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The Influence of Visual Representations and Context on Mathematical Word Problem Solving

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Abstract

The aim of this study is to explore problem solving performance of fifth graders on mathematical word problems in familiar and unfamiliar context with or without visual representations. 867 fifth graders in Turkish Republic of Northern Cyprus were the participants. The participants sat, on a voluntary basis, for a 30 item multiple choice test in which there were 6 operation items and 24 result-unknown type of word problems. All the participants answered the same questions under the same conditions and scored in the same manner. One-way repeated measures ANOVA results showed that the students showed better performances when solving familiar word problems than solving comparable unfamiliar word problems. The results also showed that the presence of visual representations in word problems strongly influenced students' problem solving performances in a positive way. It was observed that visual representations contributed a lot especially when the context of the word problems was unfamiliar.

Key Words: *Visual representations, Context, Problem solving.*

Bağlam ve Görsel Anlatımların Matematiksel Sözel Problem Çözümüne Etkisi

Özet

Bu çalışmanın amacı, görsel anlatımların sözel problemlerde yer alıp almaması ve problemlerde alışılagelmiş/tanıdık veya alışılmışın dışında/aşina olunmayan bağlamların kullanılmasının ilkokul 5. sınıf öğrencilerinin matematiksel sözel problem çözümlerini ne şekilde etkilediğini incelemektir. Bu amaçla Kuzey Kıbrıs Türk Cumhuriyeti'ndeki 867 beşinci sınıf öğrencisine, gönüllülük esasına göre, aynı ortam ve koşullarda, 6'sı işlem, 24'ü sözel problem olan 30 soruluk bir test uygulanmıştır. Tekrarlı ölçümlere sahip tek faktörlü varyans analizinden elde edilen bulgular, öğrencilerin alışılagelmiş/tanıdık bağlamlı sorulardaki performanslarının alışılmışın dışında/aşina olunmayan sorulardan çok daha iyi olduğunu göstermiştir. Bulgular aynı zamanda sözel problemlerin görsel anlatımlarla desteklenmesinin problem çözümüne olumlu katkısı olduğunu göstermiştir. Diğer yandan görsel anlatımların en çok alışılmışın dışında/aşina olunmayan bağlamlı problem çözümüne katkısı olduğu gözlenmiştir.

Anahtar Sözcükler: *Görsel anlatımlar, Bağlam, Problem çözme.*

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Introduction

A common view among most of the researchers, mathematics teachers, students and parents is that, doing mathematics is solving problems and solving problems is considered as the heart of mathematics (Baykul ve Sulak, 2006; Cockcroft, 1982, Kaur, 1997; NCTM, 2000; Schoenfeld, 1985). However, the nature of problem solving and difficulties with problem solving are still need to be investigated. In this study we mainly focus on some difficulty factors on word problem solving. Research on word problem solving, at both arithmetic and algebra levels has pointed the difficulty of word problems (Altun, 2005; Carpenter, Kepper, Corbit, Linquist & Reys, 1980; Koedinger & Nathan, 2004; Nathan, Kintsch & Young, 1992). For example, Geary (1994, p.96) states that "children make errors when solving word problems than solving comparable number problems."

Although there are many factors causing problem difficulty, related literature mainly focuses on *unknown values* and *presentation format* (Nathan, Koedinger & Tabachneck, 1997) as important sources of difficulties in problem solving. It is argued that problem difficulty is strongly affected by the unknown quantity within the problem statement (e.g., Carpenter et al., 1994; Riley & Greeno, 1988). Riley and Greeno (1988) found that while students were 100% correct on result-unknown problems, they were 33% correct on start unknown problems. Start-unknown problems tend to subvert simple modeling and direct calculation. On the other hand the format in which a problem is presented also bears on problem difficulty (Koedinger & Nathan, 2004; Tabachneck et al., 1994). In order to find more specific results in this study, we take the *presentation format* into consideration. To reduce other difficulty factors we focused on start-unknown type of word problems which can be solved by multiplication followed by an addition or subtraction in the context of natural numbers.

