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AN APPROACH TO PRE-SEPARATION AXIOMS IN NEUTROSOPHIC SOFT TOPOLOGICAL SPACES

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Abstract. In this study, we introduce the concept of neutrosophic soft pre-open (neutrosophic soft pre-closed) sets and pre-separation axioms in neutrosophic soft topological spaces. In particular, the relationship between these separation axioms are investigated. Also, we give a new definition for neutrosophic soft topological subspace and define neutrosophic soft pre irresolute soft and neutrosophic pre irresolute open soft functions.

1. Introduction

In 2005, Smarandache introduced the concept of a neutrosophic set [20] as a generalization of classical sets, fuzzy set theory [20] (see also [10]) , intuitionistic fuzzy set theory [4] (see also [14]) etc. By using this theory of neutrosophic set, some scientists made researches in many areas of mathematics [7, 18]. Many inherent difficulties exist in classical methods for the inadequacy of the theories of parametrization tools. So, classical methods are insufficient in dealing with several practical problems in some other disciplines such as economics, engineering, environment, social science, medical science, etc. In 1999, Molodtsov [16] pointed out the inherent difficulties of these theories. A different approach was initiated by Molodtsov for modeling uncertainties. This approach was applied in some other directions such as smoothness of functions, game theory, operations research, Riemann integration, Perron integration and so on. The theory of soft topological spaces was introduced by Shabir and Naz [19] for the first time in 2011. Soft topological spaces were defined over an initial universe with a fixed set of parameters and showed that a soft topological space gives a parameterized family of topological

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spaces. In [1, 2, 5, 6, 9, 11, 13], some scientists made researches and did theoretical studies in soft topological spaces. In 2013, Maji [15] defined the concept of neutrosophic soft sets for the first time. Then, Deli and Broumi [12] modified this concept. In 2017, Bera presented neutrosophic soft topological spaces in [8].

In this study, our pupose is to adapt the concepts of neutrosophic pre open soft set, neutrosophic pre closed soft set to neutrosophic soft topological spaces. Then, we define neutrosophic soft pre interior point, neutrosophic soft pre cluster point, neutrosophic soft pre interior operator and neutrosophic soft pre closure operator. By using these definitions and concepts, the concept of pre-separation axioms of neutrosophic soft topological spaces is introduced. Furthermore, we analyze properties of neutrosophic soft pre T_i -spaces ($i = 0, 1, 2, 3, 4$) and focus on some relations between them. Characterization theorems of them are also proved. We hope that, the findings in this document will help scientists to enhance and promote the further studies on neutrosophic soft topology to carry out a general framework for their applications in practical life.

2. Preliminaries

In this section, we present the basic definitions and theorems related to neutrosophic soft set theory.

Definition 1. *[20] A neutrosophic set A on the universe set X is defined as:*
$A = \{\langle x, T_A(x), I_A(x), F_A x\rangle : x \in X\}$,
where
T,I, $F : X \to]^-0, 1^+[$ *and* $^-0 \leq T_A(x) + I_A(x) + F_A(x) \leq 3^+$.

Definition 2. *[16] Let X be an initial universe, E be a set of all parameters, and $P(X)$ denote the power set of X. A pair (F, E) is called a soft set over X, where F is a mapping given by $F : E \to P(X)$. In other words, the soft set is a parameterized family of subsets of the set X. For $e \in E$, $F(e)$ may be considered as the set of e-elements of the soft set (F, E), or as the set of e-approximate elements of the soft set, i.e.*
$(F, E) = \{(e, F(e)) : e \in E,\ F : E \to P(X)\}$.

After the neutrosophic soft set was defined by Maji [15], this concept was modified by Deli and Broumi [12] as given below:

Definition 3. *[12] Let X be an initial universe set and E be a set of parameters. Let $P(X)$ denote the set of all neutrosophic sets of X. Then a neutrosophic soft set $\left(\widetilde{F}, E\right)$ over X is a set defined by a set valued function $\widetilde{F}$ representing a mapping $\widetilde{F} : E \to P(X)$, where $\widetilde{F}$ is called the approximate function of the neutrosophic soft set $\left(\widetilde{F}, E\right)$. In other words, the neutrosophic soft set is a parametrized family of some elements of the set $P(X)$ and therefore it can be written as a set of ordered pairs:*

$\left(\widetilde{F}, E\right) = \left\{\left(e,\ \left\langle x,\ T_{\widetilde{F}(e)}(x), I_{\widetilde{F}(e)}(x), F_{\widetilde{F}(e)}(x)\right\rangle : x \in X\right) : e \in E\right\}$

where $T_{\widetilde{F}(e)}(x)$, $I_{\widetilde{F}(e)}(x)$, $F_{\widetilde{F}(e)}(x) \in [0,1]$ *are respectively called the truth-membership,*
indeterminacy-membership and falsity-membership function of $\widetilde{F}(e)$. *Since the supremum of each* T, I, F *is* 1, *the inequality*

$0 \leq T_A(x) + I_A(x) + F_A(x) \leq 3$

is obvious.

Definition 4. *[8] Let* $\left(\widetilde{F}, E\right)$ *be a neutrosophic soft set over the universe set* X. *The complement of* $\left(\widetilde{F}, E\right)$ *is denoted by* $\left(\widetilde{F}, E\right)^c$ *and is defined by:*

$\left(\widetilde{F}, E\right)^c = \left\{\left(e,\ \left\langle x,\ F_{\widetilde{F}(e)}(x),\ 1 - I_{\widetilde{F}(e)}(x),\ T_{\widetilde{F}(e)}(x)\right\rangle : x \in X\right) : e \in E\right\}.$

It is obvious that $\left[\left(\widetilde{F}, E\right)^c\right]^c = \left(\widetilde{F}, E\right)$.

Definition 5. *[15] Let* $\left(\widetilde{F}, E\right)$ *and* $\left(\widetilde{G}, E\right)$ *be two neutrosophic soft sets over the universe set* X. $\left(\widetilde{F}, E\right)$ *is said to be a neutrosophic soft subset of* $\left(\widetilde{G}, E\right)$ *if*

$T_{\widetilde{F}(e)}(x) \leq T_{\widetilde{G}(e)}(x)$, $I_{\widetilde{F}(e)}(x) \leq I(x)$, $F_{\widetilde{F}(e)}(x) \geq F_{\widetilde{G}(e)}(x)$, $\forall e \in E$, $\forall x \in X$.

It is denoted by $\left(\widetilde{F}, E\right) \subseteq \left(\widetilde{G}, E\right)$. $\left(\widetilde{F}, E\right)$ *is said to be neutrosophic soft equal to* $\left(\widetilde{G}, E\right)$ *if* $\left(\widetilde{F}, E\right) \subseteq \left(\widetilde{G}, E\right)$ *and* $\left(\widetilde{G}, E\right) \subseteq \left(\widetilde{F}, E\right)$. *It is denoted by* $\left(\widetilde{F}, E\right) = \left(\widetilde{G}, E\right)$.

