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Research Article

Stability Analysis of Neutral-Type Hopfield Neural Networks with Discrete Delays

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Abstract

This research paper deals with the stability problem for a class of neutral-type Hopfield neural networks that involves discrete time delays in the states of neurons and discrete neutral delays in the time derivatives of the states of neurons. By constructing a novel suitable Lyapunov functional, an easily verifiable algebraic condition for global asymptotic stability of this type of Hopfield neural systems is presented. This stability condition is absolutely independent of the discrete time and neutral delays. An instructive example is given to demonstrate the applicability of the proposed condition.

Keywords: Neutral Systems, Hopfield Neural Networks, Lyapunov Functionals, Stability Analysis.

Ayrık Gecikmeli Nötral-Tip Hopfield Yapay Sinir Ağlarının Kararlılık Analizi

Öz-Türkçe

Bu araştırma makalesi, nöron durumlarının ayrık zaman gecikmeleri ve nöron durumlarının türevlerinin ayrık nötral gecikmeler içerdiği nötral-tip Hopfiled yapay sinir ağlarının kararlılık problemi ile ilgilenmektedir. Yeni ve uygun bir Lyapunov fonksiyonu kullanılarak, bu tip Hopfield yapay sinir ağlarının kararlılığı için, yeni ve kolayca doğrulanabilir cebirsel olarak ifade edilen bir koşul sunulmaktadır. Bu kararlılık koşulu kesinlikle hem ayrık zaman gecikmeleri hem de ayrık nötral gecikmelerinden bağımsızdır. Elde edilen kararlılık koşulunun uygulanabilirliğini göstermek için öğretici bir sayısal örnek verilmiştir.

Anahtar Kelimeler: Nötral Sistemler, Hopfield Yapay Sinir Ağları, Lyapunov Fonksiyonları, Kararlılık Analizi.

1. Introduction

Recently, the class of Hopfield neural networks has been used in many critical engineering applications associated with image processing, pattern recognitions and optimization related problems [1]-[5]. In these typical engineering applications of this neural network, the main problem is to know the requirement for the desired dynamical behavior of this neural network. For instance, in case of optimization problems, the critical point is that this neural network must converge some unique and globally asymptotically stable equilibrium points. A critical issue is that the dynamics of a neural network can be changed by different external parameters. Specially, the electronically implemented neural networks can show undesired dynamical activities due to the time delays caused by finite switching speed of electronic elements and signal processing times of neurons. Therefore, it would be appropriate to represent these

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time delays in the dynamical modelling of these systems. Presently, many research papers have studied the stability of Hopfield neural networks involving discrete time delays [6]-[12]. It is important to mention that the neural networks including time delays may not always reveal the desired dynamics of neuronal reaction process because of some strange complicated dynamical activities of interactions taking place between the neurons. Thus, It is of crucial importance to introduce the meaningful information associated with the time derivatives of states of the neurons when establishing the dynamical representations of these systems for identifying the complete dynamics of these types of complex neuronal interactions. This task is carried out by presenting the additional delays to time derivatives of states of neurons. Neural networks whose mathematical models involve both different time delays in states of neurons and different neutral delays in time derivatives of states of neurons are called neutral-type neural networks. These types of networks have been proved to be effective systems in many applications in the fields of the population ecology, distributed networks involving lossless transmission lines [13]-[15].

This paper will analyze a neutral-type Hopfield neural network which involves different discrete time delays in states of neurons and different discrete neutral delays time derivatives of the states of neurons. Such a neural network possesses a dynamics that is governed by the dynamical equations:

$$\dot{x}_i(\mathrm{t}) + \sum_{j=1}^{n} e_{ij}\,\dot{x}_j(\mathrm{t}-\zeta_j) = -c_i x_i(t) + \sum_{j=1}^{n} a_{ij} f_j\left(x_j(t)\right) + \sum_{j=1}^{n} b_{ij} f_j\left(x_j(t-\tau_j)\right) + u_i,\ \ i = 1,2,\dots,n \tag{1}$$

where $x_i(t)$ is a state variable representing ith neuron, c_i represent some positive constants. The constants a_{ij} and b_{ij} are interconnection parameters. Discrete time delays are denoted by τ_j and discrete neutral delays are denoted by ζ_j, $1 \le j \le n$. The e_{ij} are the constant parameters associated with time derivatives of the states having discrete neutral delays. The $f_j(x_j(t))$ are the activation functions and u_i are the inputs. In neutral-type Hopfield neural network given by (1), denote $\tau = \max\{\tau_j\}$, $\zeta = \max\{\zeta_j\}$, $1 \le j \le n$, and $\Omega = \max\{\tau,\zeta\}$. Thus, neural network (1) can be defined by the initial conditions of $x_i(t) = \varphi_i(t)$ and $\dot{x}_i(t) = \vartheta_i(t)$ in $C([-\Omega,0],R)$. We also note that $C([-\Omega,0],R)$ include the real valued functions which are assumed to be defined from $[-\Omega,0]$ to R.

