adv} \left\langle \vec{u}^{(k)}(t+\tau_k^{adv}), \dot{\vec{u}}^{(k)}(t+\tau_k^{adv}) \right\rangle}{\Delta_{(k)adv}^4};$$

$$\left\langle \xi^{(k)ret}, \frac{d\lambda^{(k)ret}}{ds_{ret}} \right\rangle_4 = D_k^{ret} \left(\frac{\left\langle \vec{\xi}^{(k)ret}, \dot{\vec{u}}^{(k)}(t-\tau_k^{ret}) \right\rangle}{\Delta_{(k)ret}^2} + \frac{\left\langle \vec{\xi}^{(k)ret}, \vec{u}^{(k)}(t-\tau_k^{ret}) \right\rangle - c^2 \tau_k^{ret}}{\Delta_{kn}^4} \left\langle \vec{u}^{(k)}(t-\tau_k^{ret}), \dot{\vec{u}}^{(k)}(t-\tau_k^{ret}) \right\rangle \right);$$

$$\left\langle \lambda^{(k)}, \frac{d\lambda^{(k)ret}}{ds_{ret}} \right\rangle_4 = \frac{D_k^{ret}}{\Delta_k} \left(\frac{\left\langle \vec{u}^{(k)}(t), \dot{\vec{u}}^{(k)}(t-\tau_k^{ret}) \right\rangle}{\Delta_{(k)ret}^2} + \frac{\left\langle \vec{u}^{(k)}(t), \vec{u}^{(k)}(t-\tau_k^{ret}) \right\rangle - c^2 \tau_k^{ret}}{\Delta_{(k)ret}^4} \left\langle \vec{u}^{(k)}(t), \dot{\vec{u}}^{(k)}(t-\tau_k^{ret}) \right\rangle \right);$$

$$\left\langle \xi^{(k)adv}, \frac{d\lambda^{(k)adv}}{ds_{adv}} \right\rangle_4 = D_k^{adv} \left(\frac{\left\langle \vec{\xi}^{(k)adv}, \dot{\vec{u}}^{(k)}(t+\tau_k^{adv}) \right\rangle}{\Delta_{(k)adv}^2} + \frac{\left\langle \vec{\xi}^{(k)adv}, \vec{u}^{(k)}(t+\tau_k^{adv}) \right\rangle - c^2 \tau_k^{adv}}{\Delta_{(k)adv}^4} \left\langle \vec{u}^{(k)}(t+\tau_k^{adv}), \dot{\vec{u}}^{(k)}(t+\tau_k^{adv}) \right\rangle \right);$$

$$\left\langle \lambda^{(k)}, \frac{d\lambda^{(k)adv}}{ds_{adv}} \right\rangle_4 = \frac{D_k^{adv}}{\Delta_k} \left(\frac{\left\langle \vec{u}^{(k)}(t), \dot{\vec{u}}^{(k)}(t+\tau_k^{adv}) \right\rangle}{\Delta_{(k)adv}^2} + \frac{\left\langle \vec{u}^{(k)}(t), \vec{u}^{(k)}(t+\tau_k^{adv}) \right\rangle - c^2 \tau^{(k)adv}}{\Delta_{(k)adv}^4} \left\langle \vec{u}^{(k)}(t), \dot{\vec{u}}^{(k)}(t+\tau_k^{adv}) \right\rangle \right);$$

$$H_{kn} = \left[1 + D_{kn} \left(\frac{\left\langle \vec{\xi}^{(kn)}, \dot{\vec{u}}^{(n)} \right\rangle}{\Delta_{kn}^2} + \frac{\left(\left\langle \vec{\xi}^{(kn)}, \vec{u}^{(n)} \right\rangle - c^2 \tau_{kn} \right) \left\langle \vec{u}^{(n)}, \dot{\vec{u}}^{(n)} \right\rangle}{\Delta_{kn}^4} \right) \right] \Delta_{kn}^2,$$

$$H_k^{ret} = \left[1 + D_k^{ret} \left(\frac{\left\langle \vec{\xi}^{(k)ret}, \dot{\vec{u}}^{(k)ret} \right\rangle}{\Delta_{(k)ret}^2} + \frac{\left(\left\langle \vec{\xi}^{(k)ret}, \vec{u}^{(k)ret} \right\rangle - c^2 \tau_{(k)ret} \right) \left\langle \vec{u}^{(k)ret}, \dot{\vec{u}}^{(k)ret} \right\rangle}{\Delta_{(k)adv}^4} \right) \right] \Delta_{(k)ret}^2,$$

$$H_k^{adv} = \left[1 + D_k^{adv} \left(\frac{\left\langle \vec{\xi}^{(k)adv}, \dot{\vec{u}}^{(k)adv} \right\rangle}{\Delta_{(k)adv}^2} + \frac{\left(\left\langle \vec{\xi}^{(k)adv}, \vec{u}^{(k)adv} \right\rangle - c^2 \tau_{(k)adv} \right) \left\langle \vec{u}^{(k)adv}, \dot{\vec{u}}^{(k)adv} \right\rangle}{\Delta_{(k)adv}^4} \right) \right] \Delta_{(k)adv}^2,$$

for $(k=1,2,3)$, $n\neq k$. Then the equations of motion (2.1) and (2.2) become $\alpha=1,2,3;\ k=1,2,3$:

