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Research Article

Multi-representation raised by prospective mathematics teachers in expressing algebra

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Abstract

This research investigated types of multi-representations raised by prospective teachers in expressing algebra. This study used a qualitative approach with case study methods. As many as 112 prospective mathematics education teachers from Universitas Negeri Malang of Indonesia participated. Researchers used Algebra Task Sheet and Interview Guide Sheet as data collection tools. Then an analysis is carried out so that the following categories are obtained: subjects that symbolically express algebra, pictorial, and geometric. The results obtained showed that 74% of subjects expressed symbolically algebra, while 15% of subjects expressed pictorially, and 11% of subjects expressed geometrically. The research findings showed that there are three forms of representation raised by the subject in expressing algebra, namely the representation of algebraic symbols, image representations, and geometric representations. Most of the participants produced algebraic symbolic representations and some of them experienced obstacles in producing pictorial representations and geometric representations, also researcher found similar patterns in producing geometric representation namely, perception, appearance, strategy, and re-examination. Researchers recommend geometric representations for further research because they tend to be done by subjects with high mathematical abilities and rarely found research that produces geometrical representations when solving algebraic problems.

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Introduction

Algebra is one of the important material in the domain of mathematics learning. One of the four mathematical problem domains that have the highest percentage in TIMSS is algebra problems, which is 30%, but the percentage of students who answered correctly is still low, namely only 18% (Mullis et al. 2012). The survey results in the Program for International Student Assessment (PISA) published in his article, Lailiyah et al. (2018) that Indonesia is ranked 64th out of 65 countries. The results of previous studies also revealed that there are still many students who experience difficulties regarding the use of symbolic algebra (Lutaif et al. 2019). Generally, students who can solve algebra problems are students who have high mathematical abilities (Covington et al. 2019). The development of students' cognitive abilities capable of completing algebra is more complex and comprehensive (Lepak et al. 2018), and students tend to and efficiently use arithmetic or recursive strategies than algebraic strategies when completing algebra (Walkington et al. 2012). For that reason, the focus of this research is algebra material.

Algebra problem is the main focus of several studies in various countries today. In the United States, most students have understood the main ideas related to algebra problems including understanding equivalent meanings, expressions, similarities, and inequalities, developing arithmetic generalizations, building functional thinking in

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problem-solving, and understanding variables (Blanton et al. 2015; Jacobs et al. 2015). Whereas in the west coast of America, algebraic understanding is used as a reference to identify the development and cognitive abilities of students at the secondary level (Lepak et al. 2018). Furthermore, in Brazil, most students have understood algebraic procedures and are able to link algebraic representations with visual representations and it is important for prospective teacher students to know the form of representation that students emerge in completing algebra (David et al. 2014). In Northeastern United States, students have developed mental strategies in algebra specifically on the problem of equivalence, expressions, equations, and inequalities are able to utilize the representation of algebraic symbols (Blanton et al. 2015). The country of Romania has also researched students' related algebraic knowledge through text-based problems (Máté, 2019). Unlike the findings in Southeast Asia (Malaysia, Thailand, Singapore and Indonesia) which tend to focus on the difficulties experienced by students. Namely, difficulties in understanding algebraic expressions, implementing numerical operations, understanding the different meanings of signs equal to and understanding variables (Jupri et al. 2014). The factors that cause students' difficulties in solving algebra problems are the inadequacy of ideas or understanding they have in linking mathematical concepts as well as between mathematical representations.

Representation is one of the important domains in mathematics learning at all mathematical levels. The importance of these representation was revealed by several researchers (Cankoy & Özder, 2011; Chang et al. 2016; Sajadi et al. 2013). It is also, according to what was recommended by (Addition, 2015) that to encourage students' mathematical understanding it is necessary to incorporate various representations (multi-representations) into the school curriculum. Mathematical ideas can not only be described through one representation, but the use of various forms of representation (external/internal) that can facilitate the improvement of mathematical understanding (Mcgee, 2016; Papadopoulos, 2019). If students can use various forms of representation, students will find it easier to express mathematical problems (Ott et al. 2018). Representation in principle refers to products and processes that are obtained externally, as well as processes that are obtained internally in the minds of individuals (Powell et al. 2019). Internal representations refer to mental actions carried out by individuals, while external representations refer to products used to communicate mathematical ideas in the form of words (verbal language), pictures, tables, graphics, and schemes (visual language), numeric and algebra (symbolic language) (Montenegro et al. 2018). However, the use of symbolic representations is more dominant compared to other representations in mathematics learning so that in transforming between representations that can support students' mathematical abstractions (Garnelo & Shanahan, 2019), and it is difficult for students to convert between representations (Presmeg, 2014). Some researchers also note that students still have difficulty integrating information between representations, for example in functions (Adu-gyamfi & Bosse, 2013), calculus (Chang et al. 2016; Mcgee, 2016), 2D and 3D geometry (Fujita et al. 2017). So that the use of representation is needed in teaching and learning mathematics.