In dealing with the dynamical analysis associated with investigated stability issues of neutral neural system represented with equation (1), the basic property that is needed to be satisfied by the activation functions $f_j(x_j(t))$ is an important concept. Therefore, it is first required to determine basic characteristics of these activation functions employed in (1). In the literature, it is customary to assume that there exist positive Lipschitz constants ℓ_i such that

$$\left|f_i(x_i(t)) - f_i(y_i(t))\right| \le \ell_i |x_i(t) - y_i(t)|, \qquad \forall x_i(t), \forall y_i(t) \in R, x_i(t) \ne y_i(t), \forall i, \tag{2}$$

The formulation of neural system (1) is of a mathematical nature that allows us to put system (1) in a form of vectors and matrices as shown in the following equation:

$$\dot{x}\,(t) + E\dot{x}\,(t-\zeta) = -Cx(t) + Af\left(x(t)\right) + Bf\left(x(t-\tau)\right) + u \tag{3}$$

where $C = diag(c_i > 0)$, $A = (a_{ij})_{nxn}$, $B = (b_{ij})_{nxn}$ and $E = (e_{ij})_{nxn}$ represent the connection matrices of system (1). $x(t) = (x_1(t), x_2(t), \dots, x_n(t))^T$, $\dot{x}(t) = (\dot{x}_1(t), \dot{x}_2(t), \dots, \dot{x}_n(t))^T$, $f(x(t)) = (f_1(x_1(t)), f_2(x_2(t)), \dots, f_n(x_n(t)))^T$, $f(x(t-\tau)) = (f_1(x_1(t-\tau)), f_2(x_2(t-\tau)), \dots, f_n(x_n(t-\tau)))^T$, $\dot{x}(t-\zeta) = (\dot{x}_1(t-\zeta), \dot{x}_2(t-\zeta), \dots, \dot{x}_n(t-\zeta))^T$, $u = (u_1, u_2, \dots, u_n)^T$

If neutral-type neural networks possess discrete delays, then the mathematical models of these neural systems can be formulated in the forms of vectors and matrices. Then, we may study the stability of these neural network models by exploiting linear matrix inequality approach combining with the other appropriate mathematical tools and methods. In [16]-[25], the stability of neutral neural-type networks defined by (6) have been studied and by constructing some classes of suitable Lyapunov functionals together with employing some lemmas and new mathematical techniques, different sets of novel stability results on the considered neutral-type neural networks of various forms of linear matrix inequalities have been presented. In [26]-[31], new global stability criteria for system (6) in the forms of different representations of linear matrix inequality formulations have been proposed by employing various proper Lyapunov functionals with the triple or four integral terms. In [32] and [33] various stability problems for neutral-type neural networks defined by (4) have been investigated, in which, by making the use of semi-free weighting matrix techniques and an augmented Lyapunov functional, some less conservative and restrictive global stability conditions via linear matrix inequalities have been presented. In [34], the stability for Hopfield neural networks of neutral-type possessing discrete delays has been suitable conducted, and by utilizing a proper Lyapunov functional that makes a combination of the descriptor model transformation, a novel stability criterion has been formulated in linear matrix inequalities. In [35], stability of neural system defined by (4) has been addressed and by proposing a appropriate Lyapunov functionals utilizing Auxiliary function-type integral inequalities and reciprocally convex method, some sets of stability results via linear matrix inequalities have been obtained. In [36], the Lagrange stability issue of neutral-type neural systems having mixed delays has been analyzed, and by utilizing the proper Lyapunov functionals and applying some appropriate linear matrix inequality techniques, various sufficient criteria have been obtained to assure Lagrange stability of this model of considered neural network system. In [37], the issues associated with stability of neutral type singular neural systems involving different delay parameters have been studied, and by exploiting a novel adequate Lyapunov functional and some rarely integral inequalities, a new global asymptotic stability condition via linear matrix inequality has been derived. In [38], dynamical issues of neural networks of neutral type possessing some various delay parameters have been analysed, and various stability results have been derived employing linear matrix inequality together with Razumikhin-type approaches.