$$
\begin{aligned}
&\frac{1}{\Delta_k^2}\dot{u}_\alpha^{(k)} + \frac{u_\alpha^{(k)}}{\Delta_k^4}\left\langle \vec{u}^{(k)},\, \dot{\vec{u}}^{(k)}\right\rangle = \sum_{n=1,n\neq k}^{3}\frac{e_n e_k}{m_k c^2}\left\{ \frac{\frac{\left(c^2-\left\langle \vec{u}^{(k)},\vec{u}^{(n)}\right\rangle\right)}{\Delta_k\Delta_{kn}}\xi_\alpha^{(kn)} - \frac{u_\alpha^{(n)}}{\Delta_{kn}}\frac{\left(c^2\tau_{kn}-\left\langle \vec{\xi}^{(kn)},\vec{u}^{(k)}\right\rangle\right)}{\Delta_k}}{\left(\frac{c^2\tau_{kn}-\left\langle \vec{\xi}^{(kn)},\vec{u}^{(n)}\right\rangle}{\Delta_{kn}}\right)^3} H_{kn} + \right.\\
&\quad +\Delta_{kn}^2\left[\frac{\frac{\left(\left\langle \vec{\xi}^{(kn)},\vec{u}^{(k)}\right\rangle - c^2\tau_{kn}\right)}{\Delta_k} D_{kn}\left(\frac{\dot{u}_\alpha^{(n)}}{\Delta_{kn}^2}+\frac{u_\alpha^{(n)}\left\langle \vec{u}^{(n)},\dot{\vec{u}}^{(n)}\right\rangle}{\Delta_{kn}^4}\right)}{\left(\left\langle \vec{\xi}^{(kn)},\vec{u}^{(n)}\right\rangle - c^2\tau_{kn}\right)^2} - \right.\\
&\quad \left.\left. - \frac{\xi_\alpha^{(kn)}\frac{D_{kn}}{\Delta_k}\left(\frac{\langle \vec{u}^{(k)},\dot{\vec{u}}^{(n)}\rangle}{\Delta_{kn}^2}+\frac{\left(\langle \vec{u}^{(k)},\vec{u}^{(n)}\rangle - c^2\right)\left\langle \vec{u}^{(n)},\dot{\vec{u}}^{(n)}\right\rangle}{\Delta_{kn}^4}\right)}{\left(\left\langle \vec{\xi}^{(kn)},\vec{u}^{(n)}\right\rangle - c^2\tau_{kn}\right)^2}\right]\right\} +\\
&\quad + \frac{e_k^2}{2m_kc^2}\left\{\frac{\xi_\alpha^{(k)ret}\frac{\langle \vec{u}^{(k)},\,\vec{u}^{(k)ret}\rangle - c^2}{\Delta_k\Delta_{(k)\,ret}} - u_\alpha^{(k)ret}\frac{\langle \vec{u}^{(k)},\vec{\xi}^{(k)\,ret}\rangle - c^2\tau_k^{ret}}{\Delta_k\Delta_{(k)\,ret}}}{\left(\frac{\langle \vec{u}^{(k)ret},\,\vec{\xi}^{(k)\,ret}\rangle - c^2\tau_k^{ret}}{\Delta_{(k)\,ret}}\right)^3} H_k^{ret} + \right.\\
&\quad +\Delta_{(k)\,ret}^2\left[\frac{\frac{\left(\langle \vec{\xi}^{(k)\,ret},\vec{u}^{(k)}\rangle - c^2\tau_k^{ret}\right)D_k^{ret}}{\Delta_k}\left(\frac{\dot{u}_\alpha^{(p)ret}}{\Delta_{(k)\,ret}^2}+\frac{u_\alpha^{(p)r}\langle \vec{u}^{(k)ret},\dot{\vec{u}}^{(k)ret}\rangle}{\Delta_{(k)\,ret}^4}\right)}{\left(c^2\tau_k^{ret} - \langle \vec{\xi}^{(k)\,ret},\vec{u}^{(k)ret}\rangle\right)^2}\right.\\
&\quad \left.\left. - \frac{\xi_\alpha^{(k)\,ret}\frac{D_k^{ret}}{\Delta_k}\left(\frac{\langle \vec{u}^{(k)},\dot{\vec{u}}^{(k)ret}\rangle}{\Delta_{(k)\,ret}^2}+\frac{\left(\langle \vec{u}^{(k)},\vec{u}^{(k)ret}\rangle - c^2\right)\langle \vec{u}^{(k)},\dot{\vec{u}}^{(k)ret}\rangle}{\Delta_{(k)\,ret}^4}\right)}{\left(c^2\tau_k^{ret} - \langle \vec{\xi}^{(k)\,ret},\vec{u}^{(k)ret}\rangle\right)^2}\right]\right\} -\\
&\quad - \frac{e_k^2}{2m_kc^2}\left\{\frac{\xi_\alpha^{(k)\,adv}\frac{\langle \vec{u}^{(k)},\,\vec{u}^{(k)\,adv}\rangle - c^2}{\Delta_k\Delta_{(k)\,adv}} - \frac{u_\alpha^{(k)adv}}{\Delta_{(k)\,adv}}\frac{\langle \vec{u}^{(k)},\vec{\xi}^{(k)\,adv}\rangle - c^2\tau_k^{adv}}{\Delta_k}}{\left(\frac{\langle \vec{u}^{(k)adv},\,\vec{\xi}^{(k)\,adv}\rangle - c^2\tau_k^{adv}}{\Delta_{(k)\,adv}}\right)^3} H_k^{adv} + \right.\\
&\quad +\Delta_{(k)\,adv}^2\left[\frac{\frac{\langle \vec{\xi}^{(p)\,adv},\vec{u}^{(k)}\rangle - c^2\tau_k^{adv}}{\Delta_k} D_k^{adv}\left(\frac{\dot{u}_\alpha^{(k)adv}}{\Delta_{(k)\,adv}^2}+\frac{u_\alpha^{(k)adv}\langle \vec{u}^{(k)adv},\dot{\vec{u}}^{(k)adv}\rangle}{\Delta_{(k)\,adv}^4}\right)}{\left(c^2\tau_k^{adv} - \langle \vec{\xi}^{(k)\,adv},\vec{u}^{(k)adv}\rangle\right)^2} - \right.\\
&\quad \left.\left. \frac{\xi_\alpha^{(k)\,adv}\frac{D_k^{adv}}{\Delta_k}\left(\frac{\langle \vec{u}^{(k)},\dot{\vec{u}}^{(k)adv}\rangle}{\Delta_{(k)\,adv}^2}+\frac{\left(\langle \vec{u}^{(k)},\vec{u}^{(k)adv}\rangle - c^2\right)\langle \vec{u}^{(k)},\dot{\vec{u}}^{(k)adv}\rangle}{\Delta_{(k)\,adv}^4}\right)}{\left(c^2\tau_k^{adv} - \langle \vec{\xi}^{(k)\,adv},\vec{u}^{(k)adv}\rangle\right)^2}\right]\right\};
\end{aligned}
\tag{3.1}
$$

$(\alpha = 1,\, 2,\, 3)$

$$
\begin{aligned}
\frac{ic}{\Delta_k^4}\left\langle \vec{u}^{(k)},\, \dot{\vec{u}}^{(k)}\right\rangle &= \frac{e_k e_n}{m_k c^2}\left\{ \frac{\frac{\left(c^2-\left\langle \vec{u}^{(k)},\vec{u}^{(n)}\right\rangle\right)}{\Delta_k\Delta_{kn}}ic\tau_{kn} - \frac{ic}{\Delta_{kn}}\frac{\left(c^2\tau_{kn}-\left\langle \vec{\xi}^{(kn)},\vec{u}^{(k)}\right\rangle\right)}{\Delta_k}}{\left(\frac{c^2\tau_{kn}-\left\langle \vec{\xi}^{(kn)},\vec{u}^{(n)}\right\rangle}{\Delta_{kn}}\right)^3} H_{kn} + \right.\\
&\quad + \Delta_{kn}^2\left[\frac{\frac{\left\langle \xi^{(kn)},\vec{u}^{(k)}\right\rangle - c^2\tau_{kn}}{\Delta_p}\ \frac{icD_{kn}}{\Delta_{pq}^4}\left\langle \vec{u}^{(n)},\, \dot{\vec{u}}^{(n)}\right\rangle}{\left(\left\langle \vec{\xi}^{(kn)},\vec{u}^{(n)}\right\rangle - c^2\tau_{kn}\right)^2} - \right.\\
&\quad \left.\left. - \frac{ic\tau_{kn}\frac{D_{kn}}{\Delta_k}\left(\frac{\left\langle \vec{u}^{(k)},\dot{\vec{u}}^{(n)}\right\rangle}{\Delta_{\tau_{kn}}^2}+\frac{\left(\left\langle \vec{u}^{(k)},\vec{u}^{(n)}\right\rangle - c^2\right)\left\langle \vec{u}^{(n)},\, \dot{\vec{u}}^{(n)}\right\rangle}{\Delta_{kn}^4}\right)}{\left(\left\langle \vec{\xi}^{(kn)},\vec{u}^{(n)}\right\rangle - c^2\tau_{kn}\right)^2}\right]\right\} +
\end{aligned}
\tag{3.2}
$$