Ineffective use of the representation of prospective teacher students in understanding a mathematical concept. This happens, because they have different ideas and still have weak connections in understanding a mathematical concept (Adu-gyamfi & Bosse, 2013). Without connections between representations, students cannot develop strategies to solve problems. The inability to translate between representations becomes the main problem in learning mathematical concepts (Leikin et al. 2013). This resulted in the failure of students to integrate information between representations (Fujita et al. 2017). Students also experience errors in constructing the construction of meaning between mathematical representations, namely representational attributes, interference thinking, pseudo-thinking, disconnected connections, implementation errors and lack of preservation of representative equivalence (Afriyani et al. 2019). There are two characteristics of students' understanding in solving mathematical problems related to multiple representations, namely flexibility and compartmentalization (Sa'dijah et al. 2018). In general, problems experienced by students or prospective teachers, due to lack of flexibility in representing mathematical ideas or the ability to translate between representations. Representation acts as a tool for thinking, presenting more concrete mathematical ideas and supporting and broadening student reasoning in solving mathematical problems. Various forms of representation that can be used in expressing or solving mathematical problems, for example symbolically and visually.

Symbolic representations are one of the most commonly used representations of students in solving mathematical problems. This is consistent with the results of research Fujita et al. (2017) which states that students are more dominant using symbolic representations than other representations in solving algebra problems. Verbal and symbolic representations are used to count, detect, correct mistakes, and justify students' answers in completing quadratic functions (Santia & Sutawidjadja, 2019). Most students are not able to show their creativity in writing problem writing geometry words (verbal), so that they are only motivated by what is already known in the textbook (Levenberg, 2014).

Symbolic representations are used as a basis for solving computational problems in students in Asia (Sari et al. 2018). Besides being symbolic, visualization is also an important aspect of solving mathematical problems.

Visualization is one of the important aspects in improving the mathematical abilities of teachers and students at all mathematical levels, for example in linear algebra (Souto-Robio, 2012), addition operations (Teppo & van den Heuvel-Panhuizen, 2014), image patterns (Montenegro et al. 2018), geometric (Llinares & Clemente, 2014), limits (Kidron & Tall, 2015) and, calculus (Natsheh & Karsenty, 2014). Souto-Robio (2012) advocating the use of visualization in mathematics at the university level, claims that visualization is very important to achieve a deep understanding that enables mathematical thinking to progress. Teppo and Van Den Heuvel-Panhuizen (2014) investigated students' understanding of number lines based on visual aspects by realizing the complexity of the mathematical meanings used. Montenegro et al. (2018) revealed that visualization is a tool to communicate mathematical ideas graphically, tabularly, and schematically for the problem of image patterns. Llinares and Clemente (2014) investigated 97 teacher candidates who were asked to solve geometric problems with the finding that the process of visualization played a role in recognizing properties, definitions, relationships in geometric and justification processes. Natsheh and Karsenty (2014) conducted a case study investigation on students and teachers found that visualization was needed to develop ideas, graphs, iconic, and diagrammatic problems in calculus. However, visualization tends to be associated with spatial aspects as a visual-spatial ability. Visual-spatial ability is the ability to understand the relationship between objects and space consisting of spatial visualization, mental rotation, spatial orientation, and perception (Bruce et al. 2017; Haciomeroglu & LaVenia, 2017; Kospentaris et al. 2016; Kotsopoulos et al. 2017). Activities related to visual-spatial specifically encourage the formation of visual representations (e.g. diagrams, graphic models, mental images) and mental processes (e.g. visual thinking, visual reasoning (Bruce et al. 2017). In this paper, Researcher, show the multi-representations that are raised prospective teacher students in expressing algebra, which can be used as a tool to solve algebra problems (de la Fuente & Deulofeu, 2016).