Note that the results of [17]-[39] employ some various classes of linear matrix inequality tolls to derive different sets of sufficient stability conditions for system (6). However, the global stability results derived via linear matrix inequality method are required to test some negative definite properties of very high dimensional matrices whose elements are established by the system parameters of neural networks. Due to these complex and costly calculation problems, it becomes necessity to obtain different stability conditions for system (6), that are not expressed in linear matrix inequality forms. In this concept, this paper will focus on the dynamical analysis of system (6) to derive some easily verifiable algebraic stability conditions.

2. Stability Analysis

The basic contribution of this section will be deriving some stability conditions ensuring the stability of neutral-type Hopfield neural system whose model is given by (1). We now proceed with a first step to simplify the proofs of the stability conditions. This step needs to transform the equilibrium points equilibrium points $x^* = (x_1^*, x_2^*, \dots, x_n^*)^T$ of Hopfield-type neural network represented by equation (1) to the origin. This will be achieved by utilizing the simple formula $z_i(t) = x_i(t) - x_i^*$, which turns neutral-type neural network (1) to an equivalent neutral-type neural network represented by the following differential equations:

$$\dot{z}_i(\mathrm{t}) + \sum_{j=1}^{n} e_{ij}\,\dot{z}_j(\mathrm{t}-\zeta) = -c_i z_i(t) + \sum_{j=1}^{n} a_{ij} g_j\left(z_j(t)\right) + \sum_{j=1}^{n} b_{ij} g_j\left(z_j(t-\tau)\right), i = 1,2,\dots,n \tag{4}$$

where the new activation functions are determined to be in the form $g_i(z_i(t)) = f_i(z_i(t) + x_i^*) - f_i(x_i^*), \forall i.$ In the light of (2), the functions $g_i(z_i(t))$ justify the following conditions :

$$|g_i(z_i(t))| \le \ell_i |z_i(t)|, \forall z_i(t) \in R, \forall i \tag{5}$$

The formulation of neural system (4) is of a mathematical nature that allows us to put system (1) in a form of vectors and matrices as shown in the following equation:

$$\dot{z}(t) + E\dot{z}(t-\zeta) = -Cz(t) + Ag(z(t) + Bg(z(t-\tau)) \tag{6}$$

where $z(t) = (z_1(t), z_2(t), \dots, z_n(t))^T$, $\dot{z}(t) = (\dot{z}_1(t), \dot{z}_2(t), \dots, \dot{z}_n(t))^T$, $g(z(t)) = (g_1(x_1(t)), g_2(z_2(t)), \dots, g_n(z_n(t)))^T$, $g(z(t-\tau)) = (g_1(z_1(t-\tau)), g_2(z_2(t-\tau)), \dots, g_n(z_n(t-\tau)))^T$, $\dot{z}(t-\zeta) = (\dot{z}_1(t-\zeta), \dot{z}_2(t-\zeta), \dots, \dot{z}_n(t-\zeta))^T$.

We are now in the position to state the contribution of the paper by a theorem stated as follows :

Theorem 1 : For neutral-type Hopfield neural system (6), assume that the activation functions $g_i(z_i(t))$ satisfy (5). Then, the origin of system (6) is globally asymptotically stable, if the following conditions hold:

$$\delta = c_m^2 - (\|A\|_2^2 + 2\|A\|_2\|B\|_2 + \|B\|_2^2)\ell_M^2 - 2c_M^2\|E\|_2 - c_M\|E\|_2(\|A\|_2 + \|B\|_2)\ell_M^2 - c_M\|E\|_2(\|A\|_2 + \|B\|_2) > 0$$

and

$$e_i = 1 - \sum_{j=1}^{n} |e_{ji}|, \quad i = 1, 2, \dots, n$$

Proof : This theorem will be proved by using the state transformation approach. To this end, we define the following:

$$y_i(t) = z_i(t) + \sum_{j=1}^{n} e_{ij} z_j(t-\zeta_j), \ \ i = 1, 2, \dots, n. \tag{7}$$

or equivalently

$$\mathrm{y}(t) = \mathrm{z}(t) + \mathrm{Ez}(\mathrm{t}-\zeta) \tag{8}$$

In this case, taking the time derivatives of both sides of equation (7) yields:

$$\dot{y}_i(t) = \dot{z}_i(t) + \sum_{j=1}^{n} e_{ij}\dot{z}_j(t-\zeta_j), \ \ i = 1, 2, \dots, n \tag{9}$$

