Turk. J. Math. Comput. Sci.
12(2)(2020) 136–150
© MatDer
DOI : 10.47000/tjmcs.751730

Fuzzy Modelling of Covid-19 in Turkey and Some Countries in The World

Harun Baldemir[1,*], Agah Akin[2], Omer Akin[3]

[1]Department of Mathematics, Faculty of Science, Cankiri Karatekin University, 18100 , Cankiri, Turkey.
[2]Bitlis Tatvan State Hospital, 13200, Bitlis, Turkey.
[3]Department of Mathematics, Faculty of Sciences and Arts, TOBB Economics and Technology University, 06650, Ankara, Turkey.

Received: 11-06-2020 • Accepted: 12-11-2020

Abstract. Coronaviruses are a large family of viruses that are found in many different species of animals and are deadly illnesses for human. In late December 2019, China first announced the outbreak of a new coronavirus: Corona Virus Disease 2019 (or COVID-19), in which the symptoms are similar to common colds and flu. However it can sometimes be more serious, particularly for the elderly as well as patients with weak immune systems. The World Health Organization declared COVID-19 a global pandemic on March 11, 2020. As of date October 14, 2020, confirmed coronavirus cases exceeded 38 million including more than one million deaths worldwide.
In this paper, we use dynamical modelling approach, namely Fuzzyfied Richards Growth Model, to understand the dynamic behaviour of the COVID-19 based on the real data and to predict possible future scenarios applying fuzzy approaches for some countries around the world including China, the United States, the top five countries with the highest population in Europe and Turkey.

2010 AMS Classification: 34A07, 92D25, 00A71

Keywords: Novel coronavirus (COVID-19), Population dynamics, Fuzzy modelling, Zadeh's extention principle.

1. Introduction

All viruses are obligate intracellular parasites; they inhabit a no-man's-land between the living and the non-living worlds, and possess characteristics of both. They are now known to differ radically from the simplest true organisms, bacteria, in a number of respects [11]: They

- cannot be observed using a light microscope,
- have no internal cellular structure,
- are incapable of replication unless occupying an appropriate living host cell and
- are incapable of metabolism individuals show no increase in size.

Coronaviruses are enveloped, single-stranded RNA viruses that can cause infection in humans and animals. They have several clinical spectrum ranging from mild colds to severe respiratory distress. Coronaviruses are a family of viruses with four subgroups known as alpha, beta, gamma and delta.

At the end of 2019, a novel coronavirus was identified as the cause of a cluster of pneumonia cases in Wuhan, a city in the Hubei Province of China. It rapidly spread, resulting in an epidemic throughout China, followed by an increasing number of cases in other countries throughout the World. In February 2020 , the World Healthy Organization designated

*Corresponding Author
Email addresses: HarunBaldemir@karatekin.edu.tr (H. Baldemir), agh_akn@hotmail.com (A. Akin), omerakin@etu.edu.tr (O. Akin)

the disease COVID-19, which stands for coronavirus disease 2019 [28]. The virus that causes COVID-19 is designated severe acute respiratory syndrome coronavirus 2 (SARS-CoV-2); previously, it was referred to as 2019-nCoV.

The incubation period for COVID-19 is thought to be within 14 days following exposure, with most cases occurring approximately four to five days after exposure [6, 10, 16]. The spectrum of symptomatic infection ranges from mild to critical; most infections are not severe. Comorbidities that have been associated with severe illness and mortality include; cardiovascular disease, diabetes mellitus, hypertension, chronic lung disease, cancer and chronic kidney disease [7, 15, 17, 19, 23, 29, 32].

The possibility of COVID-19 should be considered primarily in patients with fever and/or respiratory tract symptoms who reside in or have traveled to areas with community transmission or who have had recent close contact with a confirmed or suspected case of COVID-19. There are no specific clinical features that can yet reliably distinguish COVID-19 from other viral respiratory infections. The most common clinical features of the onset of COVID-19 disease are fever, fatigue, dry cough, anorexia, myalgia and dyspnea [26].

Full-genome sequencing and phylogenic analysis indicated that the coronavirus that causes COVID-19 is a betacoronavirus in the same subgenus as the severe acute respiratory syndrome (SARS) virus, but in a different clade. The structure of the receptor-binding gene region is very similar to that of the SARS coronavirus, and the virus has been shown to use the same receptor, the angiotensin-converting enzyme 2 (ACE2), for cell entry [31]. The Middle East respiratory syndrome (MERS) virus, another betacoronavirus, appears more distantly related [18, 34].

At the height of the 2003 SARS epidemic, 140 new infected patients were reported weekly. SARS-CoV had a mortality rate of 9.7% and the majority of infections were nosocomial. The SARS epidemic ceased in 2003 within the year with a total of 8098 reported cases with 774 deaths globally [27].

According to the data of the World Healthy Organization on October 14, 2020, more than 38 million confirmed cases and one million total deaths of COVID-19 have been reported globally. In China, 85611 cases were reported from the first reported case at the end of 2019 until October 14, 2020, with the majority of those from Hubei and surrounding provinces.

Reporting Country	Total confirmed cases	Daily new cases*	Total deaths	Daily new deaths*
China	8.5611×10^4	17	4634	0
Germany	3.4174×10^5	4375	9771	17
France	7.7906×10^5	17936	33037	85
The United Kingdom	6.5464×10^5	19722	43155	91
Italy	3.7279×10^5	7331	36289	32
Spain	9.7999×10^5	13670	33413	209
Turkey	3.4045×10^5	1671	9014	57
The United States of America	8.1579×10^6	59725	221850	969

Table 1. Some countries with reported laboratory-confirmed COVID-19 cases and deaths. Data as of October 14, 2020. * Daily new cases and deaths are shown as 7 days average data.

There are some mathematical models in the literature to study and forecast epidemics [3, 4, 8, 9]. The main goal of this paper is to apply a mathematical growth model, namely fuzzyfied Richards Model, to describe the existing data and to predict future growth of COVID-19 in Turkey and some other countries. In particular, we use fuzzy environment for modelling since we can have more information about the COVID-19 using the Richards Model in fuzzy environment than classical Richards Model. As far as our knowledge according to literature in this field, this paper is the first to study Richards Model in the fuzzy environment. We hope that people in authorities can take earlier actions to get rid of the bad effects of the COVID-19 in the light of our results estimating the growth of this disaese.

The rest of the paper is organized as follows. We start with the historical background of population growth modelling and introduce the classical Richards Model in Section 2. Section 3 is devoted to prelimineries of fuzzy modelling and to introduction of the fuzzyfied Richards Model. In Section 4, we present the outbreak of COVID-19 focusing on some countries, which are namely China, the top five countries with the highest population in Europe, the United States and Turkey. The current situations and future estimations of those countries are presented in the light of fuzzy environments. Finally, in Section 5, we discuss the results, comparisions and our conclusions.

2. The History of The Population Growth Modelling

After getting introductory information about COVID-19, we need to introduce the mathematical background of the population dynamics. Mathematical modelling in population dynamics helps us to understand the dynamic processes of viruses spread out and to make practical predictions for their future size. From the mathematical point of view, there are mainly two types of modeling techniques; namely the continuous and discrete time approaches. Some population growth models can be explained in both continuous and discrete fashion. For example, in 1798, Thomas Malthus proposed a single species model, called the *Malthusian growth model*, where the rate of population growth is proportional to the size of the population. The Malthusian growth model, an early and famous population model, is written in a continuous form as

$$\frac{dN(t)}{dt} = rN(t)$$

where $N(t)$ is the population size at time t and r is the population growth. On the other hand, it can also be written in a discrete form as

$$N_{n+1} = N_n + rN_n.$$

The Malthusian growth model can describe the population during the initial stage of growth, but it fails for longer time due to the growth of population needs not be restricted.

In 1838, a more realistic model of population growth, called the Logistic population model, was introduced by Verhulst that allows the growth rate to depend upon the size of the total population. The continuous form of this model is given by

$$\frac{dN(t)}{dt} = rN(t)\left(1 - \frac{N(t)}{K}\right) \tag{2.1}$$

where K is the carrying capacity. Here, during the initial stage, the exponential growth occurs as seen in the Malthusian model due to the term $rN(t)$. Later, as the population grows, the term $\frac{rN^2(t)}{K}$ becomes almost as large as the first term and make a *S-shaped curve*. Hence, as time goes the population reaches a plateau about K, which represents the maximum population size i.e. the *carrying capacity*.