$$
\begin{aligned}
&+\frac{e_k^2}{2m_kc^2}\left\{\frac{ic\tau_k^{ret}\frac{\left\langle\vec{u}^{(k)},\,\vec{u}^{(k)ret}\right\rangle-c^2}{\Delta_k\Delta_{(k)\,ret}}-\frac{ic}{\Delta_{(k)\,ret}}\frac{\left\langle\vec{u}^{(k)},\vec{\xi}^{(k)\,ret}\right\rangle-c^2\tau_k^{ret}}{\Delta_k}}{\left(\frac{\left\langle\vec{u}^{(k)ret},\,\vec{\xi}^{(k)\,ret}\right\rangle-c^2\tau_k^{ret}}{\Delta_{(k)\,ret}}\right)^3}H_k^{ret}+\right.\\
&+\Delta_{(k)\,ret}^2\left[\frac{\left[\frac{\left\langle\vec{\xi}^{(k)\,ret},\vec{u}^{(k)}\right\rangle-c^2\tau_k^{ret}}{\Delta_k}\quad\frac{icD_k^{ret}\left\langle\vec{u}^{(k)ret},\dot{\vec{u}}^{(k)ret}\right\rangle}{\Delta_{(k)\,ret}^4}\right.}{\left(c^2\tau_k^{ret}-\left\langle\vec{\xi}^{(k)\,ret},\vec{u}^{(k)ret}\right\rangle\right)^2}\right.\\
&\left.\left.-ic\tau_k^{ret}\frac{D_k^{ret}}{\Delta_k}\frac{\left(\frac{\left\langle\vec{u}^{(k)},\dot{\vec{u}}^{(k)ret}\right\rangle}{\Delta_{(k)\,ret}^2}+\frac{\left(\langle\vec{u}^{(k)},\vec{u}^{(k)ret}\rangle-c^2\right)\left\langle\vec{u}^{(k)ret},\dot{\vec{u}}^{(k)ret}\right\rangle}{\Delta_{(k)\,ret}^4}\right)}{\left(c^2\tau_k^{ret}-\left\langle\vec{\xi}^{(k)\,ret},\vec{u}^{(k)ret}\right\rangle\right)^2}\right]\right\}-\\
&-\frac{e_k^2}{2m_kc^2}\left\{\frac{ic\tau_k^{adv}\frac{\left\langle\vec{u}^{(k)},\,\vec{u}^{(k)adv}\right\rangle-c^2}{\Delta_k\Delta_{(k)\,adv}}-\frac{ic}{\Delta_{(k)\,adv}}\frac{\left\langle\vec{u}^{(k)},\,\vec{\xi}^{(k)adv}\right\rangle-c^2\tau_k^{adv}}{\Delta_k}}{\left(\frac{\left\langle\vec{u}^{(k)adv},\,\vec{\xi}^{(k)adv}\right\rangle-c^2\tau_k^{adv}}{\Delta_{(k)\,adv}}\right)^3}H_k^{adv}+\right.\\
&+\Delta_{(k)\,adv}^2\left[\frac{\left[\frac{\left\langle\vec{\xi}^{(k)\,adv},\vec{u}^{(k)}\right\rangle-c^2\tau_k^{adv}}{\Delta_k}\quad\frac{icD_k^{adv}\left\langle\vec{u}^{(k)adv},\dot{\vec{u}}^{(k)adv}\right\rangle}{\Delta_{(k)\,adv}^4}\right.}{\left(c^2\tau_k^{adv}-\left\langle\vec{\xi}^{(k)\,adv},\vec{u}^{(k)adv}\right\rangle\right)^2}-\right.\\
&\left.\left.-\frac{ic\tau_k^{adv}\frac{D_k^{adv}}{\Delta_k}\left(\frac{\left\langle\vec{u}^{(k)},\dot{\vec{u}}^{(k)adv}\right\rangle}{\Delta_{(k)\,adv}^2}+\frac{\left(\left\langle\vec{u}^{(k)},\vec{u}^{(k)adv}\right\rangle-c^2\right)\left\langle\vec{u}^{(k)adv},\dot{\vec{u}}^{(k)adv}\right\rangle}{\Delta_{(k)\,adv}^4}\right)}{\left(c^2\tau_k^{adv}-\left\langle\vec{\xi}^{(k)\,adv},\vec{u}^{(k)adv}\right\rangle\right)^2}\right]\right\}.
\end{aligned}
$$

One can prove as in [2] equations (3.2) are consequence of (3.1). Therefore we consider in the following only the system (3.1).

4. Reducing the System of Equations of Motion in a Suitable Form

We are able to simplify (3.1) using denotations:

$$
\begin{aligned}
A_{kn} &= \frac{H_{kn}\left(c^2-\left\langle\vec{u}^{(k)},\vec{u}^{(n)}\right\rangle\right)}{\left(c^2\tau_{kn}-\left\langle\vec{\xi}^{(kn)},\vec{u}^{(n)}\right\rangle\right)^3}-D_{kn}\frac{\Delta_{kn}^2\left\langle\vec{u}^{(k)},\dot{\vec{u}}^{(n)}\right\rangle+\left(\left\langle\vec{u}^{(k)},\vec{u}^{(n)}\right\rangle-c^2\right)\left\langle\vec{u}^{(n)},\dot{\vec{u}}^{(n)}\right\rangle}{\Delta_{kn}^2\left(\left\langle\vec{\xi}^{(kn)},\vec{u}^{(n)}\right\rangle-c^2\tau_{kn}\right)^2};\\
B_{kn} &= \frac{H_{kn}\left(c^2\tau_{kn}-\left\langle\vec{\xi}^{(kn)},\vec{u}^{(k)}\right\rangle\right)}{\left(c^2\tau_{kn}-\left\langle\vec{\xi}^{(kn)},\vec{u}^{(n)}\right\rangle\right)^3}-\frac{D_{kn}\left(\left\langle\vec{\xi}^{(kn)},\vec{u}^{(k)}\right\rangle-c^2\tau_{kn}\right)\left\langle\vec{u}^{(n)},\dot{\vec{u}}^{(n)}\right\rangle}{\Delta_{kn}^2\left(\left\langle\vec{\xi}^{(kn)},\vec{u}^{(n)}\right\rangle-c^2\tau_{kn}\right)^2};\\
C_{kn} &= \frac{D_{kn}\left(\left\langle\vec{\xi}^{(kn)},\vec{u}^{(k)}\right\rangle-c^2\tau_{kn}\right)}{\left(\left\langle\vec{\xi}^{(kn)},\vec{u}^{(n)}\right\rangle-c^2\tau_{kn}\right)^2};\\
A_{(k)ret} &= \frac{H_k^{ret}\left(c^2-\left\langle\vec{u}^{(k)},\vec{u}^{(k)ret}\right\rangle\right)}{\left(c^2\tau_k^{ret}-\left\langle\vec{\xi}^{(k)ret},\vec{u}^{(k)ret}\right\rangle\right)^3}-D_k^{ret}\frac{\Delta_{(k)ret}^2\left\langle\vec{u}^{(k)},\dot{\vec{u}}^{(k)ret}\right\rangle+\left(\left\langle\vec{u}^{(k)},\vec{u}^{(k)ret}\right\rangle-c^2\right)\left\langle\vec{u}^{(k)ret},\dot{\vec{u}}^{(k)ret}\right\rangle}{\Delta_{(k)ret}^2\left(c^2\tau_k^{ret}-\left\langle\vec{\xi}^{(k)ret},\vec{u}^{(k)ret}\right\rangle\right)^2};\\
B_{(k)ret} &= \frac{H_k^{ret}\left(c^2\tau_k^{ret}-\left\langle\vec{\xi}^{(k)ret},\vec{u}^{(k)}\right\rangle\right)}{\left(c^2\tau_k^{ret}-\left\langle\vec{\xi}^{(k)ret},\vec{u}^{(k)ret}\right\rangle\right)^3}-\frac{D_k^{ret}\left(\left\langle\vec{\xi}^{(k)ret},\vec{u}^{(k)}\right\rangle-c^2\tau_k^{ret}\right)\left\langle\vec{u}^{(k)ret},\dot{\vec{u}}^{(k)ret}\right\rangle}{\Delta_{(k)ret}^2\left(c^2\tau_k^{ret}-\left\langle\vec{\xi}^{(k)ret},\vec{u}^{(k)ret}\right\rangle\right)^2};\\
C_{(k)ret} &= \frac{D_k^{ret}\left(\left\langle\vec{\xi}^{(k)ret},\vec{u}^{(k)}\right\rangle-c^2\tau_k^{ret}\right)}{\left(c^2\tau_k^{ret}-\left\langle\vec{\xi}^{(k)ret},\vec{u}^{(k)ret}\right\rangle\right)^2};
\end{aligned}
$$