Equation (7) is equivalent to the following

$$\dot{y}(t) = \dot{z}(t) + E\dot{z}(t-\zeta) \quad (10)$$

Combining (9) with (4) leads to

$$y_i(\mathrm{t}) = -c_i z_i(t) + \sum_{j=1}^{n} a_{ij} g_j\left(z_j(t)\right) + \sum_{j=1}^{n} b_{ij} g_j\left(z_j(t-\tau)\right), i = 1,2,\dots,n. \quad (11)$$

(11) can be written in form of matrices and vectors as stated below

$$\dot{y}(t) = -Cz(t) + Ag(z(t) + Bg(z(t-\tau)) \quad (12)$$

We can now proceed further to construct a proper Lyapunov functional for the stability analysis of system (6) defined by :

$$V(t) = \sum_{i=1}^{n} c_i\, y_i^2(t) + \sum_{i=1}^{n} \int_{t-\zeta_i}^{t} \dot{y}_i^2\,(s)ds + (\alpha+\beta)\sum_{i=1}^{n}\int_{t-\tau_i}^{t} z_i^2\,(s)ds + (\alpha+\gamma)\sum_{i=1}^{n}\int_{t-\zeta_i}^{t} z_i^2\,(s)ds \quad (13)$$

In (13), α, β and γ represent some positive real constants whose appropriate numerical values will be specified in what follows. The time derivative $\dot{V}(t)$ of the Lyapunov functional $\mathrm{V}(t)$ along the trajectories of system (6) is calculated to be in the form:

$$\begin{aligned}\dot{V}(t) &= 2\sum_{i=1}^{n} c_i\, y_i(t)\,\dot{y}_i(t) + \sum_{i=1}^{n} \dot{y}_i^2\,(t) - \sum_{i=1}^{n} \dot{y}_i^2\,(t-\zeta_i) + (\alpha+\beta)\sum_{i=1}^{n} z_i^2(t)\\ &\quad -(\alpha+\beta)\sum_{i=1}^{n} z_i^2(t-\tau_i) + (\alpha+\gamma)\sum_{i=1}^{n} z_i^2(t) - (\alpha+\gamma)\sum_{i=1}^{n} z_i^2(t-\zeta_i)\\ &\le 2\sum_{i=1}^{n} c_i\, y_i(t)\,\dot{y}_i(t) + \sum_{i=1}^{n}\dot{y}_i^2\,(t) + (\alpha+\beta)\sum_{i=1}^{n} z_i^2(t)\\ &\quad -(\alpha+\beta)\sum_{i=1}^{n} z_i^2(t-\tau_i) + (\alpha+\gamma)\sum_{i=1}^{n} z_i^2(t) - (\alpha+\gamma)\sum_{i=1}^{n} z_i^2(t-\zeta_i)\end{aligned} \quad (14)$$

(14) can be rewritten in matrix and vector form by the following inequality:

$$\begin{aligned}\dot{V}(t) &\le 2y^T(t)C\,\dot{y}(t) + \dot{y}^T(t)\,\dot{y}(t) + (2\alpha+\beta+\gamma)z^T(t)z(t) - (\alpha+\beta)z^T(t-\tau)z(t-\tau) - (\alpha+\gamma)z^T(t-\zeta)z(t-\zeta)\\ &= \left(2y^T(t)C + \dot{y}^T(t)\right)\dot{y}(t) + (2\alpha+\beta+\gamma)z^T(t)z(t) - (\alpha+\beta)z^T(t-\tau)z(t-\tau) - (\alpha+\gamma)z^T(t-\zeta)z(t-\zeta)\\ &= \left(2Cy(t) + \dot{y}(t)\right)^T\dot{y}(t) + (2\alpha+\beta+\gamma)z^T(t)z(t) - (\alpha+\beta)z^T(t-\tau)z(t-\tau) - (\alpha+\gamma)z^T(t-\zeta)z(t-\zeta)\end{aligned} \quad (15)$$