There have been several contributions suggesting alternative functional forms of the right hand side of the logistic model in (2.1) while keeping the sigmoidal shape and asymptotic properties of the S-shaped curve [25]. For example, Richards suggested a growth equation to fit the empirical plant data [24]. The Richards Model, which is a modification of the logistic model, is given by

$$\frac{dN(t)}{dt} = rN(t)\left(1 - \left(\frac{N(t)}{K}\right)^{\gamma}\right)$$

where K is the carrying capacity, r is the growth rate and γ is a measure of flexibility that allows to vary the shape of the sigmoidal curve. It is seen that if γ=1, the Richards model becomes the Logistic model.

The analytical solution of the Richards Model is given by

$$N(t) = K\left(1 + Ae^{-\gamma rt}\right)^{-1/\gamma}$$

where $A = \left(\frac{K}{N_0}\right)^{\gamma} - 1$ and N_0 is the initial population i.e. $N(0) = N_0$.

In this paper, we use the fuzzyfied Richards Model to predict the epidemic situation of COVID-19. The main reason why we prefer this model is to study cases where the data shows a single or several S-shaped curves distinguished by their turning (inflection) points. Also, we can control the shape of the sigmoidal curves by varying the parameter γ in the model.

In Richards Model, we have three parameters, namely the carrying capacity K, the growth rate r and a measure of flexibility γ. The least square method, a mathematical optimization technique, is used for curve fitting to compute these parameters according to the cumulative confirmed cases of COVID-19,

In the following section, we discuss the Richards model in a fuzzy environment. Before giving an algorithm for the fuzzyfied Richards model, we first provide a short fundamental background of fuzzy logic theory focusing on fuzzy initial value problems.

3. Preliminaries of Fuzzy Modelling

In this section, we introduce the mathematical background of fuzzy environment and introduce the main algorithm for the fuzzyfied Richards model.

Basic Concepts [2]. A fuzzy set A in a universe set X is a mapping $A(x) : X \to [0,1]$. We think A is assigning to each element $x \in X$ a degree of membership, $0 \leq A(x) \leq 1$. Let us denote by $\mathcal{F}$ the class of fuzzy subsets of the real axis $A(x) : X \to [0,1]$, satisfying the following properties:

- A is a convex fuzzy set, i.e. $A(r\lambda + (1-\lambda)s) \geq min[A(r), A(s)]$, $\lambda \in [0,1]$ and $r, s \in X$;
- A is normal, i.e. $\exists\, x_0 \in X$ with $A(x_0) = 1$;
- A is upper semicontinuous, i.e. $A(x_0) \geq \lim_{x \to x_0^+} A(x)$;
- $[A]^\circ = \overline{supp(A)} = \overline{\{x \in R \quad | \quad \mu(x) \geq 0\}}$ is compact where $\overline{A}$ denotes the closure of A.

Then $\mathcal{F}$ is called the space of fuzzy numbers. If A is a fuzzy set, we define $[A]^\alpha = \overline{\{x \in R \quad | \quad \mu(x) \geq \alpha\}}$ the $\alpha-$level (cut) set of A, with $0 \leq \alpha \leq 1$. For $u, v \in \mathcal{F}$ and $\lambda \in R$ the sum $u \oplus v$ and the product $\lambda \odot u$ are defined by $[u \oplus v]^\alpha = [u]^\alpha + [v]^\alpha$ and $[\lambda \odot u]^\alpha = \lambda [u]^\alpha$, $\forall \alpha \in [0,1]$. Additionally $u \oplus v = v \oplus u$, $\lambda \odot u = u \odot \lambda$. Also if $u \in \mathcal{F}$ then $\alpha-$cut of u denoted by $[u]^\alpha = [\underline{u}^\alpha, \overline{u}^\alpha]$, $\forall \alpha \in [0,1]$. Let $D : \mathcal{F} \times \mathcal{F} \to R_+ \cup \{0\}$, $D(u,v) = sup_{\alpha \in [0,1]} \max\{|\underline{u}^\alpha - \underline{v}^\alpha|, |\overline{u}^\alpha - \overline{v}^\alpha|\}$ be the Hausdorff distance between fuzzy numbers, where $[u]^\alpha = [\underline{u}^\alpha, \overline{u}^\alpha]$ and $[v]^\alpha = [\underline{v}^\alpha, \overline{v}^\alpha]$. The following properties are well-known [20, 21].

- $D(u \oplus w, v \oplus w) = D(u, v), \quad \forall u, v, w \in \mathcal{F}$
- $D(k \odot u, k \odot v) = |k| D(u, v), \quad \forall k \in R$ and $\forall u, v \in \mathcal{F}$
- $D(u \oplus v, w \oplus e) \leq D(u, v) + D(w, e), \quad \forall u, v, w, e \in \mathcal{F}$

and $(\mathcal{F}, D)$ is a complete metric space.

Zadeh's Extension Principle. In [30], Zadeh proposed the extension principle, which has become an important tool in fuzzy theory and its applications. With extension principle, we can find the fuzzyness of output of a function which has fuzzy input variables or fuzzy input variables and fuzzy relation simultaneously.

Let function $f(x_1, x_2, \ldots, x_n) : X \to Y$. Then fuzzy set B in Y can be obtained by function f and fuzzy sets $A_1, A_2, \ldots, A_n$ as follows

$$\mu_B(y) = \begin{cases} \max\limits_{y = f(x_1, x_2, \ldots, x_n)} [\min(\mu_{A_1}(x_1), \ldots, \mu_{A_n}(x_n))] & , f^{-1}(y) \neq 0 \\ 0 & , f^{-1}(y) = 0. \end{cases}$$

If f is one-to-one correspondence function [14, 30]

$$\mu_B(y) = \mu_A(x) = \mu_A(f^{-1}(y)); f^{-1}(y) \neq 0.$$

Fuzzy Initial Value Problem [2]. Consider the initial value problem

$$X'(t) = f(t, X(t)), \quad X(0) = X_0, \quad T = (a, b) \tag{3.1}$$

where X_0 is a fuzzy initial value and $f : [0,T] \times \mathcal{F} \to \mathcal{F}$ is obtained from such a continuous function $g : [0,T] \times U \to R^n, U \subset R^n$ by using Zadeh's extension principle. While solving (3.1) we consider fuzzy differential equation as a deterministic differential equation then we solve the deterministic differential equation. After getting deterministic solution, the fuzzy solution can be obtained by applying the extension principle to deterministic solution [20, 21].

For our aim in this paper, it is necessary to consider the problem given above in (3.1) and enough to examine the following IVP

$$x'(t) = g(t, x(t)), \quad x(0) = x_0. \tag{3.2}$$

Assume that (3.2) has the solution $x(t, x_0)$. Then if we apply Zadeh's extension principle, given in details in the following section, to $x(t, x_0)$ with respect to parameter x_0 we can find the fuzzy solution $x(t) = \tilde{x}(t, x_0)$, for all t.

An Algorithm for the Fuzzyfied Richards Model. To catch our point, it is necessary that we have to give the following fundamental knowledge for the sake use in this algorithm.

Definition 3.1 (Fuzzy Derivative [1])**.** Let $\overline{x}(t) : I \to \mathcal{F}$ be a fuzzy function, where I is an open subset of R, and $[\overline{x}(t)]^\alpha = [x_1(t,\alpha), x_2(t,\alpha)]$ be the $\alpha-$cut set of the fuzzy function $\overline{x}(t)$ for all $\alpha \in [0,1]$. We define an $\alpha-$cut set of the derivative $\overline{x}'(t)$ of the fuzzy function $\overline{x}(t)$ as follows:

$$[\overline{x}'(t)]^\alpha = [x_1'(t,\alpha), x_2'(t,\alpha)]$$

where

$$\overline{x_1}'(t,\alpha) = \min\{x_1'(t,\alpha), x_2'(t,\alpha)\}$$
$$\overline{x_2}'(t,\alpha) = \max\{x_1'(t,\alpha), x_2'(t,\alpha)\}.$$

Then we call the function $\overline{x}'(t)$ as the first order fuzzy derivative of the function $x(t)$. In a similar manner, the $n-th$ order fuzzy derivatives can be defined iteratively. It is known that the $\alpha-$cuts of the fuzzy function $\overline{x}(t)$ are representations of the fuzzy function $\overline{x}(t)$.