$$A_{(k)adv} = \frac{H_k^{adv}\left(c^2 - \left\langle \vec{u}^{(k)}, \vec{u}^{(k)adv}\right\rangle\right)}{\left(c^2\tau_k^{adv} - \left\langle \vec{\xi}^{(k)adv}, \vec{u}^{(k)adv}\right\rangle\right)^3} - D_k^{adv}\frac{\Delta^2_{(k)adv}\left\langle \vec{u}^{(k)}, \dot{\vec{u}}^{(k)adv}\right\rangle + \left(\left\langle \vec{u}^{(k)}, \vec{u}^{(k)adv}\right\rangle - c^2\right)\left\langle \vec{u}^{(k)adv}, \dot{\vec{u}}^{(k)adv}\right\rangle}{\Delta^2_{(k)adv}\left(c^2\tau_k^{adv} - \left\langle \vec{\xi}^{(k)adv}, \vec{u}^{(k)adv}\right\rangle\right)^2};$$

$$B_{(k)adv} = \frac{H_k^{adv}\left(c^2\tau_k^{adv} - \left\langle \vec{\xi}^{(k)adv}, \vec{u}^{(k)}\right\rangle\right)}{\left(c^2\tau_k^{adv} - \left\langle \vec{\xi}^{(k)adv}, \vec{u}^{(k)adv}\right\rangle\right)^3} - \frac{D_k^{adv}\left(\left\langle \vec{\xi}^{(k)adv}, \vec{u}^{(k)}\right\rangle - c^2\tau_k^{adv}\right)\left\langle \vec{u}^{(k)adv}, \dot{\vec{u}}^{(k)adv}\right\rangle}{\Delta^2_{(k)adv}\left(c^2\tau_k^{adv} - \left\langle \vec{\xi}^{(k)adv}, \vec{u}^{(k)adv}\right\rangle\right)^2};$$

$$C_{(k)adv} = \frac{D_k^{adv}\left(\left\langle \vec{\xi}^{(k)adv}, \vec{u}^{(k)}\right\rangle - c^2\tau_k^{adv}\right)}{\left(c^2\tau_k^{adv} - \left\langle \vec{\xi}^{(k)adv}, \vec{u}^{(k)adv}\right\rangle\right)^2}, \ (k = 1, 2, 3).$$

Then we reach the system

$$\begin{aligned}\dot{u}_\alpha^{(1)}(t) &+ \frac{u_\alpha^{(1)}(t)}{\Delta_1^2}\left\langle \vec{u}^{(1)}(t), \dot{\vec{u}}^{(1)}(t)\right\rangle = \frac{e_1e_2\Delta_1}{m_1c^2}\left(A_{12}\xi_\alpha^{(12)} - B_{12}u_\alpha^{(2)} + C_{12}\dot{u}_\alpha^{(2)}\right) + \\ &+ \frac{e_1e_3\Delta_1}{m_1c^2}\left(A_{13}\xi_\alpha^{(13)} - B_{13}u_\alpha^{(3)} + C_{13}\dot{u}_\alpha^{(3)}\right) + \frac{e_1^2\Delta_1}{2m_1c^2}\left(A_{(1)ret}\xi_\alpha^{(1)ret} - B_{(1)ret}u_\alpha^{(1)} + \right. \\ &\quad \left. + C_{(1)ret}u_\alpha^{(1)ret} - A_{(1)adv}\xi_\alpha^{(1)adv} + B_{(1)adv}u_\alpha^{(1)} - C_{(1)adv}u_\alpha^{(1)adv}\right);\end{aligned}$$

$$\begin{aligned}\dot{u}_\alpha^{(2)}(t) &+ \frac{u_\alpha^{(2)}(t)}{\Delta_2^2}\left\langle \vec{u}^{(2)}(t), \dot{\vec{u}}^{(2)}(t)\right\rangle = \frac{e_2e_1\Delta_2}{m_2c^2}\left(A_{21}\xi_\alpha^{(21)} - B_{21}u_\alpha^{(1)} + C_{21}\dot{u}_\alpha^{(1)}\right) \\ &+ \frac{e_2e_3\Delta_2}{m_2c^2}\left(A_{23}\xi_\alpha^{(23)} - B_{23}u_\alpha^{(3)} + C_{23}\dot{u}_\alpha^{(3)}\right) + \frac{e_2^2\Delta_2}{2m_2c^2}\left(A_{(2)ret}\xi_\alpha^{(2)ret} - B_{(2)ret}u_\alpha^{(2)} + \right. \\ &\left. + C_{(2)ret}u_\alpha^{(2)ret} - A_{(2)adv}\xi_\alpha^{(2)adv} + B_{(2)adv}u_\alpha^{(2)} - C_{(2)adv}u_\alpha^{(2)adv}\right)\end{aligned}$$

$$\begin{aligned}\dot{u}_\alpha^{(3)}(t) &+ \frac{u_\alpha^{(3)}(t)}{\Delta_3^2}\left\langle \vec{u}^{(3)}(t), \dot{\vec{u}}^{(3)}(t)\right\rangle = \frac{e_3e_1\Delta_3}{m_3c^2}\left(A_{31}\xi_\alpha^{(31)} - B_{31}u_\alpha^{(1)} + C_{31}\dot{u}_\alpha^{(1)}\right) + \\ &+ \frac{e_3e_2\Delta_3}{m_3c^2}\left(A_{32}\xi_\alpha^{(32)} - B_{32}u_\alpha^{(2)} + C_{32}\dot{u}_\alpha^{(2)}\right) + \frac{e_3^2\Delta_3}{2m_3c^2}\left(A_{(3)ret}\xi_\alpha^{(3)ret} - B_{(3)ret}u_\alpha^{(3)} + \right. \\ &\left. + C_{(3)ret}u_\alpha^{(3)ret} - -A_{(3)adv}\xi_\alpha^{(3)adv} + B_{(3)adv}u_\alpha^{(3)} - C_{(3)adv}u_\alpha^{(3)adv}\right).\end{aligned}$$