Definition 6. *[3] Let* $\left(\widetilde{F}_1, E\right)$ *and* $\left(\widetilde{F}_2, E\right)$ *be two neutrosophic soft sets over the universe set* X. *Then their union is denoted by* $\left(\widetilde{F}_1, E\right) \cup \left(\widetilde{F}_2, E\right) = \left(\widetilde{F}_3, E\right)$ *and is defined by:*

$\left(\widetilde{F}_3, E\right) = \left\{\left(e,\ \left\langle x,\ T_{\widetilde{F}_3(e)}(x), I(x), F_{\widetilde{F}_3(e)}(x)\right\rangle : x \in X\right) : e \in E\right\},$

where

$T_{\widetilde{F}_3(e)}(x) = max\left\{T_{\widetilde{F}_1(e)}(x),\ T_{\widetilde{F}_2(e)}(x)\right\},$

$I_{\widetilde{F}_3(e)}(x) = max\left\{I_{\widetilde{F}_1(e)}(x),\ I_{\widetilde{F}_2(e)}(x)\right\},$

$F_{\widetilde{F}_3(e)}(x) = min\left\{F_{\widetilde{F}_1(e)}(x),\ F_{\widetilde{F}_2(e)}(x)\right\}.$

Definition 7. *[3] Let* $\left(\widetilde{F}_1, E\right)$ *and* $\left(\widetilde{F}_2, E\right)$ *be two neutrosophic soft sets over the universe set* X. *Then their* intersection *is denoted by* $\left(\widetilde{F}_1, E\right) \cap \left(\widetilde{F}_2, E\right) = \left(\widetilde{F}_3, E\right)$ *and is defined by:*

$\left(\widetilde{F}_3, E\right) = \left\{\left(e,\ \left\langle x,\ T_{\widetilde{F}_3(e)}(x), I(x), F_{\widetilde{F}_3(e)}(x)\right\rangle : x \in X\right) : e \in E\right\},$

where

$$T_{\widetilde{F}_3(e)}(x) = min\left\{T_{\widetilde{F}_1(e)}(x), T_{\widetilde{F}_2(e)}(x)\right\},$$

$$I_{\widetilde{F}_3(e)}(x) = min\left\{I_{\widetilde{F}_1(e)}(x), I_{\widetilde{F}_2(e)}(x)\right\},$$

$$F_{\widetilde{F}_3(e)}(x) = max\left\{F_{\widetilde{F}_1(e)}(x), F_{\widetilde{F}_2(e)}(x)\right\}.$$

Definition 8. *[3] A neutrosophic soft set* $\left(\widetilde{F}, E\right)$ *over the universe set* X *is said to be a null neutrosophic soft set if* $T_{\widetilde{F}(e)}(x) = 0$, $I_{\widetilde{F}(e)}(x) = 0$, $F_{\widetilde{F}(e)}(x) = 1$; $\forall e \in E$, $\forall x \in X$. *It is denoted by* $0_{(X,E)}$.

Definition 9. *[3] A neutrosophic soft set* $\left(\widetilde{F}, E\right)$ *over the universe set* X *is said to be an absolute neutrosophic soft set if* $T_{\widetilde{F}(e)}(x) = 1$, $I_{\widetilde{F}(e)}(x) = 1$, $F_{\widetilde{F}(e)}(x) = 0$; $\forall e \in E$, $\forall x \in X$ *It is denoted by* $1_{(X,E)}$.

Clearly $0^c_{(X,E)} = 1_{(X,E)}$ *and* $1^c_{(X,E)} = 0_{(X,E)}$.

Definition 10. *[3] Let* $NSS(X, E)$ *be the family of all neutrosophic soft sets over the universe set* X *and* $\tau \subset NSS(X, E)$. *Then* τ *is said to be a neutrosophic soft topology on* X *if:*

1. $0_{(X,E)}$ *and* $1_{(X,E)}$ *belong to* τ,
2. *the union of any number of neutrosophic soft sets in* τ *belongs to* τ,
3. *the intersection of a finite number of neutrosophic soft sets in* τ *belongs to* τ.

Then (X, τ, E) *is said to be a neutrosophic soft topological space over* X. *Each member of* τ *is said tobe a neutrosophic soft open set [3].*

Definition 11. *[3] Let* (X, τ, E) *be a neutrosophic soft topological space over* X *and* $\left(\widetilde{F}, E\right)$ *be a neutrosophic soft set over* X. *Then* $\left(\widetilde{F}, E\right)$ *is said to be a neutrosophic soft closed set iff its complement is a neutrosophic soft open set.*

Definition 12. *[3] Let* $NSS(X, E)$ *be the family of all neutrosophic soft sets over the universe set* X. *Then neutrosophic soft set* $x^e_{(\alpha,\beta,\gamma)}$ *is called* a neutrosophic soft point *for every* $x \in X$, $0 < \alpha, \beta, \gamma \leq 1$, $e \in E$ *and is defined as follows:*

$$x^e_{(\alpha,\beta,\gamma)}(e')(y) = \begin{cases} (\alpha, \beta, \gamma), & \text{if } e' = e \text{ and } y = x \\ (0, 0, 1), & \text{if } e' \neq e \text{ or } y \neq x \end{cases}$$

It is clear that every neutrosophic soft set is the union of its neutrosophic soft points.

Definition 13. *[3] Let* $\left(\widetilde{F}, E\right)$ *be a neutrosophic soft set over the universe set* X. *We say that* $x^e_{(\alpha,\beta,\gamma)} \in \left(\widetilde{F}, E\right)$ *read as belonging to the neutrosophic soft set* $\left(\widetilde{F}, E\right)$ *whenever*
$\alpha \leq T_{\widetilde{F}(e)}(x)$, $\beta \leq I_{\widetilde{F}(e)}(x)$ *and* $\gamma \geq F_{\widetilde{F}(e)}(x)$.

Definition 14. *[3] Let* $x^{e}_{(\alpha,\beta,\gamma)}$ *and* $y^{e'}_{(\alpha',\beta',\gamma')}$ *be two neutrosophic soft points. For the neutrosophic soft points* $x^{e}_{(\alpha,\beta,\gamma)}$ *and* $y^{e'}_{(\alpha',\beta',\gamma')}$ *over a common universe* X*, we say that the neutrosophic soft points are distinct points if* $x^{e}_{(\alpha,\beta,\gamma)} \cap y^{e'}_{(\alpha',\beta',\gamma')} = 0_{(X,E)}$*. It is clear that* $x^{e}_{(\alpha,\beta,\gamma)}$ *and* $y^{e'}_{(\alpha',\beta',\gamma')}$ *are distinct neutrosophic soft points if and only if* $x \neq y$ *or* $e \neq e'$*.*

Definition 15. *[7] Let* $\left(\widetilde{F}, E_1\right)$,$\left(\widetilde{G}, E_2\right)$ *be two neutrosophic sets over the universal set* X*. Then their cartesian product is another neutrosophic set* $\left(\widetilde{K}, E_3\right) = \left(\widetilde{F}, E_1\right) \times \left(\widetilde{G}, E_2\right)$*, where* $E_3 = E_1 \times E_2$ *and* $\widetilde{K}(e_1, e_2) = \widetilde{F}(e_1) \times \widetilde{G}(e_2)$*. The truth, indeterminacy and falsity membership of* $\left(\widetilde{K}, E_3\right)$ *are given by* $\forall e_1 \in E_1$, $\forall e_2 \in E_2$, $\forall x \in X$,

$$T_{\widetilde{K}(e_1,e_2)}(x) = min\left\{T_{\widetilde{F}(e_1)}(x)\,,\, T_{\widetilde{G}(e_2)}(x)\right\},$$

$$I_{\widetilde{K}(e_1,e_2)}(x) = min\left\{I_{\widetilde{F}(e_1)}(x)\,,\, I_{\widetilde{G}(e_2)}(x)\right\},$$

$$F_{\widetilde{K}(e_1,e_2)}(x) = max\left\{F_{\widetilde{F}(e_1)}(x)\,,\, F_{\widetilde{G}(e_2)}(x)\right\}.$$

This definition can be extended for more than two neutrosophic soft sets.