Taking into account the definition of this fuzzy derivative, we can give an algorithm of the fuzzyfied Richards model:

$$\begin{aligned} \overline{N}'(t) &= \overline{r}\overline{N}(t)\left(1 - \left(\frac{\overline{N}(t)}{\overline{K}}\right)^{\overline{\gamma}}\right) \\ \overline{N}(0) &= \overline{N_0} \end{aligned} \tag{3.3}$$

where $\overline{r}, \overline{K}, \overline{\gamma}, \overline{N_0}$ are fuzzy numbers and $\overline{N}(t)$ is a fuzzy solution, and their $\alpha-$cut sets are as follows:

$$[\overline{r}]^\alpha = [r_1(\alpha), r_2(\alpha)], \quad [\overline{K}]^\alpha = [K_1(\alpha), K_2(\alpha)], \quad [\overline{\gamma}]^\alpha = [\gamma_1(\alpha), \gamma_2(\alpha)], \quad [\overline{N_0}]^\alpha = [N_{0_1}(\alpha), N_{0_2}(\alpha)]$$

and

$$[\overline{N}(t)]^\alpha = [N_1(t,\alpha), N_2(t,\alpha)], \quad [\overline{N}'(t)]^\alpha = [N_1'(t,\alpha), N_2'(t,\alpha)].$$

Hence we get from (3.3):

$$[N_1'(t,\alpha), N_2'(t,\alpha)] = [r_1(\alpha), r_2(\alpha)][N_1(t,\alpha), N_2(t,\alpha)]\left(1 - \frac{[N_1(t,\alpha), N_2(t,\alpha)]}{[K_1(\alpha), K_2(\alpha)]}^{[\gamma_1(\alpha),\gamma_2(\alpha)]}\right)$$

As we know from the interval analysis in [13],

$$N_1'(t,\alpha) = \min\left\{[r_1(\alpha), r_2(\alpha)][N_1(t,\alpha), N_2(t,\alpha)]\left(1 - \frac{[N_1(t,\alpha), N_2(t,\alpha)]}{[K_1(\alpha), K_2(\alpha)]}^{[\gamma_1(\alpha),\gamma_2(\alpha)]}\right)\right\}$$

and

$$N_2'(t,\alpha) = \max\left\{[r_1(\alpha), r_2(\alpha)][N_1(t,\alpha), N_2(t,\alpha)]\left(1 - \frac{[N_1(t,\alpha), N_2(t,\alpha)]}{[K_1(\alpha), K_2(\alpha)]}^{[\gamma_1(\alpha),\gamma_2(\alpha)]}\right)\right\}.$$

Hence we have a set of min and max problems. It is not an easy task to determine the result of min and max problems. Thus we propose the following algorithm:

Step 1:

The problem is considered as a crisp problem and is solved.

Step 2:

Since $N(t)$ is a differentiable function, the behaviour of the crisp solution is investigated and according to the domains the following cases are considered:

- Let B_1 be the domain for which $N(t) \geq 0$ and $N'(t) \geq 0$.
- Let B_2 be the domain for which $N(t) \geq 0$ and $N'(t) \leq 0$.

Step 3:

After investigating the alternatives stated above, the $\alpha-$cut sets are for $\overline{N}(t)$ and $\overline{N}'(t)$ are to be investigated. They can be summarised in the following cases:

- $[N_1(t,\alpha), N_2(t,\alpha)]$ and $[N_1'(t,\alpha), N_2'(t,\alpha)]$ in domain B_1.
- $[N_1(t,\alpha), N_2(t,\alpha)]$ and $[N_2'(t,\alpha), N_1'(t,\alpha)]$ in domain B_2.

Step 4:

Next substitute the $\alpha-$cut sets obtained above in the fuzzyfied Richards model. Therefore, the fuzzyfied Richards model can be solved by means of $\alpha-$cut set operations.

4. Outbreak of COVID-19

Coronaviruses are a large family of viruses that are found in many different species of animals and deadly illnesses for humans such as severe acute respiratory syndrome (SARS) and Middle East respiratory syndrome (MERS). For example, there were 8098 reported cases and 774 deaths of SARS and 2494 reported cases and 858 deaths of MERS.

In late December 2019, China first announced the outbreak of a new coronavirus: Corona Virus Disease 2019 (abbreviated as COVID-19), the symptoms of which look very similar to common colds and flu. However it can sometimes be more serious, particularly for the elderly and patients with weak immune systems. The World Health Organization declared COVID-19 a global pandemic on March 11, 2020 when there were more than 100k reported cases and 4k deaths across 114 countries. Moreover, confirmed coronavirus cases exceeded 38 million worldwide including at least one million deaths until October 14, 2020.

4.1. **Epidemic Trends in China.** The outbreak of COVID-19 in humans occurred in Wuhan, China and initially was linked to a seafood market, which sells live animals. The market was closed on January 1, 2020. However, thousands of people in China, including many provinces such as Hubei, Zhejiang, Guangdong, Henan, Hunan, infected within a month. Furthermore, the disease traveled to other countries.

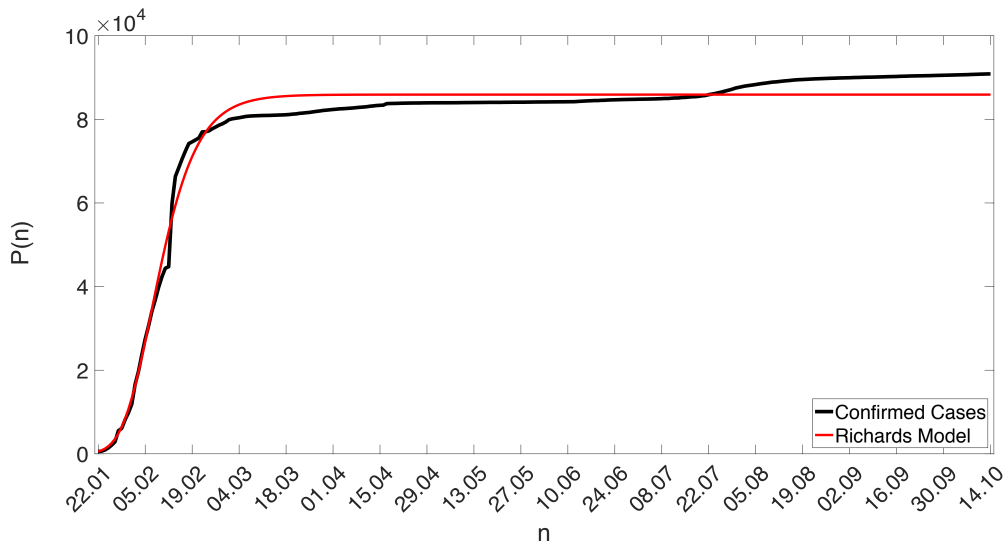

Figure 1. Comparison of the cumulative confirmed COVID-19 cases (black curve) and Richards model fitting curve (red curve). Here, the values of parameters are the carrying capacity K=8.5916×10^4, the growth rate r=0.6572 and the flexibility rate γ=0.2085 where the initial population N_0=548. x-axis represents the time as *dd.mm* (day.month) style and y-axis represents the cumulative number of the virus population.

In this section, we use mathematical modelling in population dynamics, namely Richards model, in order to describe the data and the possible scenarios with a range of initial *unreported cases* applying fuzzy approaches. In other words, we consider different possible scenarios using the reported number initial case as a fuzzy number. Before this, we estimate the parameters with the best fit curve according to the given data using curve-fitting techniques.