Visualization which has a close relation with the *presentation format* of problems has been investigated for many years (Koedinger & Nathan, 2004; Lowrie, 1996, Lowrie & Kay, 2001; Presmeg and Canas-Balderas, 2001; Rieber, 1995). Most of the research findings emphasize drawing pictures or figures in the problem solving process rather than making use of the given pictures or figures (e.g., Overholt, Aaberg & Lindsey, 1990; Posamentier & Krulik, 1998; Tertemiz, 1994). Therefore in this study we focused on the effect of the presence of visual representations in word problem solving. The context which is related to *presentation format* was considered as another factor which has a potential to affect problem difficulty (e.g., Brown, Collins & Duguid, 1989; Moyer, 2000; NCTM, 2000). In the present study the context was considered as being familiar or unfamiliar. The familiar context in this study was considered as textbook-like cases that could be solved or answered with a standard algorithm or procedure and problems which were related to the students' life. On the other hand unfamiliar context was considered as the cases that often were not used in the classroom practices, textbooks and were not directly related to the students' life and interests. These considerations are parallel with the NCTM (2000) standards. The context and presentation format can also be thought as important agents in reducing the level of cognitive load associated with the problem solving task (Pawley, Ayres, Cooper, & Sweller, 2005; Sweller & Low, 1992). In this sense, lack of research on how visual representations, like pictures or figures, given in word problems affect problem difficulty (e.g., Garderen & Montague, 2003; Polya, 1957, 1973) makes this study important. So, in this study we sought answers to the following questions.

1. Does the context of a mathematical word problem affect students' problem solving performance?
2. Does the presence of visual representations affect students' problem solving performance?
3. Are there any interaction effects of the context and the presence of visual representations on students' problem solving performance?

Education in Northern Cyprus In Northern Cyprus, education at the elementary and secondary school level is highly centralized and under the control of the Ministry of Education. Students enter elementary school at age 6 and leave at age 11 (from first grade to fifth grade). At the end of each school year, most fifth graders (nearly one third of all elementary school graduates) in Northern Cyprus take the Entrance Examination for the Middle Schools (EEMS), for which the general medium of instruction is English. The examination is considered by the majority of families in Northern Cyprus as the most important key

in the future academic life of students. The EEMS is prepared and administered once a year by the Ministry of Education. Because of this high-stakes standardized testing, which usually begins at the fourth through the fifth grade, instructional approaches in elementary schools of Northern Cyprus are geared mostly to teaching to the test.

Methodology

In this descriptive study, a one-way repeated measures ANOVA design was used. Presence of visual representations in problems (visual representation vs. no visual representation) was considered as the between subjects factor and the context (familiar v. unfamiliar) of the problems was considered as the within subjects factor. Exploring the effects of the presence of visual representations and the familiarity of the problem context in word problem solving was aimed. Four variables were considered, namely, *visual familiar, visual unfamiliar, non-visual familiar and non-visual unfamiliar problem* scores. Since the same arithmetical structure was used throughout the problems the variables were thought to be measured repeatedly.

Participants

The population of this study consists of all fifth graders in Northern Cyprus. The participants were 867 fifth graders, representing 30 percent of the population ($N = 3645$), from four regions (strata) in Northern Cyprus, namely, Lefkoşa ($n = 261$), Mağusa ($n = 364$), Güzelyurt ($n = 164$), and Girne ($n = 88$) enrolled in the academic year of 2010-2011. The participants were the students who were planning to take the EEMS and they sat for the Mathematical Performance Test (MPT) on a voluntary basis which was a sub-test of a practice test developed by the researchers for the EEMS.

Data collection tools

In order to explore the performance of fifth graders on problems in familiar and non-familiar context with or without visual representations, the Mathematical Performance Test (MPT), in multiple choice format, was developed by the researchers consisting of 24 word problems which can be solved in two steps (multiplication followed by addition or subtraction) plus 6 items including multiplication operations followed by addition or subtraction. A pilot test of MPT administered by the researchers on 60 fifth graders revealed an alpha reliability coefficient of 0.90. Six of the items of the MPT were designed to explore if the participants of the present study were able to perform the operations which were used as the arithmetical structure of the word problems. The word problems part of the MPT was the main concern of this study including four sub-tests, namely, Familiar Word Problems With Visual Representations (FWVR), Familiar Word Problems Without Visual Representations (FWOVR), Unfamiliar Word Problems With Visual Representations (UFWVR) and Unfamiliar Word Problems Without Visual Representations (UFWOVR) tests each including six, result-unknown, word problems. In this study, a familiar mathematics problem represented a textbook-like problem that could be solved or answered with a standard algorithm or procedure. For familiar mathematics problems, the student had to implement only a limited number of steps. The familiar problems were also closely related to the students' life. However, for unfamiliar mathematics problems, the students did not have to apply any formal algorithms. In unfamiliar mathematics problems, the contexts that the students used often were not used in the classroom practices, textbooks and were not directly related to the students' life and interests. The alpha reliability coefficients of the four sub-tests ranged from 0.64 to 0.81. An expert in language teaching and 2 experienced elementary school teachers were asked to judge the linguistic complexity of the problems. All of them concluded that the texts in the problems were all suitable for fifth graders. The arithmetical structure, $a \times b \pm c$, used in operation items, where a,b and c are natural numbers, was also used as the arithmetical structure of all word problems to eliminate arithmetical structure as a source of problem difficulty (see Table 1). Pictures (visual representations) given in two of the sub-tests (FWVR and UFWVR tests) were given in such a format that the problems can also be solved without that pictures. The MPT was administered in a 50-minute period.