Using (8) and (10) in (15) results in

$$\begin{aligned}\dot{V}(t) &\le (2\mathrm{C}(\mathrm{z(t)} + \mathrm{E}z(t-\zeta)) - \mathrm{Cz(t)} + Ag(z(t)) + Bg(z(t-\tau)))^T(-\mathrm{Cz(t)} + Ag(z(t)) + Bg(z(t-\tau)))\\ &\quad +(2\alpha+\beta+\gamma)z^T(t)z(t) - (\alpha+\beta)z^T(t-\tau)z(t-\tau) - (\alpha+\gamma)z^T(t-\zeta)z(t-\zeta)\\ &= (\mathrm{Cz(t)} + 2\mathrm{CE}z(t-\zeta) + Ag(z(t)) + Bg(z(t-\tau)))^T(-\mathrm{Cz(t)} + Ag(z(t)) + Bg(z(t-\tau)))\\ &\quad +(2\alpha+\beta+\gamma)z^T(t)z(t) - (\alpha+\beta)z^T(t-\tau)z(t-\tau) - (\alpha+\gamma)z^T(t-\zeta)z(t-\zeta)\\ &= -z^T(t)C^2z(t) - 2z^T(t-\zeta)E^TC^2z(t) + 2z^T(t-\zeta)E^TCAg\big(z(t)\big) + 2z^T(t-\zeta)E^TCBg(z(t-\tau)))\\ &\quad +g^T(z(t))A^TAg(z(t)) + g^T(z(t-\tau)))\,B^TBg(z(t-\tau))) + 2g^T\big(z(t)\big)A^TBg(z(t-\tau)))\\ &\quad +(2\alpha+\beta+\gamma)z^T(t)z(t) - (\alpha+\beta)z^T(t-\tau)z(t-\tau) - (\alpha+\gamma)z^T(t-\zeta)z(t-\zeta)\end{aligned} \quad (16)$$

First note the inequalities

$$-z^T(t)C^2z(t) \le -c_m^2\|z(t)\|_2^2 \quad (17)$$

$$-2z^T(t-\zeta)E^TC^2z(t) \le 2c_M^2\,\|E\|_2\|z(t)\|_2\|z(t-\zeta)\|_2 \le c_M^2\|E\|_2\|z(t)\|_2^2 + c_M^2\|E\|_2\|z(t-\zeta)\|_2^2 \quad (18)$$

$$2z^T(t-\zeta)E^TCAg(z(t)) \le 2c_M\,\|A\|_2\|E\|_2\|g(z(t))\|_2\|z(t-\zeta)\|_2$$

$$\leq c_M \|A\|_2\|E\|_2\|g(z(t))\|_2^2 + c_M \|A\|_2\|E\|_2\|z(t-\zeta)\|_2^2 \tag{19}$$

$$\begin{aligned} 2z^T(t-\zeta)E^TCBg(z(t-\tau)) &\leq 2c_M \|B\|_2\|E\|_2\|g(z(t-\tau))\|_2\|z(t-\zeta)\|_2 \\ &\leq c_M \|B\|_2\|E\|_2\|g(z(t-\tau))\|_2^2 + c_M \|B\|_2\|E\|_2\|z(t-\zeta)\|_2^2 \end{aligned} \tag{20}$$

$$g^T(z(t))A^TAg(z(t)) \leq \|A\|_2^2\|g(z(t))\|_2^2 \tag{21}$$

$$g^T(z(t-\tau))B^TBg(z(t-\tau)) \leq \|B\|_2^2\|g(z(t-\tau))\|_2^2 \tag{22}$$

$$\begin{aligned} 2g^T(z(t))A^TBg(z(t-\tau)) &\leq 2\|A\|_2\|B\|_2\|g(z(t))\|_2\|g(z(t-\tau))\|_2 \\ &\leq \|A\|_2\|B\|_2\|g(z(t))\|_2^2 + \|A\|_2\|B\|_2\|g(z(t-\tau))\|_2^2 \end{aligned} \tag{23}$$

where $c_m = \min\{c_i\}$ and $c_M = \max\{c_i\}$. Inserting (17)-(23) into (16) yields:

$$\begin{aligned} \dot{V}(t) \leq &-c_m^2\|z(t)\|_2^2 + c_M^2\|E\|_2\|z(t)\|_2^2 + c_M^2\|E\|_2\|z(t-\zeta)\|_2^2 + c_M \|A\|_2\|E\|_2\|g(z(t))\|_2^2 + c_M \|A\|_2\|E\|_2\|z(t-\zeta)\|_2^2 \\ &+ c_M \|B\|_2\|E\|_2\|g(z(t-\tau))\|_2^2 + c_M \|B\|_2\|E\|_2\|z(t-\zeta)\|_2^2 + \|A\|_2^2\|g(z(t))\|_2^2 + \|B\|_2^2\|g(z(t-\tau))\|_2^2 \\ &+ \|A\|_2\|B\|_2\|g(z(t))\|_2^2 + \|A\|_2\|B\|_2\|g(z(t-\tau))\|_2^2 \\ &+(2\alpha+\beta+\gamma)z^T(t)z(t) - (\alpha+\beta)z^T(t-\tau)z(t-\tau) - (\alpha+\gamma)z^T(t-\zeta)z(t-\zeta) \end{aligned} \tag{24}$$

Since $\|g(z(t))\|_2^2 \leq \ell_M^2\|z(t)\|_2^2$ and $\|g(z(t-\tau))\|_2^2 \leq \ell_M^2\|z(t-\tau)\|_2^2$, (24) can be written as

$$\begin{aligned} \dot{V}(t) \leq &-c_m^2\|z(t)\|_2^2 + c_M^2\|E\|_2\|z(t)\|_2^2 + c_M^2\|E\|_2\|z(t-\zeta)\|_2^2 + c_M \|A\|_2\|E\|_2\ell_M^2\|z(t)\|_2^2 \\ &+c_M \|A\|_2\|E\|_2\|z(t-\zeta)\|_2^2 + c_M \|B\|_2\|E\|_2\ell_M^2\|z(t-\tau)\|_2^2 + c_M \|B\|_2\|E\|_2\|z(t-\zeta)\|_2^2 \\ &+\|A\|_2^2\ell_M^2\|z(t)\|_2^2 + \|B\|_2^2\ell_M^2\|z(t-\tau)\|_2^2 + \|A\|_2\|B\|_2\ell_M^2\|z(t)\|_2^2 + \|A\|_2\|B\|_2\ell_M^2\|z(t-\tau)\|_2^2 \\ &+(2\alpha+\beta+\gamma)\|z(t)\|_2^2 - (\alpha+\beta)\|z(t-\tau)\|_2^2 - (\alpha+\gamma)\|z(t-\zeta)\|_2^2 \end{aligned} \tag{25}$$

where $\ell_M = \max\{\ell_i\}$. We make the following choices for the values of β and γ :

$$\beta = c_M \|B\|_2\|E\|_2\ell_M^2 + \|B\|_2^2\ell_M^2 + \|A\|_2\|B\|_2\ell_M^2 \tag{26}$$

and

$$\gamma = c_M^2\|E\|_2 + c_M \|A\|_2\|E\|_2 + c_M \|B\|_2\|E\|_2 \tag{27}$$

Inserting (26) and (27) into (25) yields

$$\begin{aligned} \dot{V}(t) &\leq (-c_m^2 + c_M^2\|E\|_2 + c_M \|A\|_2\|E\|_2\ell_M^2 + \|A\|_2^2\ell_M^2 + \|A\|_2\|B\|_2\ell_M^2)\|z(t)\|_2^2 \\ &\quad+(c_M \|B\|_2\|E\|_2\ell_M^2 + \|B\|_2^2\ell_M^2 + \|A\|_2\|B\|_2\ell_M^2 + c_M^2\|E\|_2 + c_M \|A\|_2\|E\|_2 + c_M \|B\|_2\|E\|_2)\|z(t)\|_2^2 \\ &\quad+2\alpha\|z(t)\|_2^2 - \alpha\|z(t-\tau)\|_2^2 - \alpha\|z(t-\zeta)\|_2^2 \\ &= -(c_m^2 - (\|A\|_2^2 + 2\|A\|_2\|B\|_2 + \|B\|_2^2)\ell_M^2 + 2c_M^2\|E\|_2 + c_M\|E\|_2(\|A\|_2 + \|B\|_2)(\ell_M^2+1))\|z(t)\|_2^2 \\ &\quad+2\alpha\|z(t)\|_2^2 - \alpha\|z(t-\tau)\|_2^2 - \alpha\|z(t-\zeta)\|_2^2 \\ &= -\delta\|z(t)\|_2^2 + 2\alpha\|z(t)\|_2^2 - \alpha\|z(t-\tau)\|_2^2 - \alpha\|z(t-\zeta)\|_2^2 \end{aligned} \tag{28}$$

(28) satisfies

$$\dot{V}(t) \leq -\delta\|z(t)\|_2^2 + 2\alpha\|z(t)\|_2^2 = -(\delta - 2\alpha)\|z(t)\|_2^2 \tag{29}$$

In (29), the choice $2\alpha < \delta$ implies that $\dot{V}(t)$ will be negative definite for all $z(t) \neq 0$.