Problem of Research

Research on the representation of prospective teachers so far emphasizes the translation between the same types of representations, for example symbolic representations to symbolic representations, visual to visual representations, etc. In the context of teaching and learning student's algebra problems, so students understand in depth algebraic knowledge. So the prospective teacher students are required not only to be able to make translations on the same type of representation, but also between representations (multi-representation). Research on multi-representation that is raised by prospective teachers in expressing algebra problems has never been done by other researchers before.

Thus, researchers focus on problem of research to investigate the types of multi-representation that are raised by prospective teachers in expressing algebraic problem. Meanwhile, sub-problems of research which formulated throughout this study are:

- Understanding the types of multi-representation that is raised by prospective teachers in expressing algebraic problem
- Investigate the process of forming multi-representation types that are raised by prospective teachers in expressing algebraic problem
- Investigate the causes of the formation of multi-representation types that are raised by prospective teachers in expressing algebraic problem

Methodology

Research Design

This research used a case study with a qualitative approach. The case in this study is a phenomenon of several prospective mathematics teachers who express algebraic problems with different types of representations. Meanwhile the qualitative approach was chosen because the researcher wanted to get natural data. The intended natural data is data that describes the condition of the subject as it is and is obtained without doing any treatment (Creswell, 2012). Figure 1 below shows the research scheme carried out in the study.

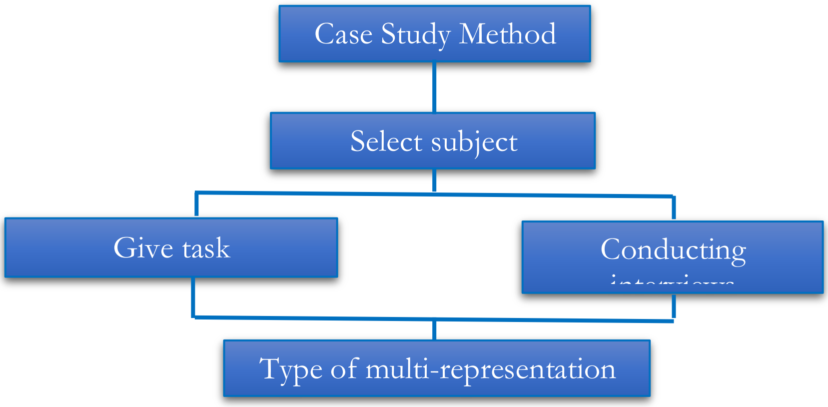

Figure 1.
Research Scheme

Participants

The participants in this research were 112 prospective mathematics education teachers with an age range of 22-23 years from five classes in the mathematics education study program at Universitas Negeri Malang. The participants consisted of 59 (16 men and 43 women) in semester 6 and 53 (18 men and 35 women) in semester 8. The researcher determined the level of this semester because the participants had taken the Algebra Abstract course 1. This it is important to ensure that participants have sufficient knowledge and competence in solving algebra problems.

The participants were selected using purposive sampling. It is intended that the selection of subjects is based on the relevance of information in the purpose of the question or deliberately choose individuals who are rich in information and have good or communicative communication skills (Creswell, 2012). Identification procedures applied through the task of algebra. Next, classify the results of the algebra task based on the type of representation that the subject produced in algebraic expressions. The participants were selected using purposive sampling. It is intended that the selection of subjects is based on the relevance of information in the purpose of the question or deliberately choose individuals who are rich in information and have good or communicative communication skills (Creswell, 2012). Identification procedures applied through the task of algebra. Next, classify the results of the algebra task based on the type of representation that the subject produced in algebraic expressions.

Data Collection Tools

The data collection tool was carried out using an algebra task sheet and an interview guide sheet.

Algebra Task Sheet (ATS)

AST consists of an algebra problem that is used to find the form of multi-representation expressed by prospective teachers. ATS was modified from one of the research instrument items from Huang & Kulm (2012) about determining factors from a quadratic equation. The modifications made consisted of (1) making the quadratic equation into a general form; (2) changing the problem request command to a request to express the quadratic equation to another form; (3) asking prospective teachers to provide arguments.