Procedures

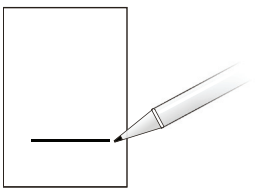
Before conducting the study we analyzed 5 different mathematics textbooks and previous standardized tests used in the last five years in Northern Cyprus elementary schools. We observed that nearly 9% of the word problems were in unfamiliar context enriched by visual representations. This led us to investigate the effect of problem context and presence of visual representations on problem solving performance. Textbook and

standardized tests analysis was considered in developing and regulating the context and visual representation aspects of the word problems of the MPT. Before conducting the study we sought permission from a private tutoring center which administers nationwide standardized practice tests for the EEMS. We agreed to provide feedback to the private tutor center and the center offered permission and assistance for us (the researchers) to conduct the present study and administer the MPT to 867 students. All the students answered the same questions (MPT) under the same conditions and scored in the same manner. The data collected were analyzed.

Data analysis

In order to observe any potential differences amongst the problem formats regarding the fifth graders performances, one-way repeated measures ANOVA procedures were used. Partial eta squared (η^2) measures were used to see how much variance was explained by each factor. In interpreting partial eta squared values, Cohen's (1977) measures were used. Cohen characterized $\eta^2 = .01$ as a small effect size, $\eta^2 = .06$ as a medium effect size, and $\eta^2 = .14$ as a large effect size. On the other hand Cohen's *d* measures were used when paired samples *t*-tests were performed. Cohen characterized $d = 0.2$ as a small effect size, $d = 0.3$ as a medium effect size, and $d = 0.5$ as a large effect size. In order to explore if the performance of students in operation items frequency measures were used. The level of significance used throughout the study was .05.

Table 1. Sample problems from MPT

Context	No Visual Representation	Visual Representation Available
Familiar	In a box there are 23 bottles. What is five more than 45 times of the number of bottles?	In a village there are 34 houses. What is 6 more than 25 times of the number of houses? 
Unfamiliar	Ali, took 36 boxes of drinks and paid 53 TL for each box. If he still has 7 TL in his pocket, how much money did he have at the beginning?	Fatma, solved correctly all the problems of a 52 item test in which each item was 15 points. If the teacher gave her 8 points as a bonus. Find the total score she has got. 

Findings

In order to eliminate the influence of the arithmetical structure, *multiply-add/subtract*, of the word problems a sub-test of the MPT including six operation items in the format multiplication followed by an addition or subtraction was used. The analysis revealed that 81% of 867 students obtained a total of 5 or 6 points from the sub-test. Since the maximum expected point was 6, we did not consider the arithmetical structure of the

word problems as a source of difficulty for the participants of this study.

A paired samples *t* test showed that mean score (M = 9.14, SD = 3.36) on items in familiar mathematics word problems was significantly higher than was the mean score (M = 6.58, SD = 3.22) on unfamiliar mathematics word problems, t (866) = 34.58, $p < .05$. That finding also was confirmed by a large effect size ($d = 1.17$). Another paired samples *t* test showed that mean score (M = 8.69, SD = 3.34) on items in mathematics word problems with pictorial representations was significantly higher than was the mean score (M = 7.64, SD = 3.17) on mathematics word problems without pictorial representations, t (866) = 25.09, $p < .05$, $d = 0.85$.

A within-subjects repeated ANOVA revealed significant differences among the sub-tests mean scores, F (3, 2598) = 700.791, $p = .001$, $\eta_p^2 = .447$. The analysis showed that the mean score obtained from the FWVR sub-test was the highest and the mean score obtained from the UNFWOVR sub-test was the lowest (see Figure 1).