Introduce denotations $(k = 1, 2, 3\,;\ \alpha = 1, 2, 3)$for the Lorentz forces

$$G_\alpha^{(12)} = \frac{e_1e_2\Delta_1}{m_1c^2}\left(A_{12}\xi_\alpha^{(12)} - B_{12}u_\alpha^{(2)} + C_{12}\dot{u}_\alpha^{(2)}\right)\ ;\ G_\alpha^{(13)} = \frac{e_1e_3\Delta_1}{m_1c^2}\left(A_{13}\xi_\alpha^{(13)} - B_{13}u_\alpha^{(3)} + C_{13}\dot{u}_\alpha^{(3)}\right)\ ;$$

$$G_\alpha^{(21)} = \frac{e_2e_1\Delta_2}{m_2c^2}\left(A_{21}\xi_\alpha^{(21)} - B_{21}u_\alpha^{(1)} + C_{21}\dot{u}_\alpha^{(1)}\right)\ ;\ G_\alpha^{(23)} = \frac{e_2e_3\Delta_2}{m_2c^2}\left(A_{23}\xi_\alpha^{(23)} - B_{23}u_\alpha^{(3)} + C_{23}\dot{u}_\alpha^{(3)}\right)\ ;$$

$$G_\alpha^{(31)} = \frac{e_3e_1\Delta_3}{m_3c^2}\left(A_{31}\xi_\alpha^{(31)} - B_{31}u_\alpha^{(1)} + C_{31}\dot{u}_\alpha^{(1)}\right)\ ;\ G_\alpha^{(32)} = \frac{e_3e_2\Delta_3}{m_3c^2}\left(A_{32}\xi_\alpha^{(32)} - B_{32}u_\alpha^{(2)} + C_{32}\dot{u}_\alpha^{(2)}\right);$$

$$\begin{aligned}G_\alpha^{(k)rad} &= \frac{e_k^2\Delta_k}{2m_kc^2}\left(A_{(k)ret}\xi_\alpha^{(k)ret} - B_{(k)ret}u_\alpha^{(k)} + C_{(k)ret}u_\alpha^{(k)ret} - A_{(k)adv}\xi_\alpha^{(k)adv}\right. \\ &\quad \left. + B_{(k)adv}u_\alpha^{(k)} - C_{(k)adv}u_\alpha^{(k)adv}\right).\end{aligned}$$

We write down the last system in the form

$$\dot{u}_\alpha^{(1)}(t) + \frac{u_\alpha^{(1)}(t)}{\Delta_1^2}\left\langle \vec{u}^{(1)}(t), \dot{\vec{u}}^{(1)}(t)\right\rangle = G_\alpha^{(12)} + G_\alpha^{(13)} + G_\alpha^{(1)rad};$$

$$\dot{u}_\alpha^{(2)}(t) + \frac{u_\alpha^{(2)}(t)}{\Delta_2^2}\left\langle \vec{u}^{(2)}(t), \dot{\vec{u}}^{(2)}(t)\right\rangle = G_\alpha^{(21)} + G_\alpha^{(23)} + G_\alpha^{(2)rad};$$

$$\dot{u}_\alpha^{(3)}(t) + \frac{u_\alpha^{(3)}(t)}{\Delta_3^2}\left\langle \vec{u}^{(3)}(t), \dot{\vec{u}}^{(3)}(t)\right\rangle = G_\alpha^{(31)} + G_\alpha^{(32)} + G_\alpha^{(3)rad}.$$

Recall our basic **Assumption (C):** All velocities satisfy the inequalities $\left|u_\alpha{}^{(p)}(t)\right| \le \sqrt{\left\langle u^{(p)}, u^{(p)}\right\rangle} \le \bar{c} < c$ and then $c^2 - \left\langle u^{(p)}, u^{(p)}\right\rangle \ge c^2 - \bar{c}^2 > 0$. Therefore, the determinant of the above system is $\delta = c^2/\Delta_k^2 > 0$ and consequently we can solve the last system with respect to $\dot{u}_\alpha^{(k)}(t)$, $(k = 1,2,3;\ \alpha = 1,2,3)$:

$$
\begin{aligned}
\dot{u}_1^{(1)}(t) &= U_1^{(1)} \equiv \frac{c^2 - \left(u_1^{(1)}\right)^2}{c^2}\left(G_1^{(12)} + G_1^{(13)} + G_1^{(1)rad}\right) - \frac{u_1^{(1)}u_2^{(1)}}{c^2}\left(G_2^{(12)} + G_2^{(13)} + G_2^{(2)rad}\right) \\
&\quad - \frac{u_1^{(1)}u_3^{(1)}}{c^2}\left(G_3^{(12)} + G_3^{(13)} + G_3^{(3)rad}\right); \\
\dot{u}_2^{(1)}(t) &= U_2^{(1)} \equiv -\frac{u_1^{(1)}u_2^{(1)}}{c^2}\left(G_1^{(12)} + G_1^{(13)} + G_1^{(1)rad}\right) + \frac{c^2 - (u_2^{(1)})^2}{c^2}\left(G_2^{(12)} + G_2^{(13)} + G_2^{(2)rad}\right) - \\
&\quad - \frac{u_2^{(1)}u_3^{(1)}}{c^2}\left(G_3^{(12)} + G_3^{(13)} + G_3^{(3)rad}\right); \\
\dot{u}_3^{(1)}(t) &= U_3^{(1)} \equiv -\frac{u_1^{(1)}u_3^{(1)}}{c^2}\left(G_1^{(12)} + G_1^{(13)} + G_1^{(1)rad}\right) - \frac{u_2^{(1)}u_3^{(1)}}{c^2}\left(G_2^{(12)} + G_2^{(13)} + G_2^{(2)rad}\right) + \\
&\quad + \frac{c^2 - (u_3^{(1)})^2}{c^2}\left(G_3^{(12)} + G_3^{(13)} + G_3^{(3)rad}\right); \\
\dot{u}_1^{(2)}(t) &= U_1^{(2)} \equiv \frac{c^2 - \left(u_1^{(2)}\right)^2}{c^2}\left(G_1^{(21)} + G_1^{(23)} + G_1^{(2)rad}\right) - \frac{u_1^{(2)}u_2^{(2)}}{c^2}\left(G_2^{(21)} + G_2^{(23)} + G_2^{(2)rad}\right) - \\
&\quad - \frac{u_1^{(2)}u_3^{(2)}}{c^2}\left(G_3^{(21)} + G_3^{(23)} + G_3^{(2)rad}\right); \\
\dot{u}_2^{(2)}(t) &= U_2^{2} \equiv -\frac{u_1^{(2)}u_2^{(2)}}{c^2}\left(G_1^{(21)} + G_1^{(23)} + G_1^{(2)rad}\right) + \frac{c^2 - (u_2^{(2)})^2}{c^2}\left(G_2^{(21)} + G_2^{(23)} + G_2^{(2)rad}\right) - \\
&\quad - \frac{u_2^{(2)}u_3^{(2)}}{c^2}\left(G_3^{(21)} + G_3^{(23)} + G_3^{(2)rad}\right); \\
\dot{u}_3^{(2)}(t) &= U_3^{2} \equiv -\frac{u_1^{(2)}u_3^{(2)}}{c^2}\left(G_1^{(21)} + G_1^{(23)} + G_1^{(2)rad}\right) - \frac{u_2^{(2)}u_3^{(2)}}{c^2}\left(G_2^{(21)} + G_2^{(23)} + G_2^{(2)rad}\right) + \\
&\quad + \frac{c^2 - (u_3^{(2)})^2}{c^2}\left(G_3^{(21)} + G_3^{(23)} + G_3^{(2)rad}\right); \\
\dot{u}_1^{(3)}(t) &= U_1^{3} \equiv \frac{c^2 - \left(u_1^{(3)}\right)^2}{c^2}\left(G_1^{(31)} + G_1^{(32)} + G_1^{(3)rad}\right) - \frac{u_1^{(3)}u_2^{(3)}}{c^2}\left(G_2^{(31)} + G_2^{(32)} + G_2^{(3)rad}\right) - \\
&\quad - \frac{u_1^{(3)}u_3^{(3)}}{c^2}\left(G_3^{(31)} + G_3^{(32)} + G_3^{(3)rad}\right); \\
\dot{u}_2^{(3)}(t) &= U_2^{3} \equiv -\frac{u_1^{(3)}u_2^{(3)}}{c^2}\left(G_1^{(31)} + G_1^{(32)} + G_1^{(3)rad}\right) + \frac{c^2 - (u_2^{(3)})^2}{c^2}\left(G_2^{(31)} + G_2^{(32)} + G_2^{(3)rad}\right) - \\
&\quad - \frac{u_2^{(3)}u_3^{(3)}}{c^2}\left(G_3^{(31)} + G_3^{(32)} + G_3^{(3)rad}\right); \\
\dot{u}_3^{(3)}(t) &= U_3^{3} \equiv -\frac{u_1^{(3)}u_3^{(3)}}{c^2}\left(G_1^{(31)} + G_1^{(32)} + G_1^{(3)rad}\right) - \frac{u_2^{(3)}u_3^{(3)}}{c^2}\left(G_2^{(31)} + G_2^{(32)} + G_2^{(3)rad}\right) + \\
&\quad + \frac{c^2 - (u_3^{(3)})^2}{c^2}\left(G_3^{(31)} + G_3^{(32)} + G_3^{(3)rad}\right),\ t \ge 0.
\end{aligned}
\tag{4.1}
$$

5. A Transformation of the Radiation Terms under Dirac Assumption

Here we follow the Dirac assumption $\tau_k^{ret} = \tau_k^{adv} = \tau$, τ is a small parameter. This assumption is motivated by the fact $\tau = \tau_0\sqrt{1-\beta^2}$, $\left(\tau_0 = r_e/c \approx 10^{-24}\,\text{sec}\right)$. Since $u_\alpha^{(k)}(t)$ are infinitely smooth functions, using the Taylor expansions we recall the reasoning from [7],[14]:

$$\xi_\alpha^{(k)adv} = x_\alpha^{(k)}(t+\tau) - x_\alpha^{(k)}(t) = \tau\, u_\alpha^{(k)}(t) + \tfrac{\tau^2}{2!}\dot{u}_\alpha^{(k)}(t) + \ldots \Rightarrow\ \xi_\alpha^{(k)adv} = \tau\, u_\alpha^{(k)}(t) + O(\tau^2)$$

$$\Rightarrow\ \xi_\alpha^{(k)adv} \approx \tau\, u_\alpha^{(k)}(t);\ \xi_\alpha^{(k)ret} = x_\alpha^{(k)}(t) - x_\alpha^{(k)}(t-\tau) \approx u_\alpha^{(k)}(t)\tau;$$

$$u_\alpha^{(k)}(t+\tau) = u_\alpha^{(k)}(t) + \tfrac{\tau}{1!}\dot{u}_\alpha^{(k)}(t) + \tfrac{\tau^2}{2!}\ddot{u}_\alpha^{(k)}(t) + \tfrac{\tau^3}{3!}\dddot{u}_\alpha^{(k)}(t) + \tfrac{\tau^4}{4!}\left(u_\alpha^{(k)}(t)\right)^{IV} + \ldots$$

$$\Rightarrow\ u_\alpha^{(k)}(t+\tau) = u_\alpha^{(k)}(t) + O(\tau);$$

$$u_\alpha^{(k)}(t-\tau) = u_\alpha^{(k)}(t) - \tfrac{\tau}{1!}\dot{u}_\alpha^{(k)}(t) + \tfrac{\tau^2}{2!}\ddot{u}_\alpha^{(k)}(t) - \tfrac{\tau^3}{3!}\dddot{u}_\alpha^{(k)}(t) + \tfrac{\tau^4}{4!}\left(u_\alpha^{(k)}(t)\right)^{IV} - \ldots$$

$$\Rightarrow\ u_\alpha^{(k)}(t-\tau) = u_\alpha^{(k)}(t) - O(\tau);$$

$$u_\alpha^{(k)}(t)u_\alpha^{(k)}(t+\tau) = \left(u_\alpha^{(k)}(t)\right)^2 + \tfrac{\tau^2}{1!}\dot{u}_\alpha^{(k)}(t)u_\alpha^{(k)}(t) + \tfrac{\tau^2}{2!}\ddot{u}_\alpha^{(k)}(t)u_\alpha^{(k)}(t) + \ldots$$

$$\Rightarrow\ u_\alpha^{(k)}(t)u_\alpha^{(k)}(t+\tau) = \left(u_\alpha^{(k)}(t)\right)^2 + O(\tau);$$

$$u_\alpha^{(k)}(t)u_\alpha^{(k)}(t-\tau) = \left(u_\alpha^{(k)}(t)\right)^2 - \tfrac{\tau}{1!}\dot{u}_\alpha^{(k)}(t)u_\alpha^{(k)}(t) + \tfrac{\tau^2}{2!}\ddot{u}_\alpha^{(k)}(t)u_\alpha^{(k)}(t) - \ldots$$

$$\Rightarrow\ u_\alpha^{(k)}(t)u_\alpha^{(k)}(t-\tau) = \left(u_\alpha^{(k)}(t)\right)^2 - O(\tau);$$

$$\left\langle \vec{u}^{(k)}, \vec{u}^{(k)adv}\right\rangle = \left\langle \vec{u}^{(k)}, \vec{u}^{(k)}(t+\tau)\right\rangle = \sum_{\gamma=1}^{3} u_\gamma^{(k)}(t)u_\gamma^{(k)}(t+\tau) \approx \sum_{\gamma=1}^{3} u_\gamma^{(k)}(t)u_\gamma^{(k)}(t) = \left\langle \vec{u}^{(k)}, \vec{u}^{(k)}\right\rangle;$$