Definition 16. *[7] A neutrosophic soft relation* $\widetilde{R}$ *between two neutrosophic soft sets* $\left(\widetilde{F}, E_1\right)$ *and* $\left(\widetilde{G}, E_2\right)$ *over the common universe* X *is the neutrosophic soft subset of* $\left(\widetilde{F}, E_1\right) \times \left(\widetilde{G}, E_2\right)$*. Clearly, it is another neutrosophic soft set* $\left(\widetilde{R}, E_3\right)$ *where* $E_3 \subset E_1 \times E_2$ *and*
$\widetilde{R}(e_1, e_2) = \widetilde{F}(e_1) \times \widetilde{G}(e_2)$ *for* $(e_1, e_2) \in E_3$*.*

Definition 17. *[7] Let* $\left(\widetilde{F}, E_1\right)$, $\left(\widetilde{G}, E_2\right)$ *be two neutrosophic sets over the universal set* X *and* f *be a neutrosophic soft relation defined on* $\left(\widetilde{F}, E_1\right) \times \left(\widetilde{G}, E_2\right)$*. Then* f *is called neutrosophic soft function if* f *associates each element of* $\left(\widetilde{F}, E_1\right)$ *with the unique element of* $\left(\widetilde{G}, E_2\right)$*. We write* $f : \left(\widetilde{F}, E_1\right) \to \left(\widetilde{G}, E_2\right)$ *as a neutrosophic soft function or a mapping. For* $x^{e}_{(\alpha,\beta,\gamma)} \in \left(\widetilde{F}, E_1\right)$ *and* $y^{e'}_{(\alpha',\beta',\gamma')} \in \left(\widetilde{G}, E_2\right)$ *when* $x^{e}_{(\alpha,\beta,\gamma)} \times y^{e'}_{(\alpha',\beta',\gamma')} \in f$*, we denote it by* $f\left(x^{e}_{(\alpha,\beta,\gamma)}\right) = y^{e'}_{(\alpha',\beta',\gamma')}$*. Here* $\left(\widetilde{F}, E_1\right)$ *and* $\left(\widetilde{G}, E_2\right)$ *are called domain and codomain respectively and* $y^{e'}_{(\alpha',\beta',\gamma')}$ *is the image of* $x^{e}_{(\alpha,\beta,\gamma)}$ *under* f*.*

Definition 18. *[8] Let (X,τ,E) be a neutrosophic soft topological space and $\left(\widetilde{F},E\right) \in NSS\,(X,E)$ be arbitrary. Then the interior of $\left(\widetilde{F},E\right)$ is denoted by $\left(\widetilde{F},E\right)^{\circ}$ and is defined as:*

$$\left(\widetilde{F},E\right)^{\circ} = \bigcup\left\{\left(\widetilde{G},E\right) : \left(\widetilde{G},E\right) \subset \left(\widetilde{F},E\right),\ \left(\widetilde{G},E\right) \in \tau\right\}$$

i.e., it is the union of all open neutrosophic soft subsets of $\left(\widetilde{F},E\right)$.

Definition 19. *[8] Let (X,τ,E) be a neutrosophic soft topological space and $\left(\widetilde{F},E\right) \in NSS\,(X,E)$ be arbitrary. Then the closure of $\left(\widetilde{F},E\right)$ is denoted by $\overline{\left(\widetilde{F},E\right)}$ and is defined as:*

$$\overline{\left(\widetilde{F},E\right)} = \bigcap\left\{\left(\widetilde{G},E\right) : \left(\widetilde{G},E\right) \subset \left(\widetilde{F},E\right),\ \left(\widetilde{G},E\right)^{c} \in \tau\right\}$$

i.e., it is the intersection of all closed neutrosophic soft super sets of $\left(\widetilde{F},E\right)$.

3. Some Properties

Definition 20. *A subset $\left(\widetilde{F},E\right)$ of a neutrosophic soft topological space (X,τ,E) is said to be neutrosophic pre open soft, if $\left(\widetilde{F},E\right) \subset \left[\overline{\left(\widetilde{F},E\right)}\right]^{\circ}$. The family of all neutrosophic pre open soft sets of (X,τ,E) is denoted by $NSPO\,(X)$. The family of all neutrosophic pre open soft sets of (X,τ,E) containing a neutrosophic soft point $x^{e}_{(\alpha,\beta,\gamma)}$ is denoted by $NSPO\left(X, x^{e}_{(\alpha,\beta,\gamma)}\right)$.*

Definition 21. *A neutrosophic soft point $x^{e}_{(\alpha,\beta,\gamma)}$ of a neutrosophic soft topological space*
(X,τ,E) is said to be neutrosophic soft pre interior point of a neutrosophic soft set $\left(\widetilde{F},E\right)$, if there exists $\left(\widetilde{G},E\right) \in NSPO\left(X, x^{e}_{(\alpha,\beta,\gamma)}\right)$ such that $x^{e}_{(\alpha,\beta,\gamma)} \nsubseteq \left(\widetilde{G},E\right)^{c}$ and $\left(\widetilde{G},E\right) \subset \left(\widetilde{F},E\right)$.

Definition 22. *The set of all neutrosophic soft pre interior points of $\left(\widetilde{F},E\right)$ is said to be neutrosophic soft pre interior of $\left(\widetilde{F},E\right)$ and denoted by $NSPint\left(\widetilde{F},E\right)$.*

Definition 23. *The complement of a neutrosophic pre open soft set is called neutrosophic pre closed soft. The intersection of all neutrosophic pre closed soft sets containing a neutrosophic soft set $\left(\widetilde{F},E\right)$ is called neutrosophic pre closure of $\left(\widetilde{F},E\right)$ and is denoted by $NSPcl\left(\widetilde{F},E\right)$.*

Definition 24. *A neutrosophic soft point $x^{e}_{(\alpha,\beta,\gamma)}$ of a neutrosophic soft topological space*

(X,τ,E) is said to be neutrosophic soft pre cluster point of a neutrosophic soft set $\left(\widetilde{F},E\right)$, if $\left(\widetilde{G},E\right) \nsubseteq \left(\widetilde{F},E\right)^c$ for any $\left(\widetilde{G},E\right) \in NSPO\left(X, x^e_{(\alpha,\beta,\gamma)}\right)$.