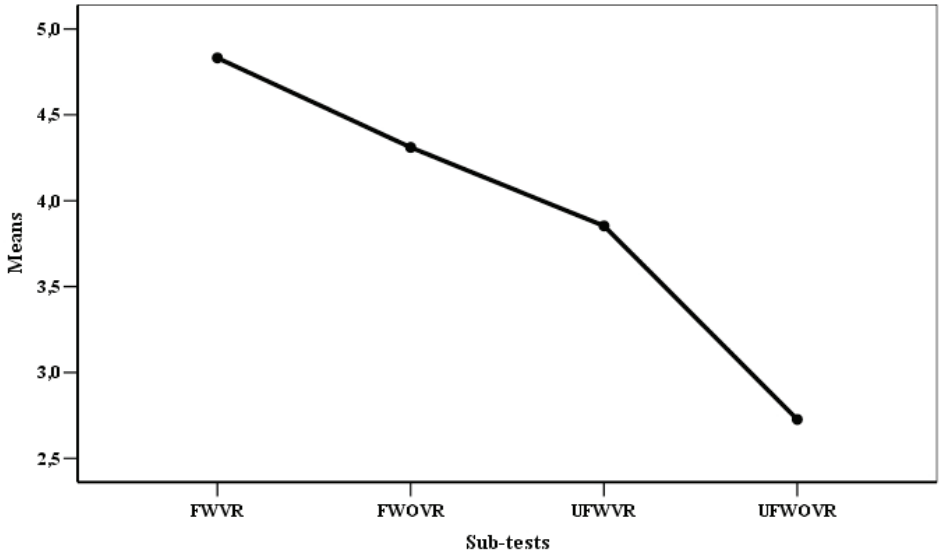


Figure 1. Sub-tests means

A one-way repeated measures ANOVA, in which the presence of visual representations in problems (visual representation vs. no visual representation) was considered as the between subjects factor and the context (familiar vs. unfamiliar) of the problems was considered as the within subjects factor, revealed a significant context of problems x presence of visual representations interaction effect (see Table 4) beside context and presence of visual representations main effects (see Table 2, 3 and 4). Larger effect size ($\eta_p^2 = 0.45$) for the context compared to the effect size for the presence of visual representations ($\eta_p^2 = 0.06$) revealed that the context has a larger effect than the presence of visual representations on students' word problem scores.

Table 2. Tests of within-subjects effects for the performance in answering familiar and unfamiliar word problems with or without visual representations

Variable	*SS*	*df*	*MS*	*F*	*P*	η_p^2
Context	1422.388	1	1422.388	1388.988	0.001	0.45
Context x Visual Representations	79.477	1	79.477	77.611	0.001	0.04
Error	1773.636	1732	1.024	-	-	-

Table 3. Tests of between-subjects effects for the performance in answering familiar and unfamiliar word problems with or without visual representations

Variable	*SS*	*df*	*MS*	*F*	*P*	η_p^2
Visual Representations	294.412	1	294.412	111.022	0.001	0.06
Error	4592.964	1732	2.652	-	-	-

Table 4. Descriptive statistics for familiar and unfamiliar word problems with or without visual representations

	Presence of Visual Representations				
	No Visual Representation		Visual Representation Available		
Problem Context	M	SD	M	SD	*N*
Familiar	4.31	1.82	4.83	1.72	867
Unfamiliar	2.72	1.69	3.85	1.87	867

Discussion

The findings of the present study revealed that problem context can be considered as an important source of problem difficulty. Many other researchers previously stated the importance of context in mathematics learning and problem solving (Choi & Hannafin, 1997; Ross, Mccormick, & Krisak, 1986) which supports the results of the present study about problem context. For example Wiest (2002) stated that problem context gave meaning to the mathematical content in a problem which influences the problem solving stage of understanding a problem. In line with this, the results of the present study also showed that the students' problem solving performances on familiar word problems were better than their performances on unfamiliar word problems. So, it seems that the familiarity of the context used was also an important factor. Hembree (1992) in a meta-analysis of 44 studies explored six pairs of problem context and he concluded that familiar contexts strongly influenced students' problem solving performances in a positive way. This is consistent with the results of the present study. In many research studies it was noticed that familiar contexts enhance word problem solving by increasing the meaningfulness of contexts and motivating students to solve the problems (Cordova & Lepper, 1996; Lopez & Sullivan, 1992; Ku & Sullivan, 2002). Therefore, it can be concluded that the familiarity of a word problem might reduce problem difficulty and enhance problem solving.