The modified ATS was then validated using expert judgment technique. ATS was validated by two professors in the field of mathematics education. Through a series of revisions to expert suggestions, the results of the ATS validity on are said to be feasible to use. Then, to check the reliability of ATS, a field test was conducted. Field tests were tested on 10 prospective mathematics education teachers who were considered to have criteria as research subjects. The results of the field tests showed that more than 50% of prospective mathematics education teachers express algebraic problems in a non-diverse manner. The researcher follows up the results of the validity and reliability of this ATS by changing the problem instructions from the closed ended to the open ended. ATS used in this study can be seen in Appendix-1.

Interview Guide Sheet (IGS)

IGS is used to clarify the process of forming multi-representation after solving problems with ATS. The questions at IGS are semi-structured, meaning that the researcher has a plan of questions and at the same time asks to follow the responses of prospective teachers. These questions are based on the problem-solving process by Polya (2004), which is understanding the problem, planning the idea of resolution, implementing the idea plan, checking the solution again. In using IGS, researchers also use field notes to write down key points that occurred during the interview process.

Similar to ATS, the validation process at IGS is carried out using expert judgment techniques by two professors in the field of mathematics education and the reliability is done by field trials. The IGS validation results showed that the interview guidelines are suitable for use with revisions. The revision is making the interview items more flexible or less formal. Meanwhile, at the time of the IGS field trial it was shown that it was able to dig up information about the process of the formation of multi-representation types. IGS used in this study can be seen in Appendix-2.

Data Analysis

Data analysis in research is carried out inductively. Inductive analysis is based on data obtained in the field and generalized to a certain conclusion or a certain general pattern (Zayyadi, 2020). The data analysis procedures in this study are illustrated below.

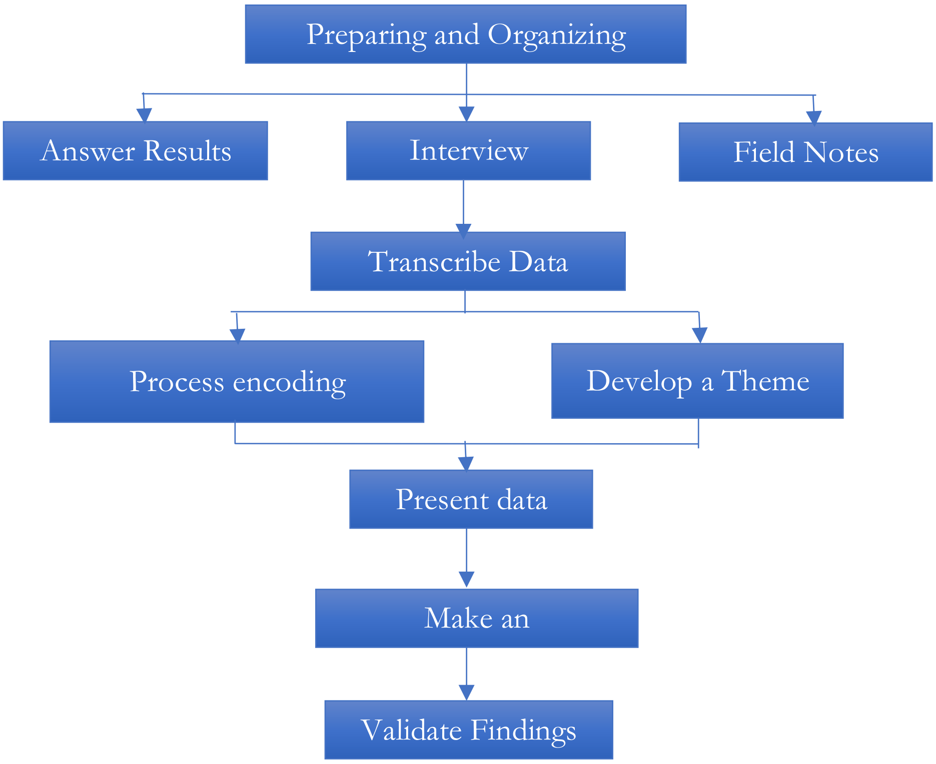

Figure 2.
Analysis Procedures

Data analysis procedures carried out in this research, through six stages, namely (1) preparing and organizing data for analysis, the data collected was recorded by digitizing both the results of the subject's written answers and field notes. The results of written answers and other data are arranged by grouping the forms of the subject's work based on the forms of representation produced and in paying attention to the truth of the subject's answers. (2) initial data exploration through the coding process, (3) using code to develop more general descriptions of data descriptions and themes, (4) representing findings through narration and visuals, (5) making interpretations of the meaning of results with personal reflection about the impact of findings and literature that might inform the findings; and finally (6) validates the accuracy of the findings.