Definition 25. *A neutrosophic soft topological space (X,τ,E) is said to be a neutrosophic soft pre T_0-space if for every pair of distinct neutrosophic soft points $x^e_{(\alpha,\beta,\gamma)}$, $y^{e'}_{(\alpha',\beta',\gamma')}$ there exist neutrosophic pre-open soft sets $\left(\widetilde{F},E\right)$, $\left(\widetilde{G},E\right)$ such that $x^e_{(\alpha,\beta,\gamma)} \in \left(\widetilde{F},E\right)$,*
$y^{e'}_{(\alpha',\beta',\gamma')} \in \left(\widetilde{F},E\right)^c$ or $x^e_{(\alpha,\beta,\gamma)} \in \left(\widetilde{G},E\right)^c$, $y^{e'}_{(\alpha',\beta',\gamma')} \in \left(\widetilde{G},E\right)$.

Definition 26. *Let (X,τ,E) be a neutrosophic soft topological space and $Y \subseteq X$. Let $\left(\widetilde{H},E\right)$ be a neutrosophic soft set over Y such that*

$$T_{\widetilde{H}(e)}(x) = \begin{cases} 1, & \text{if } x \in Y \\ 0, & \text{if } x \notin Y \end{cases}$$

$$I_{\widetilde{H}(e)}(x) = \begin{cases} 1, & \text{if } x \in Y \\ 0, & \text{if } x \notin Y \end{cases}$$

$$F_{\widetilde{H}(e)}(x) = \begin{cases} 1, & \text{if } x \in Y \\ 0, & \text{if } x \notin Y \end{cases}$$

for any $e \in E$.

Let $\tau_Y = \left\{\left(\widetilde{H},E\right) \cap \left(\widetilde{F},E\right) : \left(\widetilde{F},E\right) \in \tau\right\}$, then (Y,τ_Y,E) is called neutrosophic soft subspace of (X,τ,E). If $\left(\widetilde{H},E\right) \in \tau$ (resp. $\left(\widetilde{H},E\right)^c \in \tau$), then (Y,τ_Y,E) is called neutrosophic open (resp.closed) soft subspace of (X,τ,E).

Theorem 27. *A neutrosophic soft subspace (Y,τ_Y,E) of a neutrosophic soft pre T_0-space*
(X,τ,E) is neutrosophic soft pre T_0.

Proof. Let $x^e_{(\alpha,\beta,\gamma)}$, $y^{e'}_{(\alpha',\beta',\gamma')}$ be two distinct neutrosophic soft points in (Y,τ_Y,E). Then, these neutrosophic soft points are also in (X,τ,E). Hence, there exist neutrosophic pre-open soft sets $\left(\widetilde{F},E\right)$, $\left(\widetilde{G},E\right)$ in τ such that $x^e_{(\alpha,\beta,\gamma)} \in \left(\widetilde{F},E\right)$, $y^{e'}_{(\alpha',\beta',\gamma')} \in \left(\widetilde{F},E\right)^c$ or $x^e_{(\alpha,\beta,\gamma)} \in \left(\widetilde{G},E\right)^c$, $y^{e'}_{(\alpha',\beta',\gamma')} \in \left(\widetilde{G},E\right)$. Let $\left(\widetilde{H},E\right)$ be a neutrosophic soft set over Y as described in Definition 26. Thus, $\left(\widetilde{H},E\right) \cap \left(\widetilde{F},E\right)$ and $\left(\widetilde{H},E\right) \cap \left(\widetilde{G},E\right)$ are neutrosophic pre-open soft sets in (Y,τ_Y,E) such that $x^e_{(\alpha,\beta,\gamma)} \in \left(\widetilde{H},E\right) \cap \left(\widetilde{F},E\right)$, $y^{e'}_{(\alpha',\beta',\gamma')} \in \left[\left(\widetilde{H},E\right) \cap \left(\widetilde{F},E\right)\right]^c$ or

$x^{e}_{(\alpha,\beta,\gamma)} \in \left[\left(\widetilde{H},E\right) \cap \left(\widetilde{G},E\right)\right]^{c}$, $y^{e'}_{(\alpha',\beta',\gamma')} \in \left(\widetilde{H},E\right) \cap \left(\widetilde{G},E\right)$. Therefore, (Y,τ_Y,E) is neutrosophic soft pre T_0. □

Definition 28. *A neutrosophic soft topological space (X,τ,E) is said to be a neutrosophic soft pre T_1-space if for every pair of distinct neutrosophic soft points $x^{e}_{(\alpha,\beta,\gamma)}$, $y^{e'}_{(\alpha',\beta',\gamma')}$ there exists neutrosophic pre-open soft sets $\left(\widetilde{F},E\right)$ and $\left(\widetilde{G},E\right)$ such that $x^{e}_{(\alpha,\beta,\gamma)} \in \left(\widetilde{F},E\right)$,*
$y^{e'}_{(\alpha',\beta',\gamma')} \in \left(\widetilde{F},E\right)^{c}$ and $x^{e}_{(\alpha,\beta,\gamma)} \in \left(\widetilde{G},E\right)^{c}$, $y^{e'}_{(\alpha',\beta',\gamma')} \in \left(\widetilde{G},E\right)$.

Theorem 29. *A neutrosophic soft subspace (Y,τ_Y,E) of a neutrosophic soft pre T_1-space*
(X,τ,E) is neutrosophic soft pre T_1.

Proof. It is similar to the proof of Theorem 27. □

Theorem 30. *Every neutrosophic soft point with the truth-membership value 1, the*
indeterminacy-membership value 1 and falsity-membership value 0, is neutrosophic pre-closed soft in a neutrosophic soft topological space (X,τ,E) if and only if (X,τ,E) is neutrosophic soft pre T_1.

Proof. (⇒) Suppose that $x^{e}_{(\alpha,\beta,\gamma)}$ and $y^{e'}_{(\alpha',\beta',\gamma')}$ be two distinct neutrosophic soft points of (X,τ,E). Then, $x^{e}_{(\alpha,\beta,\gamma)} \subset x^{e}_{(1,1,0)}$ and $y^{e'}_{(\alpha',\beta',\gamma')} \subset y^{e'}_{(1,1,0)}$. By hypothesis, $y^{e'}_{(1,1,0)}$ and $y^{e'}_{(1,1,0)}$ are neutrosophic pre-closed soft sets. Then, $\left[x^{e}_{(1,1,0)}\right]^{c}$ and $\left[y^{e'}_{(1,1,0)}\right]^{c}$ are neutrosophic pre-open soft sets such that $x^{e}_{(\alpha,\beta,\gamma)} \in \left[y^{e'}_{(1,1,0)}\right]^{c}$, $y^{e'}_{(\alpha',\beta',\gamma')} \in \left[\left[y^{e'}_{(1,1,0)}\right]^{c}\right]^{c}$ and $x^{e}_{(\alpha,\beta,\gamma)} \in \left[\left[x^{e}_{(1,1,0)}\right]^{c}\right]^{c}$, $y^{e'}_{(\alpha',\beta',\gamma')} \in \left[x^{e}_{(1,1,0)}\right]^{c}$. Therefore, (X,τ,E) is neutrosophic soft pre T_1.