In order to fit the reported cases data to the solution of the Richards model in (4.1) we use the function `lsqcurvefit` in MATLAB software, which solves nonlinear data-fitting problems with the least-squares method. For example, in

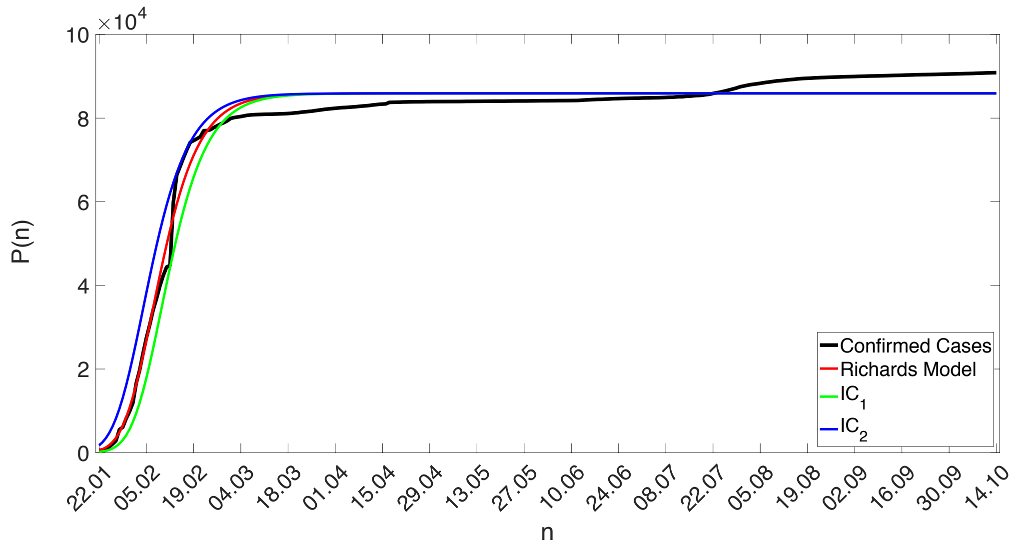

Figure 2. Fuzzyfied Richards model solutions where the green and blue curves represent the solutions of the model which are computed by taking the lowest (IC_1=175) and the highest (IC_2=1750) values of the $\alpha-$cuts for $\overline{N_0}$, respectively.

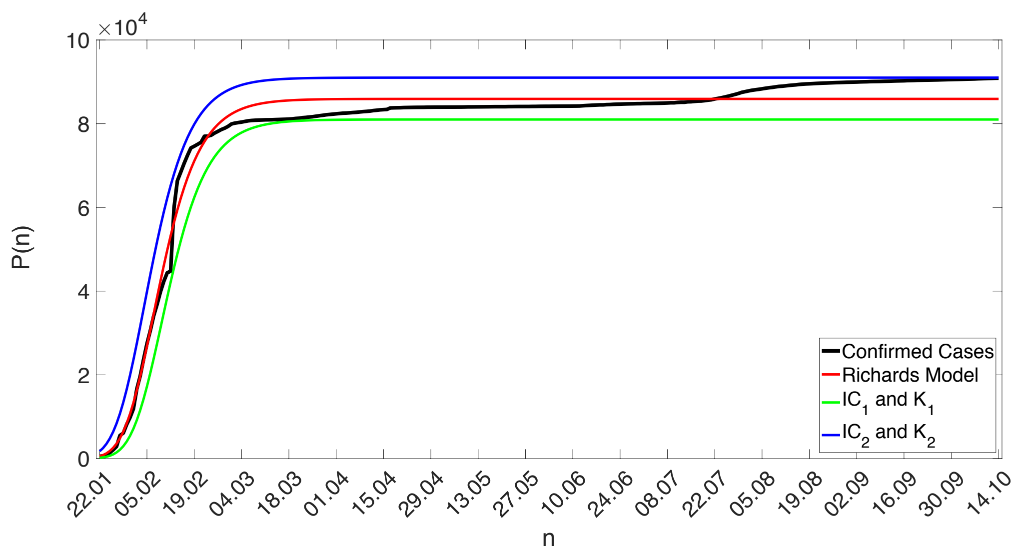

Figure 3. Fuzzyfied Richards model solutions where the green and blue curves represent the solutions of the model, which are computed by taking the lowest (IC_1=175 and K_1=8.1×10^4) and the highest (IC_2=1750 and K_2=9.1×10^4) values of the $\alpha-$cuts for $\overline{N_0}$ and $\overline{K}$, respectively.

Figure 1, we used the confirmed cases data (black curve) to estimate the parameters; the carrying capacity K, the growth rate r and the flexibility rate γ in the Richards model given by the function

$$N(t) = K\left(1 + Ae^{-\gamma rt}\right)^{-1/\gamma}. \tag{4.1}$$

Thus, using the cumulative confirmed cases data of COVID-19 in China, we calculated the carrying capacity K=8.5916×10^4 , the growth rate r=0.6572 and the flexibility rate γ=0.2085 where the initial population is N_0=548.

However, in the context of COVID-19, the initial number for confirmed cases cannot be precise due to the lack of information or even misleading information about the number of the infected people. In other words, the numbers of the confirmed cases are the numbers of the people who took the COVID-19 test and got positive result. On the other hand, there can be more people who are infected but have not been tested yet. Additionally, although COVID-19 test results of some people are positive, the fact that is they are not infected. This is just a failure of tests due to the some circumstances such as the reliability of tests. Therefore, it makes sense to consider the initially infected people in fuzzy sense.

Considering the fuzzy numbers for the initial value N_0, we can say more about the behaviour of the model. For example, in Figure 2, we consider the lowest and the highest values of the $\alpha-$cuts for $\overline{N_0}$, which are IC_1=175 and IC_2=1750, respectively. In fuzzy theory, we can state this as $[\overline{548}]^\alpha = [175 + 373\alpha, 1750 - 1202\alpha]$ where $0 \leq \alpha \leq 1$.

In terms of the population dynamics, then we can interpret the lower initial population due to the false information about the exact number of cases as mentioned above. The result of fuzzy initial values is shown in Figure 2. Assuming there were 175 infected people initially, we can draw the green curve (IC_1=175). Also assuming there were 1750 infected people initially, we can draw the blue curve (IC_2=1750). It can be seen from the Figure 2 that the reported data, black curve, is between the curves considering the lowest and highest initial population, which are the lowest and the highest values of the $\alpha-$cuts for $\overline{548}$ in fuzzy sense.

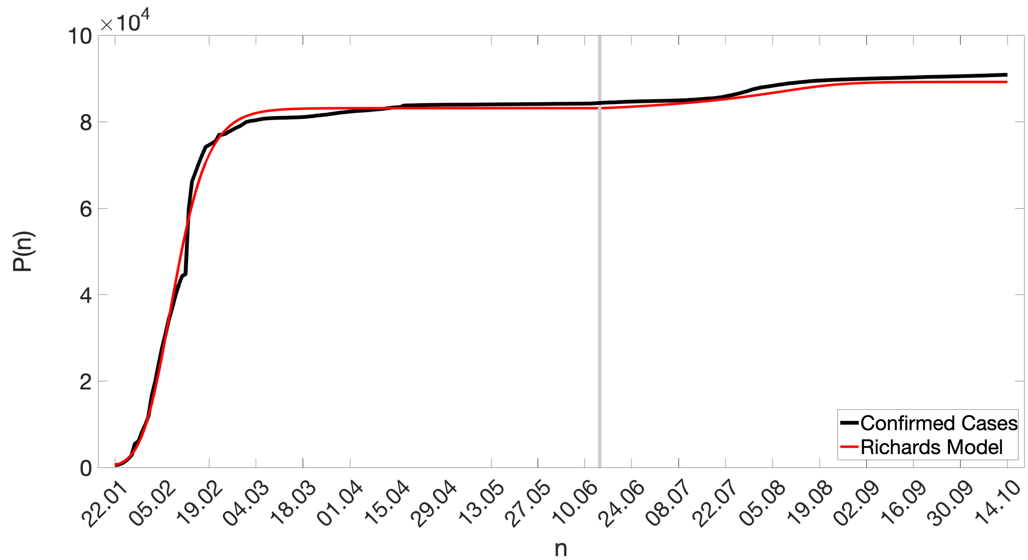

FIGURE 4. Two-stage solutions of the classical Richards model. Black and red curves represents the cumulative confirmed cases and multi-stage Richards model predictions, respectively. The horizontal line is where the first-stage of the outbreak ends (June 14) and the second-stage starts (June 15). Here, the values of parameters are the carrying capacity K=8.3158×10^4, the growth rate r=0.4308, the flexibility rate γ=0.3880 for the first-phase and K=8.9218e×10^4, r=0.0271 and γ=5.2353 for the second-phase.