$$\left\langle \vec{u}^{(k)}, \vec{u}^{(k)ret}\right\rangle \approx \left\langle \vec{u}^{(k)}, \vec{u}^{(k)}\right\rangle;\ \left\langle \vec{u}^{(k)adv}, \vec{u}^{(k)adv}\right\rangle \approx \left\langle \vec{u}^{(k)}, \vec{u}^{(k)adv}\right\rangle;\ \left\langle \vec{u}^{(k)ret}, \vec{u}^{(k)ret}\right\rangle \approx \left\langle \vec{u}^{(k)}, \vec{u}^{(k)ret}\right\rangle;$$

$$c^2\tau_k^{ret} - \left\langle \vec{\xi}^{(k)ret}, \vec{u}^{(k)ret}\right\rangle = c^2\tau - \tau\left\langle \vec{u}^{(k)}(t), \vec{u}^{(k)}(t-\tau)\right\rangle \approx \tau\left(c^2 - \left\langle \vec{u}^{(k)}(t), \vec{u}^{(k)}(t)\right\rangle\right);$$

$$c^2\tau_k^{adv} - \left\langle \vec{\xi}^{(k)adv}, \vec{u}^{(k)adv}\right\rangle = c^2\tau - \tau\left\langle \vec{u}^{(k)}(t), \vec{u}^{(k)}(t+\tau)\right\rangle \approx \tau\left(c^2 - \left\langle \vec{u}^{(k)}(t), \vec{u}^{(k)}(t)\right\rangle\right).$$

Then

$$\begin{aligned} G_\alpha^{(k)rad} &= \frac{e_k^2\Delta_k}{2m_kc^2}\Bigg[\Bigg(\frac{\Delta^2_{(k)ret}\left(c^2-\left\langle \vec{u}^{(k)},\vec{u}^{(k)ret}\right\rangle\right)}{\left(c^2\tau-\left\langle \vec{\xi}^{(k)ret},\vec{u}^{(k)ret}\right\rangle\right)^3} + \\ &+\frac{\left(c^2-\left\langle \vec{u}^{(k)},\vec{u}^{(k)ret}\right\rangle\right)\langle \vec{\xi}^{(k)ret},\dot{\vec{u}}^{(k)ret}\rangle - \left(c^2\tau-\left\langle \vec{\xi}^{(k)ret},\vec{u}^{(k)ret}\right\rangle\right)\left\langle \vec{u}^{(k)},\dot{\vec{u}}^{(k)ret}\right\rangle}{\left(c^2\tau-\left\langle \vec{\xi}^{(k)ret},\vec{u}^{(k)}\right\rangle\right)\left(c^2\tau-\left\langle \vec{\xi}^{(k)ret},\vec{u}^{(k)ret}\right\rangle\right)^2}\Bigg)\xi_\alpha^{(k)ret} - \\ &-\Bigg(\frac{\Delta^2_{(k)adv}\left(c^2-\langle \vec{u}^{(k)},\vec{u}^{(k)adv}\rangle\right)}{\left(c^2\tau-\langle \vec{\xi}^{(k)adv},\vec{u}^{(k)adv}\rangle\right)^3} + \frac{\left(c^2-\langle \vec{u}^{(k)},\vec{u}^{(k)adv}\rangle\right)\langle \vec{\xi}^{(k)adv},\dot{\vec{u}}^{(k)adv}\rangle - \left(c^2\tau-\langle \vec{\xi}^{(k)adv},\vec{u}^{(k)adv}\rangle\right)\left\langle \vec{u}^{(k)},\dot{\vec{u}}^{(k)adv}\right\rangle}{\left(c^2\tau-\langle \vec{\xi}^{(k)adv},\vec{u}^{(k)}\rangle\right)\left(c^2\tau-\langle \vec{\xi}^{(k)adv},\vec{u}^{(k)adv}\rangle\right)^2}\Bigg)\xi_\alpha^{(k)adv} + \\ &+\Bigg(\frac{\Delta^2_{(k)adv}\left(c^2\tau-\langle \vec{u}^{(k)},\vec{\xi}^{(k)adv}\rangle\right) - \left(c^2\tau-\left\langle \vec{\xi}^{(k)adv},\vec{u}^{(k)adv}\right\rangle\right)\left\langle \vec{\xi}^{(k)adv},\dot{\vec{u}}^{(k)adv}\right\rangle}{\left(c^2\tau-\langle \vec{u}^{(k)adv},\vec{\xi}^{(k)adv}\rangle\right)^3}\Bigg)u_\alpha^{(k)adv} - \\ &-\Bigg(\frac{\Delta^2_{(k)ret}\left(c^2\tau-\left\langle \vec{\xi}^{(k)ret},\vec{u}^{(k)}\right\rangle\right) - \left(c^2\tau-\left\langle \vec{\xi}^{(k)ret},\vec{u}^{(k)ret}\right\rangle\right)\left\langle \vec{\xi}^{(k)ret},\dot{\vec{u}}^{(k)ret}\right\rangle}{\left(c^2\tau-\left\langle \vec{\xi}^{(k)ret},\vec{u}^{(k)ret}\right\rangle\right)^3}\Bigg)u_\alpha^{(k)ret} - \\ &-\frac{\dot{u}_\alpha^{(k)adv}}{c^2\tau-\left\langle \vec{\xi}^{(k)adv},\vec{u}^{(k)adv}\right\rangle} + \frac{\dot{u}_\alpha^{(k)ret}}{c^2\tau-\left\langle \vec{\xi}^{(k)ret},\vec{u}^{(k)ret}\right\rangle}\Bigg] = \end{aligned}$$

$$= \frac{e_k^2\Delta_k}{2m_kc^2}\left[\left(\frac{\Delta^2_{(k)ret}\left(c^2-\left\langle\vec{u}^{(k)},\vec{u}^{(k)ret}\right\rangle\right)}{\tau^3\left(c^2-\left\langle\vec{u}^{(k)},\vec{u}^{(k)ret}\right\rangle\right)^3}+\frac{\left(c^2-\left\langle\vec{u}^{(k)},\vec{u}^{(k)ret}\right\rangle\right)\tau\langle\vec{u}^{(k)},\dot{\vec{u}}^{(k)ret}\rangle-\tau\left(c^2-\left\langle\vec{u}^{(k)},\vec{u}^{(k)ret}\right\rangle\right)\left\langle\vec{u}^{(k)},\dot{\vec{u}}^{(k)ret}\right\rangle}{\tau^3\left(c^2-\left\langle\vec{u}^{(k)},\vec{u}^{(k)}\right\rangle\right)\left(c^2-\left\langle\vec{u}^{(k)},\vec{u}^{(k)ret}\right\rangle\right)^2}\right)\tau\, u_\alpha^{(k)}(t)-\right.$$

$$-\left(\frac{\Delta^2_{(k)a}\left(c^2-\left\langle\vec{u}^{(k)},\vec{u}^{(k)adv}\right\rangle\right)}{\tau^3\left(c^2-\left\langle\vec{u}^{(k)},\vec{u}^{(k)adv}\right\rangle\right)^3}+\frac{\left(c^2-\left\langle\vec{u}^{(k)},\vec{u}^{(k)adv}\right\rangle\right)\tau\langle u^{(k)},\dot{u}^{(k)adv}\rangle-\tau\left(c^2-\left\langle\vec{u}^{(k)},\vec{u}^{(k)adv}\right\rangle\right)\left\langle\vec{u}^{(k)},\dot{\vec{u}}^{(k)adv}\right\rangle}{\tau^3\left(c^2-\left\langle\vec{u}^{(k)},\vec{u}^{(k)}\right\rangle\right)\left(c^2-\left\langle\vec{u}^{(k)},\vec{u}^{(k)adv}\right\rangle\right)^2}\right)\tau\, u_\alpha^{(k)}(t)-$$