(⇐) Suppose that (X,τ,E) is neutrosophic soft pre T_1. Let $x^{e}_{(1,1,0)}$ be a neutrosophic soft point with the truth-membership value 1, the indeterminacy-membership value 1 and falsity- membership value 0. Take any neutrosophic soft point $y^{e'}_{(\alpha',\beta',\gamma')} \in \left[x^{e}_{(1,1,0)}\right]^{c}$. It is easily seen that $x^{e}_{(1,1,0)}$ and $y^{e'}_{(\alpha',\beta',\gamma')}$ are distinct. From our assumption, there exist neutrosophic pre-open soft sets $\left(\widetilde{F},E\right)$ and $\left(\widetilde{G},E\right)$ such that $x^{e}_{(1,1,0)} \in \left(\widetilde{F},E\right)$, $y^{e'}_{(\alpha',\beta',\gamma')} \in \left(\widetilde{F},E\right)^{c}$ and
$x^{e}_{(1,1,0)} \in \left(\widetilde{G},E\right)^{c}$, $y^{e'}_{(\alpha',\beta',\gamma')} \in \left(\widetilde{G},E\right)$. Then, $y^{e'}_{(\alpha',\beta',\gamma')} \in \left(\widetilde{G},E\right) \subset \left[x^{e}_{(1,1,0)}\right]^{c}$. This means that $\left[x^{e}_{(1,1,0)}\right]^{c}$ is neutrosophic pre-open soft. Therefore, $x^{e}_{(1,1,0)}$ is neutrosophic pre-closed soft. □

Definition 31. *A neutrosophic soft topological space (X, τ, E) is said to be a neutrosophic soft pre T_2-space if for every pair of distinct neutrosophic soft points $x^e_{(\alpha,\beta,\gamma)}$, $y^{e'}_{(\alpha',\beta',\gamma')}$ there exists neutrosophic pre-open soft sets $\left(\widetilde{F}, E\right)$ and $\left(\widetilde{G}, E\right)$ such that $x^e_{(\alpha,\beta,\gamma)} \in \left(\widetilde{F}, E\right)$,*
$y^{e'}_{(\alpha',\beta',\gamma')} \in \left(\widetilde{F}, E\right)^c$, $y^{e'}_{(\alpha',\beta',\gamma')} \in \left(\widetilde{G}, E\right)$, $x^e_{(\alpha,\beta,\gamma)} \in \left(\widetilde{G}, E\right)^c$ and $\left(\widetilde{F}, E\right) \subset \left(\widetilde{G}, E\right)^c$.

For a neutrosophic soft topological space (X, τ, E) we have the following diagram:

$$\begin{array}{c}\text{neutrosophic soft pre } T_2\text{−space}\\ \downarrow \\ \text{neutrosophic soft pre } T_1\text{−space}\\ \downarrow \\ \text{neutrosophic soft pre } T_0\text{−space}\end{array}$$

Converse statements may not be true as shown in the examples below;

Example 32. *Let $X = \{x, y\}$ be a universe, $E = \{a.b\}$ be a parameteric set and $\left(\widetilde{F}_a, E\right)$ be a neutrosophic soft set defined as $\widetilde{F}_a(a) = \{\langle x, a, a, 1-a\rangle, \langle y, a, a, 1-a\rangle\}$ and*
$\widetilde{F}_a(b) = \{\langle x, 0, 0, 1\rangle, \langle y, a, a, 1-a\rangle\}$ for any $\alpha \in (0, 1]$. Then, the family
$\tau = \left\{0_{(X,E)},\ 1_{(X,E)}\right\} \cup \left\{\left(\widetilde{F}_a, E\right) : a \in (0, 1]\right\}$
is a neutrosophic soft topology over X. So, (X, τ, E) is a neutrosophic soft topological space. (X, τ, E) is a neutrosophic soft pre T_0 -space but not a neutrosophic soft pre T_1 –space. Because, $x^b_{(0.9,\ 0.6,\ 0.2)}$ and $y^a_{(0.8,\ 0.7,\ 0.4)}$ are distinct neutrosophic soft points in (X, τ, E) and there doesn't exist any neutrosophic pre-open soft set that contains $x^b_{(0.9,\ 0.6,\ 0.2)}$ but doesn't contain $y^a_{(0.8,\ 0.7,\ 0.4)}$.

Example 33. *Let $X = \{x, y\}$ be a universe, $E = \{a.b\}$ be a parameteric set and $\left(\widetilde{F}, E\right)$ be a neutrosophic soft set defined as $\widetilde{F}(a) = \{\langle x, 0, 0, 1\rangle,\ \langle y, 0, 0, 1\rangle\}$ and $\widetilde{F}(b) = \{\langle x, 0, 0, 1\rangle,\ \langle y, 0, 0, 0.9\rangle\}$. Then, the family $\tau = \left\{0_{(X,E)},\ 1_{(X,E)}, \left(\widetilde{F}, E\right)\right\}$ is a neutrosophic soft topology over X. So, (X, τ, E) is a neutrosophic soft topological space. (X, τ, E) is a neutrosophic soft pre T_1 –space. But, it is not a neutrosophic soft pre T_2 –space for the existence of distinct neutrosophic soft points $x^a_{(0.5,\ 0.5,\ 0.1)}$ and $y^b_{(0.4,\ 0.4,\ 0.6)}$.*

Theorem 34. *Let (X, τ, E) be a neutrosophic soft topological space. (X, τ, E) is neutrosophic soft pre T_2-space if and only if for any pair of distinct neutrosophic* *soft points* $x^{e}_{(\alpha,\beta,\gamma)}$, $y^{e'}_{(\alpha',\beta',\gamma')}$, *there exists a neutrosophic pre-open soft set* $\left(\widetilde{F},E\right)$ *such that*

$$x^{e}_{(\alpha,\beta,\gamma)} \in \left(\widetilde{F},E\right),\ y^{e'}_{(\alpha',\beta',\gamma')} \in \left(\widetilde{F},E\right)^{c} \text{ and } y^{e'}_{(\alpha',\beta',\gamma')} \in \left[NSPcl\left(\widetilde{F},E\right)\right]^{c}.$$

Proof. ($\Rightarrow$) Let $x^{e}_{(\alpha,\beta,\gamma)}$ and $y^{e'}_{(\alpha',\beta',\gamma')}$ be two distinct neutrosophic soft points in (X,τ,E) . Since (X,τ,E) is a neutrosophic soft pre T_2-space, there exist two neutrosophic pre-open soft sets $\left(\widetilde{F},E\right)$ and $\left(\widetilde{G},E\right)$ such that $x^{e}_{(\alpha,\beta,\gamma)} \in \left(\widetilde{F},E\right)$, $y^{e'}_{(\alpha',\beta',\gamma')} \in \left(\widetilde{G},E\right)$ and $\left(\widetilde{F},E\right) \subset \left(\widetilde{G},E\right)^{c}$. So, it is implied that $y^{e'}_{(\alpha',\beta',\gamma')} \in \left(\widetilde{F},E\right)^{c}$. Since $\left(\widetilde{G},E\right)^{c}$ is a neutrosophic pre-closed soft set, $NSPcl\left(\widetilde{F},E\right) \subset \left(\widetilde{G},E\right)^{c}$. This means that, $\left(\widetilde{G},E\right) \subset \left[NSPcl\left(\widetilde{F},E\right)\right]^{c}$. So,
$y^{e'}_{(\alpha',\beta',\gamma')} \in \left[NSPcl\left(\widetilde{F},E\right)\right]^{c}$.