We can see from the cumulative confirmed cases data that there is an increase about July 27, which may be considered as a second peak of the pandemic in China. Therefore, we should consider this change in data which can be control by the carrying capacity in the model. It is clear that fuzzyfying the carrying capacity K gives a broader range of estimation about the final size of COVID-19 in China. Additionally, we can consider values of two parameters for Richards model in a fuzzy environment, namely the carrying capacity K and the initial value N_0, in order to have the real data between the fuzzyfied solutions of the model.

Similar to fuzzyfying initial value N_0, we consider the lowest and highest values of the $\alpha-$cuts for $\overline{K}$ in fuzzyfied Richards model, which are K_1=8.1×10^4 and K_2=9.1×10^4, respectively. In fuzzy theory, we can state this as $\overline{[8.5532\times 10^4]}^{\alpha} = [8.1\times 10^4 + 4532\alpha, 9.1\times 10^4 - 5468\alpha]$ where $0 \le \alpha \le 1$. Also we considered the lowest and highest values of $\alpha-$cuts for $\overline{N_0}$, which were IC$_1$=175 and IC$_2$=1750, respectively (see Figure 1). In Figure 3, the reported real data (black curve) lies between the curves (green and blue) computed with different stages of fuzzyfied parameters $\overline{N_0}$ and $\overline{K}$. Hence, we can conclude that the fuzzy theory gives a broad range of estimation about the stages of COVID-19 in China.

As seen in Figure 1 that the Richards model cannot fit the cumulative data well starting from March since the data looks like having two different plateau stages i.e. two different S-shaped curve. Therefore, the classical Richards model does not fit well enough with the data. Similar type of data (i.e. multi-stages) was seen in the cumulative SARS case data of Greater Toronto area in 2003. This data was analyzed applying Richards model for each stages distinguished by their turning points [12]. Using this approach, called the multi-phase Richards model, we estimated the end date for China and the countries which will be studied throughout the rest of the paper. Taking the average of these end dates between June 10 and 20, we chose June 14 fixed as the end day for the first-phase and so June 15 as the first day of the second-phase of the cumulative data of COVID-19.

From now on we will apply the multi-stage Richards model fitting approach with COVID-19 cumulative case data of some countries.

4.2. **Epidemic Trends in Europe.** After the outbreak of COVID-19 in the China in December 2019, the virus has quickly spread to other regions of the world. By January 2020, a few cases had identified in some European countries. At the end of February 2020, Italy reported a significant increase of COVID-19 cases and the other European countries started reporting cases of of COVID-19.

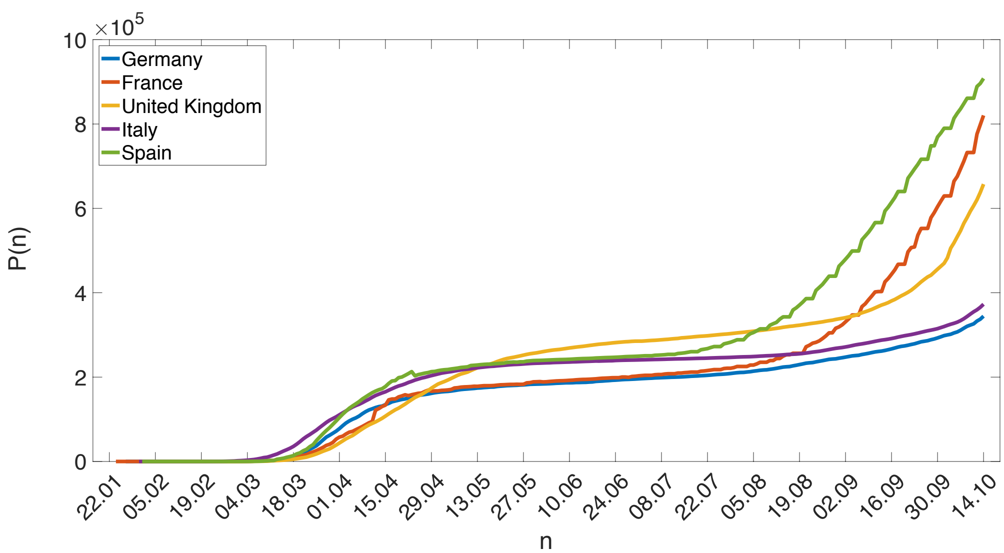

FIGURE 5. Cumulative confirmed COVID-19 cases in the top five countries with the highest population in Europe.

The World Health Organization (WHO) has declared COVID-19 outbreak a global pandemic on March 11, 2020. The WHO considered Europe the active centre of the COVID-19 pandemic on March 13, 2020 since the virus had rapidly spread through Italy and other European countries as it has slowed in China. All countries within Europe had a confirmed case of COVID-19 by 17 March.

In this section, we focus on the the top five countries with the highest population in Europe, namely Germany, France, United Kingdom, Italy and Spain. Figure 5 shows the cumulative COVID-19 cases of these countries. As seen in Figure 5, the cumulative numbers of the five countries look like plateauing between May and August. However, the number of infections has started increasing from day to day which are assumed the signs of the second phase of the pandemic in Europe.

We use the mathematical modelling techniques to investigate the possible scenarios of the outbreak of COVID-19 throughout the Europe considering these five countries. Applying the multi-stage Richards model, we first investigate the parameters; namely the carrying capacity K, the growth rate r and a measure of flexibility γ with the least square method for both the first and second stages. The computed parameters in Richards model are given in Table 2.

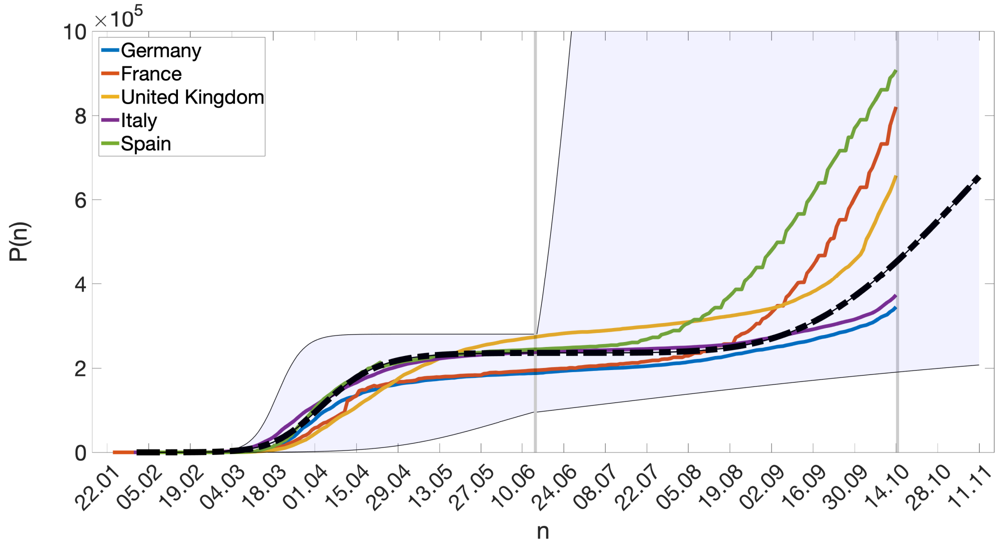

FIGURE 6. Cumulative confirmed COVID-19 cases and future predictions of the top five countries with the highest population in Europe. Black curve is drawn using the median values of the parameters, which are K=2.3589×10^5, γ=0.5435 and r=0.1759 for the first-phase and K=1.1446×10^6, γ=0.0258 and r=0.7125 for the second-phase. Shaded area shows the region computed by the lowest and the highest values of the $\alpha-$cuts for the parameters given in Table 2.