Results and Discussion

Based on the results of data analysis that has been done by researchers, we get a comparison of the number of subjects that gave rise to the type of multi-representation in expressing algebra presented in Table 1 as follows:

Table 1.
Compares the Number of Subjects in Representing Algebra Problems

Number of Subjects	Symbolic Representations of Algebra	Pictorial Representation	Geometrics Representation
83	√	-	-
17	√	√	-
12	√	√	√

The findings of this research found that the type of multi-representation of prospective teachers in expressing algebraic problems consists of symbolic representations of algebra, pictorial representation, and geometrics representation. Type of symbolic representations of algebra is the most among the other types. A total 83 of 112 subjects that gave rise to symbolic representations of algebra. Furthermore, as many as 17 other subjects produced pictorial representations. Whereas 12 other subjects, gave rise to geometric representations.

Symbolic Representations of Algebra

The results of the subject's answers in bringing up symbolic representations in expressing algebra are as follows:

Symbolic Representations of Algebra

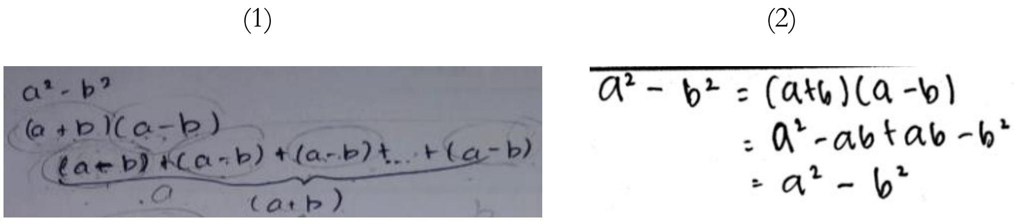

Figure 3.
Symbolic Representation of Algebra That Is Produced by the Subject

In this case, when the subject reads the information from the given algebra task, the subject generally interprets the statement "serve expressions of $(a^2 - b^2)$ in various forms" as factoring algebra. So they immediately write $(a + b)(a - b)$. They are not able to develop other expressions, because the knowledge they have has not been able to connect between symbolic representations with other representations. In addition, they are accustomed to solving problems that are symbolic in nature, thus affecting the structure of their thinking when faced with the problem of the task of algebraic expression. From the search results, they are also unable to interpret the word "expression" which shows the presence of other forms resulting from the translation process between representations.

Pictorial Representation

The results of the subject's answers in bringing up pictorial representations are as follows.