Let $z(t) = 0$. Then, from (28), we state the following inequality

$$\dot{V}(t) \leq -\alpha\|z(t-\tau)\|_2^2 - \alpha\|z(t-\zeta)\|_2^2 \leq -\alpha\|z(t-\tau)\|_2^2 \tag{30}$$

Since $\alpha > 0$, it can be directly concluded from (30) that if $z(t-\tau) \neq 0$, then $\dot{V}(t)$ will be negative definite.

Let $z(t) = 0$ and $z(t-\tau) = 0$. Then, from (28), we state the following inequality

$$\dot{V}(t) \leq -\alpha\|z(t-\zeta)\|_2^2 \tag{31}$$

Since $\alpha > 0$, it can be directly concluded from (31) that if $z(t-\zeta) \neq 0$, then $\dot{V}(t)$ will be negative definite.

Let $z(t) = 0$, $z(t-\tau) = 0$ and $z(t-\zeta) = 0$. then, from (12), it follows that $\dot{y}(t) = 0$. In this case, $\dot{V}(t)$ given by (14) takes the from:

$$\dot{V}(t) = -\|\dot{y}(t-\zeta)\|_2^2 \tag{32}$$

In (32), it is easy to observe that $\dot{V}(t) < 0$ if $\dot{y}(t-\zeta) \neq 0$, and $\dot{V}(t) = 0$ if $\dot{y}(t-\zeta) = 0$. This leads the fact of $\dot{V}(t) = 0$ if and only if $z(t) = 0$, $g(z(t)) = 0$, $z(t-\tau) = 0$, $g(z(t-\tau)) = 0$, $z(t-\zeta) = 0$ and $\dot{y}(t-\zeta) = 0$. This directly means that $\dot{V}(t) < 0$ in all the other cases. This analysis leads us to indicate that the origin of (6) is asymptotically stable. We now need to establish that system (6) is also globally stable. For this purpose, one needs to prove that $V(t)$ is radially unbounded. This is equivalent to satisfy the condition of $V(t) \to \infty$ as $\|z(t)\| \to \infty$.

Since

$$y_i(t) = z_i(t) + \sum_{j=1}^{n} e_{ij} z_j(t-\zeta_j), \quad i = 1, 2, \dots, n.$$

We can write

$$|z_i(t)| \leq |y_i(t)| + \sum_{j=1}^{n} |e_{ij}||z_j(t-\zeta_i)|, \quad i = 1, 2, \dots, n. \tag{33}$$

Now, choose a positive constant T such that $0 \leq t \leq T$. Then, (33) can be written as

$$|z_i(t)| \leq |y_i(t)| + \sum_{j=1}^{n} |e_{ij}| \sup_{0 \leq t \leq T} |z_j(t)| + \sum_{j=1}^{n} |e_{ij}| \sup_{-\Omega \leq t \leq 0} |z_j(t)|, \quad i = 1, 2, \dots, n. \tag{34}$$

(34) can be written as

$$\sup_{0 \leq t \leq T} |z_i(t)| \leq \sup_{0 \leq t \leq T} |y_i(t)| + \sum_{j=1}^{n} |e_{ij}| \sup_{0 \leq t \leq T} |z_j(t)| + \sum_{j=1}^{n} |e_{ij}| \sup_{-\Omega \leq t \leq 0} |z_j(t)|, \quad i = 1, 2, \dots, n. \tag{35}$$

From (35), we obtain

$$\sum_{i=1}^{n} \sup_{0 \leq t \leq T} |z_i(t)| \leq \sum_{i=1}^{n} \sup_{0 \leq t \leq T} |y_i(t)| + \sum_{i}^{n} \sum_{j=1}^{n} |e_{ij}| \sup_{0 \leq t \leq T} |z_j(t)| + \sum_{i}^{n} \sum_{j=1}^{n} |e_{ij}| \sup_{-\Omega \leq t \leq 0} |z_j(t)| \tag{35}$$