($\Leftarrow$)Take any pair of distinct neutrosophic soft points $x^{e}_{(\alpha,\beta,\gamma)}$, $y^{e'}_{(\alpha',\beta',\gamma')}$ in (X,τ,E). From our assumption, there exists a neutrosophic pre-open soft set $\left(\widetilde{F},E\right)$ such that $x^{e}_{(\alpha,\beta,\gamma)} \in \left(\widetilde{F},E\right)$, $y^{e'}_{(\alpha',\beta',\gamma')} \in \left(\widetilde{F},E\right)^{c}$ and $y^{e'}_{(\alpha',\beta',\gamma')} \in \left[NSPcl\left(\widetilde{F},E\right)\right]^{c}$. Since $\left[NSPcl\left(\widetilde{F},E\right)\right]^{c}$ is a neutrosophic pre-open soft set and $\left(\widetilde{F},E\right) \subset \left[\left[NSPcl\left(\widetilde{F},E\right)\right]^{c}\right]^{c}$, (X,τ,E) is neutrosophic soft pre T_2-space. □

Theorem 35. *A neutrosophic soft subspace* (Y,τ_Y,E) *of neutrosophic soft pre* T_2-*space*
(X,τ,E) *is neutrosophic soft pre* T_2.

Proof. Let (X,τ,E) be a neutrosophic soft pre T_2-space, $Y \subseteq X$ and (Y,τ_Y,E) be a neutrosophic soft subspace. Take any distinct neutrosophic soft points $x^{e}_{(\alpha,\beta,\gamma)}$ and $y^{e'}_{(\alpha',\beta',\gamma')}$ in (Y,τ_Y,E).

So, these neutrosophic soft points are also contained in (X,τ,E). Hence, there exist neutrosophic pre-open soft sets $\left(\widetilde{F},E\right)$ and $\left(\widetilde{G},E\right)$ in τ such that $x^{e}_{(\alpha,\beta,\gamma)} \in \left(\widetilde{F},E\right)$,
$y^{e'}_{(\alpha',\beta',\gamma')} \in \left(\widetilde{G},E\right)$ and $\left(\widetilde{F},E\right) \subset \left(\widetilde{G},E\right)^{c}$. Let $\left(\widetilde{H},E\right)$ be a neutrosophic soft set over Y as described in Definition 26. Then, $\left(\widetilde{H},E\right) \cap \left(\widetilde{F},E\right)$ and $\left(\widetilde{H},E\right) \cap \left(\widetilde{G},E\right)$ are neutrosophic pre-open soft sets in (Y,τ_Y,E) such that $x^{e}_{(\alpha,\beta,\gamma)} \in \left(\widetilde{H},E\right) \cap \left(\widetilde{F},E\right)$, $y^{e'}_{(\alpha',\beta',\gamma')} \in \left(\widetilde{H},E\right) \cap \left(\widetilde{G},E\right)$ and $\left(\widetilde{H},E\right) \cap \left(\widetilde{F},E\right) \subset \left[\left(\widetilde{H},E\right) \cap \left(\widetilde{G},E\right)\right]^{c}$. This means that (Y,τ_Y,E) is neutrosophic soft pre T_2. □

Definition 36. *Let (X,τ,E) be a neutrosophic soft topological space, $\left(\widetilde{G},E\right)$ be a neutrosophic pre-closed soft set and $x^e_{(\alpha,\beta,\gamma)}$ be a neutrosophic soft point such that $x^e_{(\alpha,\beta,\gamma)} \in \left(\widetilde{F},E\right)^c$. If there exist neutrosophic pre-open soft sets $\left(\widetilde{G},E\right)$ and $\left(\widetilde{K},E\right)$ such that $x^e_{(\alpha,\beta,\gamma)} \in \left(\widetilde{G},E\right)$, $\left(\widetilde{F},E\right) \subseteq \left(\widetilde{K},E\right)$ and $\left(\widetilde{K},E\right) \subset \left(\widetilde{G},E\right)^c$, then (X,τ,E) is said to be a neutrosophic soft pre regular space.*

Definition 37. *A neutrosophic soft topological space (X,τ,E) is said to be a strong neutrosophic soft pre T_1-space if every neutrosophic soft point is a neutrosophic pre-closed soft set in (X,τ,E).*

Definition 38. *A neutrosophic soft pre regular space (X,τ,E) is said to be a neutrosophic soft pre T_3-space if it is also a strong neutrosophic soft pre T_1-space.*

Theorem 39. *Every neutrosophic soft pre T_3-space is a neutrosophic soft pre T_2-space.*

Proof. Let $x^e_{(\alpha,\beta,\gamma)}$ and $y^{e'}_{(\alpha',\beta',\gamma')}$ be two distinct neutrosophic soft points of a neutrosophic soft pre T_3-space (X,τ,E). Then, $y^{e'}_{(\alpha',\beta',\gamma')}$ is neutrosophic pre-closed soft set and
$x^e_{(\alpha,\beta,\gamma)} \in \left[\, y^{e'}_{(\alpha',\beta',\gamma')}\right]^c$. From the neutrosophic soft pre-regularity, there exist disjoint neutrosophic pre-open soft sets $\left(\widetilde{G},E\right)$ and $\left(\widetilde{K},E\right)$ such that $x^e_{(\alpha,\beta,\gamma)} \in \left(\widetilde{G},E\right)$ and
$y^{e'}_{(\alpha',\beta',\gamma')} \subset \left(\widetilde{K},E\right)$. Thus, $x^e_{(\alpha,\beta,\gamma)} \in \left(\widetilde{G},E\right)$ and $y^{e'}_{(\alpha',\beta',\gamma')} \in \left(\widetilde{K},E\right)$. Therefore, (X,τ,E) is neutrosophic soft pre T_2-space □

Theorem 40. *A neutrosophic soft subspace (Y,τ_Y,E) of a neutrosophic soft pre T_3-space (X,τ,E) is neutrosophic soft pre T_3.*