Pictorial Representation

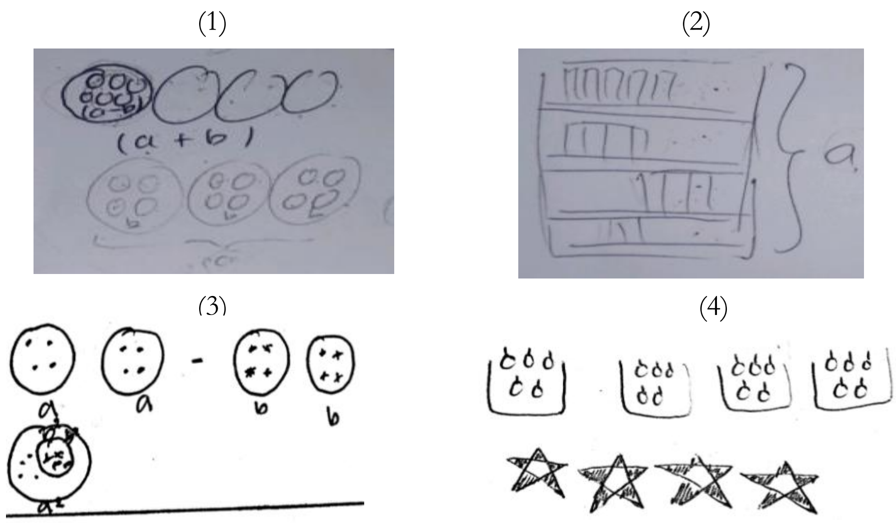

Figure 4.
Pictorial Representation of That Produced by Subject

In this case, the subject makes a picture resembling a ball, triangle, star, bookshelf, basket for expression $(a^2 - b^2)$. Some subjects use factoring information as a starting point, then view multiplication as a repetitive sum, forming recursive patterns that are represented in the number of baskets $(a + b)$ and many marbles $(a - b)$ and composing both to express $(a^2 - b^2)$. However, when traced through interviews, subjects that give rise to pictorial expressions are indirectly influenced by symbolic representations of algebra as a reference for making images that represent known forms of algebra.

Geometrics Representation

The results of the subject's answers in bringing up geometric representations are as follows:

Geometric Representation

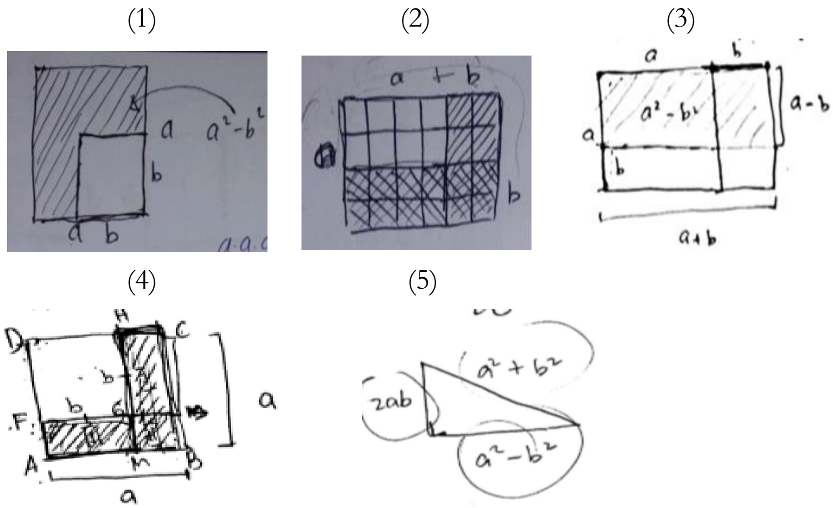

Figure 5.

Geometric Representation of That Produced by Subject

In this case, the appearance of geometric representation begins with the perception of the subject who sees the form $(a^2 - b^2)$ as the area of the unit squares. From the results of the subject's work, it was found three models of the appearance of geometric representations, namely: (1) assuming the size of area for each a^2 and b^2, making a partition between area a and area b, unifying the two expressions, and making shading that represents $(a^2 - b^2)$. (2) involves factoring, then presents a geometric shape in the form of a square that represents $a^2$and b^2, coordinates the area of square b with the area of square a, where the area of square b is contained within square a, and makes a shading that shows the expression $(a^2 - b^2)$. (3) involves symbolic representations of algebra, recursive patterns and compositions, and presents the shape of the area for square a as 4×4 units and square b as 2×2, and makes shading that shows expression $(a^2 - b^2)$.

Discussion

This research attempts to add to the research literature on representing forms of algebraic expression in several ways. Thus, the researcher discusses findings related to the form of expression that the subject raises in completing the algebra task, namely that three forms of representation are found, including symbolic algebraic representations, pictorial representations, and geometric representations. First, most subjects gave rise to symbolic representations of algebra, which is 83 out of a total of 112 subjects. In this case, when the subject reads the information from the given algebra task, the subject generally interprets the statement "serve expressions of $(a^2 - b^2)$ in various forms" as factoring algebra. So they immediately write $(a + b)(a - b)$. This, because it is influenced by learning styles (Natsheh & Karsenty, 2014), so the subject's ability to access other forms of representation is still limited. In addition, according to Fujita et al. (2017) revealed that the inability to manipulate mental representations resulted in the dominant subject expressing the representation of algebraic symbols in solving problems. This is also due to the lack of subject experience in representing a problem in various forms, for example pictorial and geometric (Adu-gyamfi & Bosse, 2013). Furthermore, subject limitations in manipulating features cause them to only use symbol representations (Herbst & Kosko, 2013). With this finding, it is possible that they are accustomed to solving problems that are symbolic in nature, thus affecting the structure of their thinking when facing the problem of the task of algebraic expression and interpreting the word "expression".