The findings of the present study revealed that the presence of visual representations in word problems can be considered as another important source of reducing problem difficulty. It was observed that students performed better in word problems with visual representations than word problems without visual representations. Related literature supports this finding. For example, many problem solving models have strongly emphasized the importance of visual processing when completing problem solving tasks (Kaufmann, 1990; Pirie & Kieren, 1991, 1992). Visual imagery has been seen as a way of storing knowledge (Presmeg, 1986) and important cognitive process used in problem solving (Antonietti, 1991; Lowrie, 1998).

On the other hand the findings of the present study revealed that the presence of visual representations does not contribute to word problem solving as much as the problem context. Consistent with this result, Lopez and Sullivan (1992) stated that especially familiar problem contexts were more effective for more demanding cognitive tasks than less demanding tasks. According to the results of the present study although the presence of visual representations in the word problems

was not as effective as problem context, it was observed that the students performed better in unfamiliar word problems with visual representations than unfamiliar word problems without visual representations. Some researchers argued that students' problem solving abilities might improve markedly if they could use working memory more efficiently (e.g., Silver, 1987; Sweller & Low, 1992; Sweller, Van Merrienboer, & Paas, 1998). In addition to this, it is also argued that visualization may reduce the level of cognitive load associated with the problem solving task which in turn may enable students to use their working memory more efficiently (Pawley, Ayres, Cooper, & Sweller, 2005; Sweller & Low, 1992). Thus, it can be concluded that visual representations especially in unfamiliar word problems may reduce problem difficulty and enhance problem solving. Many researchers and education authorities emphasize problem solving especially in unfamiliar contexts (e.g., Busbridje ve Özçelik, 1997; NCTM, 2000; Schoenfeld, 1985). For example Busbridje and Özçelik (1997) state that problem solving in familiar contexts, especially text-book like problems, could not even be considered as problem solving. So the result of the present study about the positive effects of visual representations on unfamiliar word problem solving can be considered as a support for the importance of the study. Although it was out the scope of the present study, it was observed that the students wrote some notes on the pictures given especially in the unfamiliar word problems, which can be considered as an evidence that visualization may reduce cognitive load in more challenging problem solving tasks. For example, Lowrie (2001) stated that visual imagery could provide a backup system that facilitates access to a set of cognitive process.

To summarize, the following recommendations can be offered for researchers, teachers, pre-service teachers, teacher trainers and curriculum experts in light of the findings and current practice:

Pre-service and in-service teachers should have the opportunity to view and teach problem solving in both familiar and unfamiliar contexts. Also, pre-service and in-service teachers should be educated in a way that they become more capable of using visually enriched word problems. The linkages between visualization and problem solving should be considered in all teacher training programs.

Textbooks and other instructional and assessment materials should be enriched by visual representations to reduce the level of cognitive load associated with the problem solving task. Familiar problem contexts, especially related to students' life, should be considered when the cognitive load associated with the problem solving task is high. Problems should be enriched by visual representations when the problem context is unfamiliar to the students.

Qualitative research should be conducted focusing on these and other aspects of word problem solving.

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Giriş

Eldeki araştırmanın temel amacı, matematiksel sözel problemlerde yer alan bağlamın (alışılagelmiş/tanıdık ve alışılmışın dışında/aşina olunmayan) öğrencilerin problem çözme becerilerini etkileyip etkilemediğini ve problemlerde görsel anlatımın bulunmasının öğrencilerin matematiksel problem çözme performanslarını nasıl etkilediğini ortaya koymaktır. Ayrıca, problemlerde yer alan görsel anlatım ve bağlamın öğrencilerin problem çözme performanslarına birlikte etkisini ortaya koymaktır.

Yöntem

Araştırmada betimsel yöntem kullanılmıştır. Araştırmanın evrenini Kuzey Kıbrıs'taki ilkokullarda okuyan 5. sınıf öğrencileri oluşturmaktadır. Araştırma kapsamına evrenin % 30'unu temsil eden 867 öğrenci alınmıştır. Araştırmaya alınan öğrenciler Ortaöğretime Giriş Sınavı (OGS)'na katılmayı planlayan öğrencilerdir. Araştırmada 30 sorudan oluşan matematik performans testi kullanılmıştır. Testin ön-denemesi 60 beşinci sınıf öğrencisi üzerinde yapılmış ve testin alpha güvenirlik katsayısı .90 olarak hesaplanmıştır. Testte yer alan 24 madde iki aşamada (çarpma+çıkarma/toplama) çözülebilen sözel problemlerden, 6 madde ise çarpma+çıkarma/toplama gerektiren işlemsel sorulardan oluşturulmuştur. İşlemsel sorular öğrencilerin işlemsel düzeyde yeterli önkoşula sahip olup olmadıklarını kontrol etmek için sorulmuştur. Geliştirilen test tüm öğrencilere aynı koşullarda uygulanmıştır. Elde edilen verilerin analizinde tekrarlı ölçümlere sahip tek faktörlü ANOVA ve ilişkili gruplar *t* testi analizleri kullanılmıştır. Araştırmada anlamlılık düzeyi .05 kabul edilmiştir.