	First-Stage			Second-Stage		
	K	γ	r	K	γ	r
Germany	1.8117 ×10^5	0.5862	0.1603	0.2954 ×10^6	0.0177	1.3069
France	1.8568 ×10^5	0.8868	0.1328	1.7337 ×10^6	0.0513	0.4403
United Kingdom	2.8005 ×10^5	0.3237	0.1759	3.0437 ×10^6	0.0290	0.5463
Italy	2.3595 ×10^5	0.2593	0.2529	0.4568 ×10^6	0.0258	0.7125
Spain	2.3589 ×10^5	0.5435	0.1822	1.1446 ×10^6	0.0198	1.3299

TABLE 2. Parameter estimations of the multi-stage Richards model for the top five countries with the highest population in Europe.

Due to the extreme values (outliers) in the parameters, especially K and γ, in Table 2, we prefer to consider median values since such extreme values do not affect the median as strongly as they do the mean. Then, computing the median values of the parameter values in Table 2, we can consider these as the parameter of our crisp solution (see the dashed black curve in Figure 6). Then we can make these values as fuzzy numbers as

$$[\overline{\zeta}]^{\alpha} = \left[\zeta_{\min} + \left(\frac{\zeta_{\min} + \zeta_{\max}}{2}\right)\alpha, \quad \zeta_{\max} - \left(\frac{\zeta_{\max} + \zeta_{\min}}{2}\right)\alpha\right] \tag{4.2}$$

where ζ corresponds to the parameters K, γ, r and $\zeta_{\min}, \zeta_{\max}$ are the minimum and maximum, respectively.

In Figure 6, we plot the solution of the model considering the median values of the parameters (see black dashed curve) for the first-phase, where the end date is on June 14. Then, we make the parameters fuzzy numbers as in (4.2) for this phase. The shaded region is enclosed between the curves considering the maximum and the minimum vales of fuzzy parameters as well as four weeks future population of COVID-19. Then, we consider the second-phase of the pandemic starting from June 15 and fit the cumulative number data of this phase for each country to estimate the parameters.

The shaded regions for each stages can be considered as the worst-best possible scenarios of the COVID-19 pandemic in Europe. It can be seen in Figure 6 that the last part of first-phase is well captured by Richards model due to the plateau of the data. However, a rapid increase of the pandemic in the last few weeks for some countries such as United Kingdom results a high estimate on the parameters; especially on the carrying capacity K. Therefore the upper curve of the second-phase is considerably large for the future estimates (3.0437×10^6).

4.3. **Epidemic Trends in Turkey.** The first case in Turkey was reported on March 10, 2020, much later than many European countries. The case had a history of travel from Europe. On March 13, 2020, 4 new cases were reported with a steep rise to 2433 out of 33004 tested (7%) within 14 days as of March 25, 2020, with a death rate of 2.4% (n = 59). The daily number of tests was reported to be 5035 on day 14 and by the March 27, 3629 were recorded by the WHO with 75 deaths.

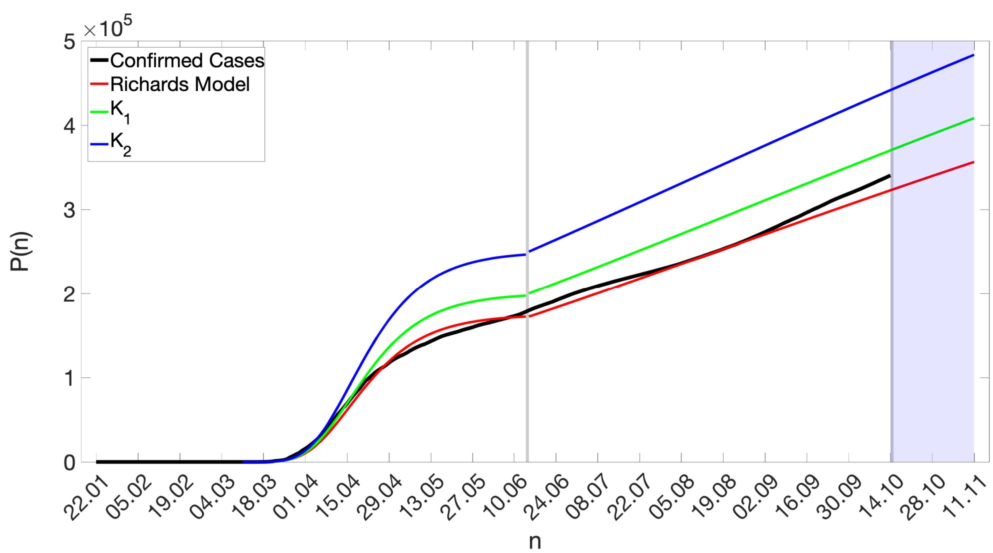

FIGURE 7. Future estimation of the COVID-19 cases in Turkey applying the multi-stage fuzzyfied Richards model. Black and red curves represent the cumulative confirmed COVID-19 cases and the model solutions, respectively. Here, the carrying capacity is K=1.7479 $\times 10^5$, the growth rate is r=5.3276 and γ=0.0135 for the first-phase and K=7.0689$\times 10^5$, r=0.3172 and γ=0.0154 for the second-phase. The green and blue curves are computed by taking the lowest (K_1=8$\times 10^5$) and the highest (K_2=9$\times 10^5$) values of the $\alpha-$cuts for $\overline{K}$, respectively. The shaded region is the future prediction for four weeks.

On day 24 of the epidemic, the total number of tests was 125556, positive cases was 18135, with 356 deaths. Istanbul has the highest number of cases (n = 8852) compared to others followed by Izmir (n = 853), Ankara (n = 712) and Konya (n = 584). On day 72 of the epidemic, the total number of tests was 2.0392 $\times 10^6$, positive cases was 1.6394 $\times 10^5$, with 4540 deaths. On day 217 (October 14, 2020), the total number of tests is about 1.19$\times 10^7$, positive cases is 3.4045 $\times 10^5$, with 9014 deaths.

The start of the epidemic shows great similarity with the rapidly rising trend observed at the early stages in Italy and Spain. Although the health authorities and the government in Turkey were quick to take action by cancelling schools and social, cultural, sports activities and scientific meetings, a nationwide lockdown was not put into practice [22].

After discussing the earlier and present stages of COVID-19 in Turkey, we can now deal with the modelling of the pandemic in order to estimate future stages of COVID-19. Similar to the previous analyses in this section, fitting the curve with the multi-phase Richards model, we find that the carrying capacity is K=1.7479×10^5, the growth rate is r=5.3276 and the flexibility rate γ=0.0135 for the first-phase and K=7.0689×10^5, r=0.3172 and γ=0.0154 for the second-phase. Then carrying the similar analysis of the prediction of COVID-19 cases for Europe, we considered Richards model for Turkey to estimate the range for the total number of future COVID-19 cases. This is done by fuzzying the parameter K for the second-phase, which corresponds to the carrying capacity, as

- $[\overline{K}]^\alpha = [K, 8\times 10^5 - 0.9311\times 10^5\alpha]$ where $0 \le \alpha \le 1$. (This corresponds to the green curve in Figure 7.)
- $[\overline{K}]^\alpha = [K, 9\times 10^5 - 1.9311\times 10^5\alpha]$ where $0 \le \alpha \le 1$. (This corresponds to the blue curve in Figure 7.)

Here K=7.0689×10^5 is the carrying capacity for the second-phase as given above.

4.4. **Epidemic Trends in the United States.** The first confirmed case of COVID-19 in the US was recorded in January, 2020 and the first known deaths happened in February, 2020. By the end of March, the COVID-19 pandemic has spread all over the US. On March 27, the US recorded highest number of COVID-19 cases globally and on April 12, the US reported the most coronavirus deaths in the world.