Second, as many as 17 other subjects produce pictorial representations. They use factoring information as a starting point, then view multiplication as a repeated sum, so that a recursive pattern is formed which is represented in the number of baskets $(a + b)$ and many marbles $(a - b)$ and composes both so that they express $(a^2 - b^2)$. In this case, the subject makes a picture resembling a ball, triangle, star, bookshelf, and basket to obtain an expression $(a^2 - b^2)$. Pictorial representation is also called iconic representation (Zahner & Corter, 2010), where the subject expresses a problem situation in the form of an image as an alternative to solving the problem. From the results obtained, several subjects first involved symbolic algebraic representations before using pictorial representations as to the main objective. This is in line with the opinion of Walkowiak (2014) where subjects that first involve symbolic representations of algebra are because they make numerical reasoning, so when they bring up pictorial representations it is caused because they do figural reasoning. The success of the subject in solving problems by involving pictorial representation depends on the flexibility of the problem at hand. This is consistent with the findings of Baek et al. (2017) that the use of pictorial representations is more effective in solving mathematical problems than other representations. However, in terms of developing mathematical abilities, several studies Boonen et al. (2014) also Pitta-pantazi and Christou (2010) argue that dominant subjects represent their understanding by involving pictorial representations that tend to be negatively correlated with their ability to solve problems. Van Lieshout and Xenidou-Dervou (2018) added that pictorial representations cause cognitive burdens compared to other representations. From this opinion, it was concluded that there were gaps which resulted in inefficient pictorial representation being applied in learning, especially at higher levels.

Whereas 12 other subjects, gave rise to geometrical representations. In this case, the appearance of geometric representation begins with the perception of the subject who views the form $(a^2 - b^2)$ as the area of the unit squares. Mamolo et al. (2015) said that this condition involves geometric reasoning and spatial visualization ability, where the process involves geometrical transformation, symmetrical, and dimensional comparison. He also added that geometric representation is useful for cognitive aspects as a tool for understanding a concept. Thom and Mcgarvey (2015) states that these conditions involve geometric thinking and spatial awareness to express a problem. In this case, the subject involves an internal cognitive scheme to bring up geometric representations that are used as mental objects. In addition, the appearance of geometric representation is caused by the magnitude of the subject's visual geometric ability when dealing with problems (Karakok, 2019). In terms of the conceptual framework of geometric c constructions, outlines three main components that cause the subject to bring up geometric representations and serve as literacy materials that are easily accessed by the teacher (Amir et al. 2019; Yang & Li, 2018). The framework is in the form of, Retrieving or recognizing, Interpreting or connecting and Reflecting or reasoning. While the results of our investigation found patterns that serve as a framework for the appearance of geometric representations, namely: perception, display, strategy, and re-check.

Researchers also find that geometric representations are difficult for most subjects to emerge. In line with the findings of Beckmann & Izsák (2015) it is argued that the appearance of geometric representations is considered difficult for some subjects, this is because they must involve knowledge of transformation and spatial. Sandoval and Possani (2016) specifically summarizes the difficulties experienced by students when representing problems geometrically, namely geometric language, description modes, problem representations, and translating and translating algebraic problems into geometric. However, this situation can be minimized by involving geometric representations in the learning process. This is in line with the findings of Fujita et al. (2017) that expressing geometric representations is seen as a process of productive reasoning that impacts on the subject's thinking. In addition, this representation can be used as a reference for long-term research, as recommended by (Karakok et al. 2014). Likewise with the opinion of Dogan-dunlap (2010) that geometrical representation helps the subject to consider various representational aspects of a flexible concept. In particular, geometrical representations enrich and provide the basis for algebraic manipulation (Sandoval & Possani, 2016).