(35) implies the following inequality

$$\sum_{i=1}^{n} (1 - \sum_{j=1}^{n} |e_{ji}|) \sup_{0 \leq t \leq T} |z_i(t)| \leq \sum_{i=1}^{n} \sup_{0 \leq t \leq T} |y_i(t)| + \sum_{i}^{n} \sum_{j=1}^{n} |e_{ij}| \sup_{-\Omega \leq t \leq 0} |z_j(t)| \tag{36}$$

Let $e_m = \min\{e_i\}$. Then, (36) takes the form

$$e_m \sum_{i=1}^{n} \sup_{0 \leq t \leq T} |z_i(t)| \leq \sum_{i=1}^{n} \sup_{0 \leq t \leq T} |y_i(t)| + \sum_{i}^{n} \sum_{j=1}^{n} |e_{ij}| \sup_{-\Omega \leq t \leq 0} |z_j(t)| \tag{37}$$

From (37), we obtain

$$e_m \sup_{0 \leq t \leq T} \|z(t)\|_1 \leq \sup_{0 \leq t \leq T} \|y(t)\|_1 + \sum_{i}^{n} \sum_{j=1}^{n} |e_{ij}| \sup_{-\Omega \leq t \leq 0} |z_j(t)| \tag{38}$$

Since the term

$$\sum_{i}^{n} \sum_{j=1}^{n} |e_{ij}| \sup_{-\Omega \leq t \leq 0} |z_j(t)|$$

is bounded, it follows form (38) that if $\|z(t)\|_1 \to \infty$, then $\|y(t)\|_1 \to \infty$. $\mathrm{V}(t)$ given by (13) ensures the following

$$\mathrm{V}(t) \geq \sum_{i=1}^{n} c_i\, y_i^2(t) \geq c_m \,\|y(t)\|_2^2$$

Since $\|y(t)\|_2^2 \geq \frac{1}{n}\|y(t)\|_1^2$, we get that

$$\mathrm{V}(t) \geq \frac{c_m}{n}\,\|y(t)\|_1^2$$

Thus, $\|z(t)\|_1 \to \infty$ also implies that $\mathrm{V}(t) \to \infty$. Q.E.D.

3. An Instructive Example

This section considers an example to demonstrate the applicability of the propose stability result.

Example: Consider the neutral system given by (1) which have the following matrices:

$$A = \frac{1}{8}\begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -1 & 1 & -1 \\ 1 & 1 & -1 & -1 \\ -1 & 1 & 1 & -1 \end{bmatrix}, \quad B = \frac{1}{8}\begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -1 & 1 & -1 \\ 1 & 1 & -1 & -1 \\ -1 & 1 & 1 & -1 \end{bmatrix}, \quad E = \begin{bmatrix} e & e & e & e \\ e & e & e & e \\ e & e & e & e \\ e & e & e & e \end{bmatrix}$$

$$C = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \quad \mathcal{L} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

where e is a positive constant. From the above matrices, we calculate : $c_m = 1, : c_M = 1, : \ell_M = 1,$ $\|A\|_2 = \frac{1}{4}, \|B\|_2 = \frac{1}{4}$ and $\|E\|_2 = 4e$. Then, the conditions of Theorem 1 are determined to satisfy the conditions:

$$\delta = 1 - \frac{1}{4} - 8\mathrm{e} - 4\mathrm{e} = \frac{3}{4} - 12\mathrm{e} > 0$$

and

$$e_i = 1 - 4\mathrm{e} > 0, \quad i = 1, 2,3,4.$$

Thus, for this example, $\mathrm{e} < \frac{1}{16}$ is determined to be a sufficient condition for stability of neural system (1).

4. Conclusions

This research work has addressed stability problem for neutral-type Hopfield neural networks involving discrete time delays in the states of neurons and discrete neutral delays in the time derivatives of the states of neurons. By utilizing an appropriate Lyapunov functional, an easily verifiable algebraic criterion for global asymptotic stability of the class of Hopfield neural systems of neutral type has been presented. This stability condition proved to absolutely independent of the discrete time and neutral delays. An instructive example has been given to demonstrate the applicability of the proposed global stability condition.

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