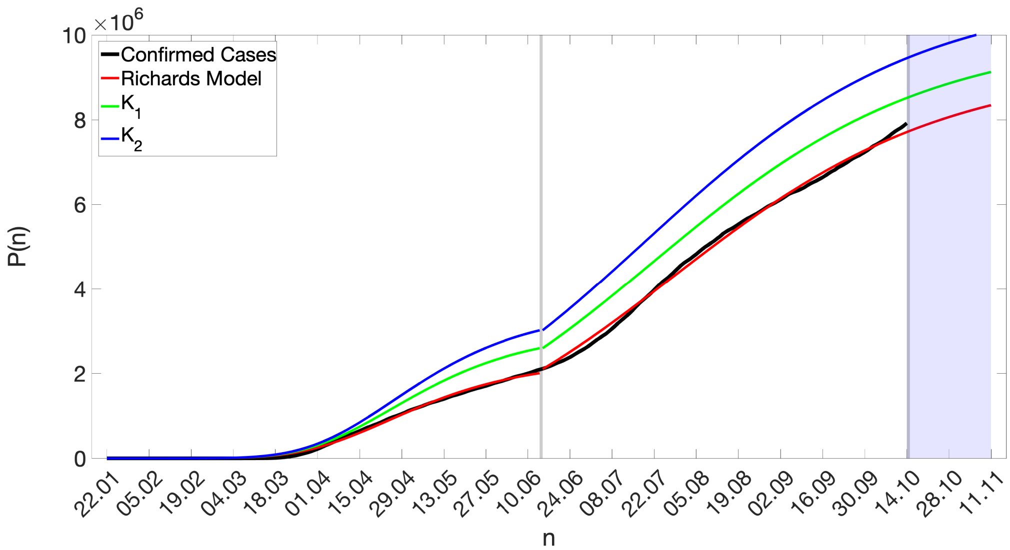

FIGURE 8. Future estimation of the COVID-19 cases in the US applying the multi-stage fuzzyfied Richards model. Black and red curves represent the cumulative confirmed COVID-19 cases and the model solutions, respectively. Here, the carrying capacity is K=2.3083×10^6, the growth rate is r=0.3383 and γ=0.1150 for the first-phase and K=9.2722×10^6, r=0.0326 and γ=0.6429 for the second-phase. The green and blue curves are computed by taking the lowest (K_1=1.0×10^7) and the highest (K_2=1.1×10^7) values of the $\alpha-$cuts for $\overline{K}$, respectively. The shaded region is the future prediction for four weeks.

Fitting the data of the US with the multi-phase Richards model, we calculate that the carrying capacity is K=2.3083×10^6, the growth rate is r=0.3383 and γ=0.1150 for the first-phase and K=9.2722×10^6, r=0.0326 and γ=0.6429 for the second-phase. In order to estimate the range for the total number of future COVID-19 cases for the US we make the parameter K fuzzyfied, which corresponds to the carrying capacity or final size of the population for the second-phase, as

- $[\overline{K}]^{\alpha} = [K, 1.0 \times 10^{7} - 0.7278 \times 10^{6}\alpha]$ where $0 \leq \alpha \leq 1$. (This corresponds to the green curve in Figure 8.)
- $[\overline{K}]^{\alpha} = [K, 1.1 \times 10^{7} - 1.7278 \times 10^{6}\alpha]$ where $0 \leq \alpha \leq 1$. (This corresponds to the blue curve in Figure 8.)

Here K=9.2722×10^6 is the carrying capacity for the second-phase as given above.

5. Comparisons, Results and Conclusions

In December 2019, a novel coronavirus, called coronavirus disease 2019 or shortly COVID-19, was identified in China. Then rapidly the virus has spread around the world which results the World Health Organization (WHO) classified it as a pandemic in March 2020. Although COVID-19 is characterized by similar symptoms to other respiratory illnesses such as the flu and common cold, it is more deadly than these diseases, especially among the elderly and those with weak immune systems.

There are many mathematical models to describe population dynamics [5] including Richards model. The cumulative number of infectious disease cases is well described by the Richards model [33]. The cumulative SARS case data of Greater Toronto area in 2003 shows a multi-staged epidemic and Richards model still could be used to understand the outbreak [12]. The cumulative number of confirmed cases of the COVID-19 for many countries behaves as a multi-phase pandemic nowadays.

Therefore, in order to understand the dynamic behaviour of the COVID-19, we apply a mathematical modelling technique, called the multi-phase Richards population model, to describe different possible scenarios using the reported cases. Due to the lack of information or misleading information about the number of reported cases of COVID-19, we could think that these reported numbers given by authorities might not be very accurate and reliable. Therefore, we apply fuzzy modelling approaches to describe possible scenarios of COVID-19.

The number of the initial confirmed cases for China is 548 although other counties had just started reporting COVID-19 cases on January 22. Since the number of the confirmed cases indicates the number of the people with positive COVID-19 test result, especially during the early time of COVID-19 in China, health experts believed many patients infected with coronavirus were nevertheless getting a negative test result. Moreover, there were many more infected people who had not been tested initially. For the fuzzy modelling purposes, therefore, we can consider the initial number of the infected people as a number in fuzzy form.

Thus, we consider the lowest and the highest values of the $\overline{N_0}$ in a fuzzy environment, which are 175 and 1750, respectively where the reported number is 548 i.e. $[\overline{548}]^{\alpha} = [175 + 373\alpha, 1750 - 1202\alpha]$, $0 \leq \alpha \leq 1$. Figure 2 clearly shows that during the a rapid expansion stage (until March), the curve of reported number of COVID-19 data in China lies between the estimated curves computed by fuzzyfying N_0. However, starting from March, the model solutions do not show good fits with the real data. For the stage starting from March, we also consider the value of the carrying capacity K in fuzzy environment. Taking into account the lowest and highest values of $\alpha-$cuts of $\overline{N_0}$ and $\overline{K}$, the reported cumulative data lies between the curves computed by the lowest and highest values of $\overline{N_0}$ and $\overline{K}$ (see Figure 3). Hence, we can conclude that the fuzzy modelling approach gives a broad range of estimation about various stages of COVID-19 in China.

Additionally, instead of applying the fuzzyfied Richards model to the whole cumulative confirmed data set at once in order to estimate some possible outcome of COVID-19 in China, we can divide the data into different stages distinguished by their turning points and Richards model can be applied each of the stages separately. We applied this technique to the cumulative reported number of COVID-19 in China for two stages (before and after June 15) and showed a great fit for both of the two stages with the classical multi-stage Richards model (see Figure 4).

After the outbreak of COVID-19 in the China, the virus has rapidly spread and the rest of the world has started be aware of COVID-19. Therefore, initial number of reported COVID-19 cases for the other counties are very small and considering these low initial values as fuzzy numbers does not make any more sense. In fuzzy modelling techniques, both the initial value, N_0, and parameters; namely the carrying capacity K, the growth rate r and a measure of flexibility γ, can be fuzzyfied. For the top five countries with the highest population in Europe, namely Germany, France, United Kingdom, Italy and Spain, we apply the Richards model to investigate the parameters with the least square method first. Then we make these parameters as fuzzy parameters considering the minimum and maximums for the lowest and the highest values of the $\alpha-$cuts. The future size of COVID-19 in Europe is estimated applying these fuzzyfied parameters.

Turkey's first coronavirus case was reported on March 10. Although, the spread of the new coronavirus outbreak slowed down in Turkey during summer, it has started increasing again. There are about total 340k confirmed cases as of October 14. The Richards model estimates the final size of COVID-19 in Turkey about 700k considering the second-phase of the data. However, one can argue that this final size seems not very reasonable since the second-phase behaves fully increasing starting from the beginning. Therefore, it could be expected to have a slow down and see an S-shaped as seen in many countries especially in Europe. For the four weeks future prediction, we fuzzyfied Richards Model to estimate final size of the pandemic taking the the carrying capacity, $\overline{K}$, as a fuzzy number. This gives us a wide range of estimation about the future and final size the virus. Fitting the data to the Richards model, we find that the carrying capacity is K=7.0689×10^5, which corresponds to the final size of the corona virus population in Turkey. However, it is better to consider this value as a fuzzy value so that we can have a range of estimation about the final size. Additionally, this fuzzyfied parameter also effects the dynamic behaviour of the model not only at the final stage but also during the middle stage. For example a higher value for the final size makes the middle stage higher and this situation can be interpreted as a lack of information about the actual size of the information due to the number of virus tests. Hence, we hope that the authorities may get benefit from the estimation of the final size of COVID-19 to take earlier actions.

Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this article.

References

[1] Akın, O., Khaniyev, T., Turksen, I., Oruc, O., *Some possible fuzzy solutions for second order fuzzy initial value problems involving forcing terms*, Applied and Computational Mathematics, (2014). 3.1

[2] Akın, O., Oruç, O., *A prey predator model with fuzzy initial values*, Hacettepe Journal of Mathematics and Statistics, **41**(3)(2012), 387-395. 3, 3

[3] Ball, F.G., *Dynamic population epidemic models*, Mathematical Biosciences, **107**(2)(1991), 299-324. 1

[4] Bauch, C.T. Lloyd-Smith, J.O., Coffee, M.P., Galvani, A.P., *Dynamically modeling SARS and other newly emerging respiratory illnesses: past, present, and future*, Epidemiology, (2005), 791-801. 1

[5] Brauer, F., Castillo-Chavez, C., Mathematical Models in Population Biology and Epidemiology. Springer, 2012. 5

[6] Chan, J.F.-W., Yuan, S., Kok, K.-H., To, K.K.-W., Chu, H., Yang, J., Xing, F., Liu, J., Yip, C.C.-Y., Poon, R.W.-S., *et al.*, *A familial cluster of pneumonia associated with the 2019 novel coronavirus indicating person-to-person transmission: A study of a family cluster*, The Lancet **395**(10223)(2020), 514-523. 1

[7] Chow, N., Fleming-Dutra, K., Gierke, R., Hall, A., Hughes, M., Pilishvili, T., Ritchey, M., et al., *Preliminary estimates of the prevalence of selected underlying health conditions among patients with coronavirus disease 2019–United States, February 12–March 28, 2020*, Morbidity and Mortality Weekly Report, **69**(13)(2020), 382. 1

[8] Chowell, G., Sattenspiel, L., Bansal, S., Viboud, C., *Mathematical models to characterize early epidemic growth: A review*, Physics of Life Reviews, **18**(2016), 66-97. 1

[9] Funk, S., Camacho, A., Kucharski, A.J., Lowe, R., Eggo, R.M., Edmunds, W.J., *Assessing the performance of real-time epidemic forecasts: A case study of ebola in the western area region of Sierra Leone, 2014-15*, PLoS Computational Biology, **15**(2)(2019). 1

[10] Guan, W., Ni, Z., Hu, Y., Liang, W., Ou, C., He, J., Liu, L., Shan, H. Lei, C., Hui, D., et al., *China medical treatment expert group for Covid-19. clinical characteristics of coronavirus disease 2019 in China*, Clinical characteristics of coronavirus disease (2019), 1708-1720. 1

[11] Hogg, S., Essential microbiology. John Wiley & Sons, 2013. 1

[12] Hsieh, Y.-H., Cheng, Y.-S., *Real-time forecast of multiphase outbreak*, Emerging Infectious Diseases, **12**(1)(2006), 122. 4.1, 5

[13] Kaleva, O., *Fuzzy differential equations*, Fuzzy Sets and Systems, **24**(3)(1987), 301-317. 3

[14] Lee, K.H., *First course on fuzzy theory and applications*, Springer Science & Business Media, **27** (2004). 3

[15] Liang, W., Guan, W., Chen, R., Wang, W., Li, J., Xu, K., Li, C., Ai, Q., Lu, W., Liang, H., et al., *Cancer patients in SARS-CoV-2 infection: A nationwide analysis in China*, The Lancet Oncology, **21**(3)(2020), 335-337. 1

[16] Li, Q., Guan, X., Wu, P., Wang, X., Zhou, L., Tong, Y., Ren, R., Leung, K.S., Lau, E.H., Wong, J.Y., et al., *Early transmission dynamics in Wuhan, China, of novel coronavirus–infected pneumonia*, New England Journal of Medicine, 2020. 1

[17] Lighter, J., Phillips, M., Hochman, S., Sterling, S., Johnson, D., Francois, F., Stachel, A., *Obesity in patients younger than 60 years is a risk factor for Covid-19 hospital admission*, Clin Infect Dis, **9**(10)(2020), 1093. 1

[18] Lu, R., Zhao, X., Li, J., Niu, P., Yang, B., Wu, H., Wang, W., Song, H., Huang, B., Zhu, N., et al., *Genomic characterisation and epidemiology of 2019 novel coronavirus: implications for virus origins and receptor binding*, The Lancet, **395**(10224)(2020), 565-574. 1

[19] Mehra, M.R., Desai, S.S., Kuy, S., Henry, T.D. , Patel, A.N., *Cardiovascular disease, drug therapy, and mortality in Covid-19*, New England Journal of Medicine, (2020). 1

[20] Mizukoshi, M.T. , Barros, L.d., Chalco-Cano, Y., Román-Flores, H., Bassanezi, R.C., *Fuzzy differential equations and the extension principle*, Information Sciences, **177**(17)(2007), 3627-3635. 3, 3

[21] Oberguggenberger, M., Pittschmann, S., *Differential equations with fuzzy parameters*, Mathematical and Computer Modelling of Dynamical Systems, **5**(3)(1999), 181-202. 3, 3

[22] Petersen, E., Gokengin, A.D., *SARS-CoV-2 epidemiology and control, different scenarios for Turkey*, Turkish Journal of Medical Sciences, **50** (2020), 509-514. 4.3

[23] Petrilli, C.M., Jones, S.A., Yang, J., Rajagopalan, H., O'Donnell, L., Chernyak, Y., Tobin, K.A., Cerfolio, R.J., Francois, F., Horwitz, L.I., *Factors associated with hospital admission and critical illness among 5279 people with coronavirus disease 2019 in New York City: Prospective cohort study*, BJM, **369**(2020). 1

[24] Richards, F., *A flexible growth function for empirical use*, Journal of Experimental Botany, **10**(2)(1959), 290-301. 2

[25] Tsoularis, A., Wallace, J., *Analysis of logistic growth models*, Mathematical Biosciences, **179**(1)(2002), 21-55. 2

[26] Wang, D., Hu, B., Hu, C., Zhu, F., Liu, X., Zhang, J., Wang, B., Xiang, H., Cheng, Z., Xiong, Y., et al., *Clinical characteristics of 138 hospitalized patients with 2019 novel coronavirus–infected pneumonia in Wuhan, China*, Jama, **323**(11)(2020), 1061-1069. 1

[27] World Health Organization et al., Consensus document on the epidemiology of severe acute respiratory syndrome (SARS), WHO, 2003. 1

[28] World Health Organization et al., *WHO director-general's remarks at the media briefing on 2019-nCoV on 11 February 2020*, WHO, (2020). 1

[29] Wu, Z., McGoogan, J.M., *Characteristics of and important lessons from the coronavirus disease 2019 (Covid-19) outbreak in China: Summary of a report of 72 314 cases from the Chinese center for disease control and prevention*, Jama, **323**(13)(2020), 1239-1242. 1

[30] Zadeh, L.A., *The concept of a linguistic variable and its application to approximate reasoning*, Information Sciences, **8**(4)(1975), 301-357. 3

[31] Zhou, P., Yang, X.-L., Wang, X.-G., Hu, B., Zhang, L., Zhang, W., Si, H.-R., Zhu, Y., Li, B., Huang, C.-L., et al., *A pneumonia outbreak associated with a new coronavirus of probable bat origin*, Nature, **579**(7798)(2020), 270-273. 1

[32] Zhou, F., Yu, T., Du, R., Fan, G., Liu, Y., Liu, Z., Xiang, J., Wang, Y., Song, B., Gu, X., et al., Clinical course and risk factors for mortality of adult inpatients with Covid-19 in Wuhan, China: A retrospective cohort study, The Lancet, 2020. 1

[33] Zhou, G., Yan, G., *Severe acute respiratory syndrome epidemic in Asia*, Emerging Infectious Diseases, **9**(12)(2003), 1608-1610. 5

[34] Zhu, N., Zhang, D., Wang, W., Li, X., Yang, B., Song, J., Zhao, X., Huang, B., Shi, W., Lu, R., et al., *A novel coronavirus from patients with pneumonia in China, 2019*, New England Journal of Medicine, (2020). 1