From the results of categorizing, Researchers found that there are four models of the appearance of geometric representations, namely: (1) assuming the size of each a^2 and b^2, making a partition between area a and area b, joining the two expressions, and making shading represent $(a^2 - b^2)$; (2) involves factoring, then presents a geometric shape in the form of a square that represents a^2 and b^2, coordinates the area of square b with the area of square a, where the area of square b is contained within square a, and makes a shading that shows the expression $(a^2 - b^2)$; (3) involves symbolic representations of algebra, recursive patterns and compositions, and presents the shape of the area size for square a as 4×4 units and square b as 2×2, and making shading that shows expression $(a^2 - b^2)$ and ; (4)

assumes the length of one side of the triangle. this is in line with the findings of Dogan-dunlap (2010) that geometrical representations do not occur independently, but require other representations as initial perception. However, the four models still need in-depth investigation, so the recommendations section will be explained in detail.

Conclusion and Recommendations

Based on the findings and discussion it can be concluded that there are three types of representations raised by the subject in completing the task of the algebra, namely algebraic symbolic representations, pictorial representations, and geometric representations. Specifically for geometric representations, three different characteristics are found, so it still needs to be studied in depth from the perspective of the process of representation, mental perception of the subject, cognitive style, the process of translation between representations, and the processing of information. The findings of this research indicate that most subjects are centred on symbolic representations of algebra and there are some of them experiencing obstacles when asked to bring up other representations.

In this research, the researcher recommends that subsequent multi-representation researchers examine specifically geometric representations of algebraic problems for mathematics teacher candidates. The findings show that the formation of geometric representations does not provide more cognitive burden than the formation of other representations, namely algebraic symbolic representations and pictorial representations. In addition, it is still rare for prospective teachers to bring up geometrical representations as well as one of the reasons. In addition, from the stages of the process of appearing geometrical representations found the same pattern and can be used for further research. The pattern is in the form, perception, appearance, strategy, and re-check. For the applicants, teachers do not realize the importance of geometrical representations, because they may feel that students are unable to complete tasks using these representations, and most importantly, they are not trained to develop tasks by involving geometric representations.

Limitations

The researcher is aware of the limitations in this study, which include: (1) the approach and research method used is qualitative, so the conclusions obtained cannot be generalized for similar cases; (2) subjects were only taken from one research location and consisted of many subjects which were still relatively small; (3) research results obtained from one task; (4) the task of investigating multi-representation of prospective teachers in expressing algebraic problems still consists of one item and is less varied; (5) in conducting the analysis, the researcher does not look at the aspects of the algebraic knowledge of the prospective teacher, so the conclusions obtained are still general.

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Appendix-1
Algebra Task Sheet (ATS)

Algebra Task Sheet
Quadratic Equation Problem

Guide
Understand the questions below, then work precisely and thoroughly.

Present expressions of, $a^2 - b^2$ in various forms! Explain your reason!

Write your answer in this areas.

Appendix-2
Interview Guide Sheet (IGS)

Interview Guide Sheet

No	Interview Criteria	Interview Question Items
The interview question items make it possible to express and clarify the process of forming multi-representation after solving algebra problems with Algebra Task Sheet (ATS)		
1	Understanding the problem	
	a. Allows the subject to reveal information that is known and asked at the ATS	• What did you think after seeing ATS? • What was your reason, so that you interpreted it like that?
	b. Allows subjects to be able to access initial ideas in bringing up representations (symbolic / pictorial / geometric)	• What was the first time you thought that you could express representation in such a form?
2	Planning the idea of resolution	
	a. Allows the subject to apply the idea of representation (symbolic / pictorial / geometric)	• Why did you involve the idea of representation in expressing algebra problems? • What was the first time you did it?
	b. Allows the subject to explain the components contained in the representation (symbolic / pictorial / geometric)	• Explain in detail when you expressed the representation?
	c. Allows the subject to break down the representation (symbolic / pictorial / geometric) results into several parts	• Why did you expressed such representations?
3	Implementing the idea plan	
	a. Allows the subject to prove the result of the representation (symbolic / pictorial / geometric) which is generated mathematically	• Why did you written the answer mathematically? • Explain in sequence so that you interpret it that way, to prove that the representation you were expressing is correct!
	b. Allows the subject to find solutions and draw conclusions	• How did you write a conclusion like this? • Try to pay attention to the ATS, whether your answer was correct!
4	Checking the solution again	
	a. Allows the subject to ensure the representation (symbolic / pictorial / geometric) produced is correct and makes sense	• Are your calculations correct? • Are you sure about your answer? Tell!