Bulgular

Araştırma bulgularına göre öğrencilerin %81'i altı sorudan oluşan çarpma + çıkarma/toplama gerektiren işlemsel sorulardan 5 veya 6 puan almışlardır. Bu nedenle problem çözmenin önkoşulu olan matematiksel işlem becerisi bakımından öğrencilerinin düzeyi yeterli seviyede bulunmuştur. İlişkili gruplar *t* testi sonuçlarına göre, öğrencilerin alışılagelmiş/tanıdık bağlamlı sözel problemlerden elde ettikleri başarı, alışılmışın dışında/aşina olunmayan problemlerden elde ettikleri başarıdan anlamlı derecede daha yüksektir ($t(866) = 34.58$, $p < .05$). Ayrıca, öğrencilerin görsel anlatımlı problemlerden elde ettikleri başarı, görsel anlatım içermeyen problemlerden elde ettikleri başarıdan anlamlı düzeyde daha yüksek bulunmuştur (t (866) = 25.09, $p < .05$).

Gruplariçi tekrarlı ölçümlere sahip tek faktörlü ANOVA sonuçlarına göre alt testlerden elde edilen ortalama puanlar arasında anlamlı bir farkın olduğu görülmektedir. Öğrenciler alışılagelmiş/tanıdık bağlama sahip ve görsel anlatımı olan problemlerden oluşan alt testten en yüksek puanı alırlarken, en düşük puanı alışılmışın dışında/aşina olunmayan ve görsel

anlatımlı olmayan problemlerden oluşan alt testten almışlardır.

Tartışma

Araştırma sonuçlarına göre problemin sahip olduğu bağlam problem zorluğunun en önemli kaynağıdır. Eldeki araştırma bulgularına benzer birçok araştırma da matematik ve problem çözmenin öğrenilmesinde bağlamın önemli olduğunu belirtmiştir.

Araştırma bulgularına göre, öğrencilerin alışılagelmiş/tanıdık bağlamlı sözel problemlerdeki problem çözme becerilerine ilişkin performansları alışılmışın dışında/aşina olunmayan sözel problemlerdeki becerilerine ilişkin performanslarından daha iyidir. Bu bağlamda alışılagelmiş/tanıdık bağlam sözel problem çözümünde önemli bir unsur olarak algılanabilir. Birçok araştırmada da belirtildiği gibi alışılagelmiş/tanıdık bağlam, problemin bağlamını daha anlamlı hale getirerek ve öğrencilerin de motivasyonunu artırarak problem çözmeyi desteklemektedir.

Araştırma bulgularına göre, görsel anlatımların sözel problemlerde yer alması problemin zorluğunu azaltan önemli bir kaynaktır. Bu bulgu birçok araştırma bulguları ile paralellik göstermektedir.

Araştırma bulgularına göre, görsel anlatımlı problemlerin problem çözme becerisini problemin bağlamından daha az desteklemektedir. Alışılmışın dışında/aşina olunmayan bağlama sahip görsel anlatımlı problemlerde öğrencilerin başarıları tanıdık bağlama sahip olan ancak görsel anlatımı olmayan problemlerdeki başarılarından daha yüksek çıkmıştır.

Bazı araştırmacılar öğrencilerin işleyen belleklerini etkilice kullanabildikleri takdirde problem çözme becerilerini önemli bir derecede geliştirebileceklerini savunmaktadırlar. Buna ek olarak, görselleştirmenin problem çözme aşamasında bilişsel yükü azalttığı ve öğrencilerin işleyen belleklerini daha etkili kullanmalarına olanak sağladığı savunulmaktadır. Buna bu araştırmadan elde edilen bulgular da katıldığında, görsel anlatımların özellikle alışılmışın dışında/aşina olunmayan bağlama sahip sözel problemlerde problemin zorluğunu önemli ölçüde azaltabileceği ve problem çözmeyi bu bağlamda destekleyebileceği söylenebilir.