$$+\left(\frac{\Delta^2_{(k)adv}\left(c^2\tau-\tau\langle\vec{u}^{(k)},\vec{u}^{(k)}\rangle\right)-\left(c^2\tau-\tau\left\langle\vec{u}^{(k)},\vec{u}^{(k)adv}\right\rangle\right)\tau\left\langle\vec{u}^{(k)},\dot{\vec{u}}^{(k)adv}\right\rangle}{\tau^3\left(c^2-\langle\vec{u}^{(k)adv},\vec{u}^{(k)}\rangle\right)^3}\right)u_\alpha^{(k)adv}-$$

$$-\left(\frac{\Delta^2_{(k)ret}\left(c^2\tau-\tau\left\langle\vec{u}^{(k)},\vec{u}^{(k)}\right\rangle\right)-\left(c^2\tau-\tau\left\langle\vec{u}^{(k)},\vec{u}^{(k)ret}\right\rangle\right)\tau\left\langle\vec{u}^{(k)},\dot{\vec{u}}^{(k)ret}\right\rangle}{\tau^3\left(c^2-\left\langle\vec{u}^{(k)},\vec{u}^{(k)ret}\right\rangle\right)^3}\right)u_\alpha^{(k)ret}-$$

$$\left.-\frac{\dot{u}_\alpha^{(k)adv}}{c^2\tau-\tau\left\langle\vec{u}^{(k)},\vec{u}^{(k)adv}\right\rangle}+\frac{\dot{u}_\alpha^{(k)ret}}{c^2\tau-\tau\left\langle\vec{u}^{(k)},\vec{u}^{(k)ret}\right\rangle}\right]=$$

$$=\frac{e_k^2\Delta_k}{2m_kc^2}\left[\left(\frac{\Delta^2_{(k)ret}}{\left(c^2-\left\langle\vec{u}^{(k)},\vec{u}^{(k)ret}\right\rangle\right)^2}-\frac{\Delta^2_{(k)adv}}{\left(c^2-\left\langle\vec{u}^{(k)},\vec{u}^{(k)adv}\right\rangle\right)^2}\right)\frac{u_\alpha^{(k)}}{\tau^2}+\frac{\Delta^2_{(k)a}-\tau\left\langle\vec{u}^{(k)},\dot{\vec{u}}^{(k)adv}\right\rangle}{\left(c^2-\left\langle\vec{u}^{(k)},\vec{u}^{(k)}\right\rangle\right)^2}\frac{u_\alpha^{(k)adv}}{\tau^2}-\right.$$

$$\left.-\frac{\Delta^2_{(k)ret}-\tau\left\langle\vec{u}^{(k)},\dot{\vec{u}}^{(k)ret}\right\rangle}{\left(c^2-\left\langle\vec{u}^{(k)},\vec{u}^{(k)}\right\rangle\right)^2}\frac{u_\alpha^{(k)ret}}{\tau^2}-\frac{\dot{u}_\alpha^{(k)adv}-\dot{u}_\alpha^{(k)ret}}{\tau\left(c^2-\left\langle\vec{u}^{(k)},\vec{u}^{(k)}\right\rangle\right)}\right]=$$

$$\approx\frac{e_k^2\Delta_k}{2m_kc^2}\left[\frac{\Delta^2_{(k)ret}-\Delta^2_{(k)adv}}{\left(c^2-\left\langle\vec{u}^{(k)},\vec{u}^{(k)}\right\rangle\right)^2}\frac{u_\alpha^{(k)}}{\tau^2}+\frac{\Delta^2_{(k)adv}-\Delta^2_{(k)ret}}{\left(c^2-\left\langle\vec{u}^{(k)},\vec{u}^{(k)}\right\rangle\right)^2}\frac{u_\alpha^{(k)}}{\tau^2}-\frac{\left\langle\vec{u}^{(k)},\dot{\vec{u}}^{(k)adv}\right\rangle-\left\langle u^{(k)},\dot{u}^{(k)ret}\right\rangle}{\tau\left(c^2-\left\langle\vec{u}^{(k)},\vec{u}^{(k)}\right\rangle\right)^2}u_\alpha^{(k)}-\right.$$

$$\left.-\frac{\dot{u}_\alpha^{(k)adv}-\dot{u}_\alpha^{(k)ret}}{\tau\left(c^2-\left\langle\vec{u}^{(k)},\vec{u}^{(k)}\right\rangle\right)}\right]=$$

$$=\frac{e_k^2\Delta_k}{m_kc^2}\left(-\frac{u_\alpha^{(k)}}{\left(c^2-\left\langle\vec{u}^{(k)},\vec{u}^{(k)}\right\rangle\right)^2}\left\langle\vec{u}^{(k)},\frac{\dot{\vec{u}}^{(k)}(t+\tau)-\dot{\vec{u}}^{(k)}(t-\tau)}{2\tau}\right\rangle-\frac{1}{c^2-\left\langle\vec{u}^{(k)},\vec{u}^{(k)}\right\rangle}\frac{\dot{u}_\alpha^{(k)}(t+\tau)-\dot{u}_\alpha^{(k)}(t-\tau)}{2\tau}\right).$$

In explicit form the radiation term becomes

$$G_\alpha^{(k)rad}=-\frac{e_k^2\Delta_k}{m_kc^2}\left(\frac{u_\alpha^{(k)}(t)}{\left(c^2-\langle\vec{u}^{(k)}(t),\vec{u}^{(k)}(t)\rangle\right)^2}\left\langle\vec{u}^{(k)}(t),\frac{\dot{\vec{u}}^{(k)}(t+\tau)-\dot{\vec{u}}^{(k)}(t-\tau)}{2\tau}\right\rangle+\right.$$

$$\left.+\frac{\dot{u}_\alpha^{(k)}(t+\tau)-\dot{u}_\alpha^{(k)}(t-\tau)}{2\tau}\frac{1}{\left(c^2-\langle\vec{u}^{(k)}(t),\vec{u}^{(k)}(t)\rangle\right)}\right)$$

or

$$G_\alpha^{(k)rad}=-\frac{e_k^2}{m_kc^2\Delta_k}\left(\frac{u_\alpha^{(k)}(t)}{{\Delta_k}^2}\sum_{\gamma=1}^{3}{u_\gamma}^{(k)}(t){\ddot{u}_\gamma}^{(k)}(t)+{\ddot{u}_\alpha}^{(k)}(t)\right).$$

6. Conclusion

We have derived the system (4.1) of equations of motion describing the 3-body electrodynamics problem. The radiation terms are obtained in Section 4. It turns out to be a nonlinear neutral system with delays depending on unknown trajectories. The difficulties to investigate such systems we overcome in the next paper by means of fixed point method. We define an operator whose fixed points are a periodic solution of the above problem.

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