Proof. Let (X,τ,E) be a neutrosophic soft pre T_3-space, $Y \subseteq X$ and (Y,τ_Y,E) be a neutrosophic soft subspace. Let $x^e_{(\alpha,\beta,\gamma)}$ be any neutrosophic soft point in (Y,τ_Y,E). It is obvious that $x^e_{(\alpha,\beta,\gamma)}$ is also a neutrosophic soft point in (X,τ,E). Since (X,τ,E) is a strong neutrosophic soft pre T_1-space, $x^e_{(\alpha,\beta,\gamma)}$ is a neutrosophic pre-closed soft set in (X,τ,E). Consider the neutrosphic soft set $\left(\widetilde{H},E\right)$ over Y defined in Definition 26. It is easily seen that $\left(\widetilde{H},E\right) \cap x^e_{(\alpha,\beta,\gamma)}$ is neutrosophic pre-closed soft in (Y,τ_Y,E). This means that (Y,τ_Y,E) is a strong neutrosophic soft pre-T_1-space. Now, we must show that (Y,τ_Y,E) is also a neutrosophic soft pre-regular space. Let $\left(\widetilde{G},E\right)$ be a neutrosophic pre-closed soft set in (Y,τ_Y,E) and $x^e_{(\alpha,\beta,\gamma)}$ be a neutrosophic soft point in (Y,τ_Y,E) such that $x^e_{(\alpha,\beta,\gamma)} \in \left(\widetilde{G},E\right)^c$. Then, $\left(\widetilde{G},E\right) = \left(\widetilde{H},E\right) \cap \left(\widetilde{F},E\right)$ for some neutrosophic pre-closed soft set $\left(\widetilde{F},E\right)$ in (X,τ,E). Hence, $x^e_{(\alpha,\beta,\gamma)} \in \left[\left(\widetilde{H},E\right)\cap\left(\widetilde{F},E\right)\right]^c$. So, $x^e_{(\alpha,\beta,\gamma)} \in \left(\widetilde{H},E\right)^c \cup \left(\widetilde{F},E\right)^c$. Because of the description of the neutrosophic soft set $\left(\widetilde{H},E\right)$ in Definition 5.2, it is clear that $x^e_{(\alpha,\beta,\gamma)} \notin \left(\widetilde{H},E\right)^c$. This means that $x^e_{(\alpha,\beta,\gamma)} \in \left(\widetilde{F},E\right)^c$. From the neutrosophic soft pre-regularity of (X,τ,E), there exists neutrosophic pre-open soft sets $\left(\widetilde{K},E\right)$ and $\left(\widetilde{L},E\right)$ such that $x^e_{(\alpha,\beta,\gamma)} \in \left(\widetilde{K},E\right)$, $\left(\widetilde{F},E\right) \subseteq \left(\widetilde{L},E\right)$ and $\left(\widetilde{K},E\right) \subseteq \left(\widetilde{L},E\right)^c$. This implies that $\left(\widetilde{H},E\right)\cap\left(\widetilde{K},E\right)$ and $\left(\widetilde{H},E\right)\cap\left(\widetilde{L},E\right)$ are neutrosophic pre-open soft sets in (Y,τ_Y,E) such that $x^e_{(\alpha,\beta,\gamma)} \in \left(\widetilde{H},E\right)\cap\left(\widetilde{K},E\right)$, $\left(\widetilde{F},E\right) \subseteq \left(\widetilde{H},E\right)\cap\left(\widetilde{L},E\right)$ and $\left(\widetilde{H},E\right)\cap\left(\widetilde{K},E\right) \subset \left(\widetilde{H},E\right)\cap\left(\widetilde{L},E\right)^c$. Therefore, (Y,τ_Y,E) is neutrosophic soft pre T_3. □

Definition 41. *Let (X,τ,E) be a neutrosophic soft topological space, $\left(\widetilde{F}_1,E\right)$ and $\left(\widetilde{F}_2,E\right)$ be neutrosophic pre-closed soft sets such that $\left(\widetilde{F}_1,E\right) \subset \left(\widetilde{F}_2,E\right)^c$. If there exist neutrosophic pre-open soft sets $\left(\widetilde{G},E\right)$ and $\left(\widetilde{K},E\right)$ such that $\left(\widetilde{F}_1,E\right) \subseteq \left(\widetilde{G},E\right)$, $\left(\widetilde{F}_2,E\right) \subseteq \left(\widetilde{K},E\right)$ and $\left(\widetilde{G},E\right) \subset \left(\widetilde{K},E\right)^c$, then (X,τ,E) is said to be a neutrosophic soft pre normal space.*

Definition 42. *A neutrosophic soft pre normal space (X,τ,E) is said to be a neutrosophic soft pre T_4-space, if it is also a strong neutrosophic soft pre T_1-space.*

Theorem 43. *Let (X,τ,E) be a fuzzy soft topological space. Then, the following statements are equivalent:*

(1) *(X,τ,E) is a neutrosophic soft pre normal space.*

(2) *For every neutrosophic pre closed soft set $\left(\widetilde{K},E\right)$ and neutrosophic preopen soft set $\left(\widetilde{L},E\right)$ such that $\left(\widetilde{K},E\right) \subseteq \left(\widetilde{L},E\right)$, there exists a neutrosophic pre open soft set $\left(\widetilde{F},E\right)$ such that $\left(\widetilde{K},E\right) \subseteq \left(\widetilde{F},E\right)$, $NSPcl\left(\widetilde{F},E\right) \subseteq \left(\widetilde{L},E\right)$.*

Proof. (1) ⇒ (2) Let $\left(\widetilde{K},E\right)$ be a pre closed soft set and $\left(\widetilde{L},E\right)$ be a fuzzy pre open soft set such that $\left(\widetilde{K},E\right) \subseteq \left(\widetilde{L},E\right)$. Then, $\left(\widetilde{K},E\right)$, $\left(\widetilde{L},E\right)^c$ are neutrosophic pre closed soft sets such that $\left(\widetilde{L},E\right)^c \subseteq \left(\widetilde{K},E\right)^c$. It follows from (1), there exist neutrosophic pre open soft sets $\left(\widetilde{F},E\right)$ and $\left(\widetilde{G},E\right)$ such that $\left(\widetilde{K},E\right) \subseteq \left(\widetilde{F},E\right)$, $\left(\widetilde{L},E\right)^c \subseteq \left(\widetilde{G},E\right)$ and $\left(\widetilde{F},E\right) \subseteq \left(\widetilde{G},E\right)^c$. Since $\left(\widetilde{G},E\right)^c$ is neutrosophic pre

closed soft, $NSPcl\left(\widetilde{F},E\right) \subseteq \left(\widetilde{G},E\right)^{c}$. So,
$NSPcl\left(\widetilde{F},E\right) \subseteq \left(\widetilde{L},E\right)$. Therefore, the neutrosophic pre open soft set $\left(\widetilde{F},E\right)$ satisfies the conditions.

(2) $\Rightarrow$ (1) Let $\left(\widetilde{F}_1,E\right)$ and $\left(\widetilde{F}_2,E\right)$ be neutrosophic pre-closed soft sets such that
$\left(\widetilde{F}_1,E\right) \subset \left(\widetilde{F}_2,E\right)^{c}$, where $\left(\widetilde{F}_2,E\right)^{c}$ is neutrosophic pre open soft. From our hypothesis, there exists a neutrosophic pre open soft set $\left(\widetilde{F},E\right)$ such that $\left(\widetilde{F}_1,E\right) \subset \left(\widetilde{F},E\right)$ and
$NSPcl\left(\widetilde{F},E\right) \subseteq \left(\widetilde{F}_2,E\right)^{c}$. So, $\left(\widetilde{F}_2,E\right) \subseteq \left[NSPcl\left(\widetilde{F},E\right)\right]^{c}$, $\left(\widetilde{F}_1,E\right) \subset \left(\widetilde{F},E\right)$ and
$\left[NSPcl\left(\widetilde{F},E\right)\right]^{c} \subseteq \left(\widetilde{F},E\right)^{c}$, where $\left[NSPcl\left(\widetilde{F},E\right)\right]^{c}$ and $\left(\widetilde{F},E\right)$ are are neutrosophic pre open soft sets. Thus, (X,τ,E) is neutrosophic soft pre normal space. □

Theorem 44. *A neutrosophic pre closed neutrosophic soft subspace (Y,τ_Y,E) of a neutrosophic soft pre normal space (X,τ,E) is neutrosophic soft pre normal.*

Proof. Let $\left(\widetilde{F},E\right)$ and $\left(\widetilde{G},E\right)$ be neutrosophic pre closed soft sets in (Y,τ_Y,E) such that $\left(\widetilde{F},E\right) \subset \left(\widetilde{G},E\right)^{c}$. Consider the neutrosphic soft set $\left(\widetilde{H},E\right)$ over Y defined in Definition 26. Then, $\left(\widetilde{H},E\right)$ is neutrosophic pre closed soft in (X,τ,E), $\left(\widetilde{F},E\right) = \left(\widetilde{H},E\right) \cap \left(\widetilde{K},E\right)$ and $\left(\widetilde{G},E\right) = \left(\widetilde{H},E\right) \cap \left(\widetilde{L},E\right)$ for some neutrosophic pre closed soft sets $\left(\widetilde{K},E\right)$ and $\left(\widetilde{L},E\right)$ in (X,τ,E). Hence, $\left(\widetilde{H},E\right) \cap \left(\widetilde{K},E\right)$ and $\left(\widetilde{H},E\right) \cap \left(\widetilde{L},E\right)$ are neutrosophic pre closed soft sets in (X,τ,E) and $\left(\widetilde{H},E\right) \cap \left(\widetilde{K},E\right) \subseteq \left[\left(\widetilde{H},E\right) \cap \left(\widetilde{L},E\right)\right]^{c}$. Since (X,τ,E) is neutrosophic soft pre normal, there exist neutrosophic pre open sets $\left(\widetilde{M},E\right)$ and $\left(\widetilde{N},E\right)$ such that
$\left(\widetilde{H},E\right) \cap \left(\widetilde{K},E\right) \subseteq \left(\widetilde{M},E\right)$, $\left(\widetilde{H},E\right) \cap \left(\widetilde{L},E\right) \subseteq \left(\widetilde{N},E\right)$ and $\left(\widetilde{M},E\right) \subseteq \left(\widetilde{N},E\right)^{c}$. So,
$\left(\widetilde{H},E\right) \cap \left(\widetilde{M},E\right)$ and $\left(\widetilde{H},E\right) \cap \left(\widetilde{N},E\right)$ are neutrosophic pre open sets in (Y,τ_Y,E) such that $\left(\widetilde{F},E\right) \subset \left(\widetilde{H},E\right) \cap \left(\widetilde{M},E\right)$, $\left(\widetilde{G},E\right) \subset \left(\widetilde{H},E\right) \cap \left(\widetilde{N},E\right)$ and
$\left(\widetilde{H},E\right) \cap \left(\widetilde{M},E\right) \subset \left[\left(\widetilde{H},E\right) \cap \left(\widetilde{N},E\right)\right]^{c}$. Therefore, (Y,τ_Y,E) is neutrosophic soft pre normal. □

Definition 45. *Let (X,τ_1,E), (Y,τ_2,K) be neutrosophic soft topological spaces and*

$f:(X,\tau_1,E)\rightarrow(Y,\tau_2,K)$ be a neutrosophic soft function. The function f is said to be neutrosophic pre irresolute soft, if $f^{-1}\left(\left(\widetilde{G},E\right)\right)\in\tau_1$ for any $\left(\widetilde{G},E\right)\in\tau_2$.

Definition 46. *Let (X,τ_1,E), (Y,τ_2,K) be neutrosophic soft topological spaces and*
$f:(X,\tau_1,E)\rightarrow(Y,\tau_2,K)$ be a neutrosophic soft function. The function f is said to be neutrosophic pre irresolute open soft, if $f\left(\left(\widetilde{F},E\right)\right)\in\tau_2$ for any $\left(\widetilde{F},E\right)\in\tau_1$.

Theorem 47. *Let (X,τ_1,E) and (Y,τ_2,K) be neutrosophic soft topological spaces and*
$f:(X,\tau_1,E)\rightarrow(Y,\tau_2,K)$ be a neutrosophic soft function which is bijective, neutrosophic pre irresolute soft and neutrosophic pre irresolute open soft. If (X,τ_1,E) is a neutrosophic soft pre normal space, then (Y,τ_2,K) is also a neutrosophic soft pre normal space.

Proof. Let $\left(\widetilde{F},E\right)$ and $\left(\widetilde{G},E\right)$ be neutrosophic pre closed soft sets in (Y,τ_2,K) such that
$\left(\widetilde{F},E\right)\subset\left(\widetilde{G},E\right)^c$. Since f is neutrosophic pre irresolute soft, then $f^{-1}\left(\left(\widetilde{F},E\right)\right)$ and
$f^{-1}\left(\left(\widetilde{G},E\right)\right)$ are neutrosophic pre closed soft sets in (X,τ_1,E) such that
$f^{-1}\left(\left(\widetilde{F},E\right)\right)\subset\left[f^{-1}\left(\left(\widetilde{G},E\right)\right)\right]^c$. Since (X,τ_1,E) is a neutrosophic soft pre normal space, there exist neutrosophic pre open soft sets $\left(\widetilde{K},E\right)$ and $\left(\widetilde{L},E\right)$ such that
$f^{-1}\left(\left(\widetilde{F},E\right)\right)\subset\left(\widetilde{K},E\right)$, $f^{-1}\left(\left(\widetilde{G},E\right)\right)\subset\left(\widetilde{L},E\right)$ and $\left(\widetilde{K},E\right)\subset\left(\widetilde{L},E\right)^c$. It follows that
$\left(\widetilde{F},E\right)=f\left[f^{-1}\left(\left(\widetilde{F},E\right)\right)\right]\subset f\left(\left(\widetilde{K},E\right)\right)$, $\left(\widetilde{G},E\right)=f\left[f^{-1}\left(\left(\widetilde{G},E\right)\right)\right]\subset f\left(\left(\widetilde{L},E\right)\right)$ and $f\left(\left(\widetilde{K},E\right)\right)\subset f\left(\left(\widetilde{L},E\right)^c\right)=\left[f\left(\left(\widetilde{L},E\right)\right)\right]^c$. From the fact that f is neutrosophic pre irresolute open soft, $f\left(\left(\widetilde{K},E\right)\right)$ and $f\left(\left(\widetilde{L},E\right)\right)$ are neutrosophic pre open soft sets such that $\left(\widetilde{F},E\right)\subset f\left(\left(\widetilde{K},E\right)\right)$, $\left(\widetilde{G},E\right)\subset f\left(\left(\widetilde{L},E\right)\right)$ and $f\left(\left(\widetilde{K},E\right)\right)\subset\left[f\left(\left(\widetilde{L},E\right)\right)\right]^c$. This means that (Y,τ_2,K) is a neutrosophic soft pre normal space. □

4. Conclusion

The notions of neutrosophic pre open soft sets, neutrosophic pre closed soft sets, neutrosophic pre soft interior, neutrosophic pre soft closure, neutrosophic soft pre-interior point, neutrosophic soft pre-cluster point and neutrosophic soft pre separation axioms are introduced, and some properties of the notions are studied. Also, several interesting properties have been established. Additionally, a new approach is made to the concept of neutrosophic soft topological subspace. Since topological structures on neutrosophic soft sets have been introduced by many scientists, we generalize the pre topological properties to the neutrosophic soft sets which may be useful in some other disciplines. For the existence of compact connections between soft sets and information systems [17, 21] the results obtained from the studies on neutrosophic soft topological space can be used to develop these connections. We hope that many researchers will benefit from the findings in this document to further their studies on neutrosophic soft topology to carry out a general framework for their applications in practical life.

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