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ON SOME PROPERTIES OF INTUITIONISTIC FUZZY MAGNIFIED TRANSLATION IN A Γ-SEMIGROUP

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Abstract: In this paper some properties of intuitionistic fuzzy magnified translation in Γ -Semigroups have been verified.

Key words: Intuitionistic fuzzy magnified translation, intuitionistic fuzzy subsemigroup, intuitionistic fuzzy left and right ideal, intuitionistic fuzzy interior ideal, intuitionistic fuzzy bi-ideal

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Γ-YARI GRUPLARDA SEZGİSEL BULANIK GENİŞLETİLMİŞ ÖTELEMENİN BAZI ÖZELLİKLERİ ÜZERİNE

Özet: Bu çalışmada Γ-yarı gruplarda sezgisel bulanık genişletilmiş ötelemenin bazı özellikleri çalışıldı.

Anahtar kelimeler: Sezgisel bulanık genişletilmiş öteleme, sezgisel bulanık alt yarı grup, sezgisel bulanık sol ve sağ ideal, sezgisel bulanık iç ideal, sezgisel bulanık bi-ideal

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1. INTRODUCTION

The notion of fuzzy sets was introduced by ZADEH (1965). The concept of intuitionistic fuzzy set was introduced by ATANASSOV (1986) and ATANASSOV & STOEVA (1983), as a generalization of the notion of fuzzy set. KANDASAMY (2003) introduced the concept of fuzzy translation and fuzzy multiplication. The idea of fuzzy magnified translation has been introduced by MAJUMDER & SARDAR (2008) The notion of intuitionistic fuzzy magnified translation and some of its properties in a semigroup has been introduced by MAJUMDER & SARDAR (2008). KUROKI (1981) discussed different properties of fuzzy ideals in a semigroup. SEN & SAHA (1986) introduced the concept of Γ-semigroups. UCKUN et al. (2007) investigated some properties of ideals in Γ-semigroups. The aim of this paper is to verify some properties of intuitionistic fuzzy magnified translation in a Γ-semigroup.

2. PRELIMINARIES

Definition 2.1. *A fuzzy subset of a non-empty set* X *is a function* $\mu : X \rightarrow [0,1]$ (ZADEH 1965).

Definition 2.2. *An intuitionistic fuzzy set* A *in a non-empty set* X *is an object having the form* $A = \{(x, \mu_A(x), \nu_A(x)) / x \in X\}$ *where the functions* $\mu_A : X \rightarrow [0,1]$ *and* $\nu_A : X \rightarrow [0,1]$ *denote the degree of membership and the degree of non-membership respectively, and* $0 \leq \mu_A(x) + \nu_A(x) \leq 1$ *for all* $x \in X$ (ATANASSOV 1986, ATANASSOV & STOEVA 1983).

Definition 2.3. *Let* μ *be a fuzzy subset of a set* X *and* $\alpha \in [0, 1 - \sup\{\mu(x) : x \in X\}]$ *and* $\beta \in [0,1]$. *A mapping* $\mu_{\beta\alpha}{}^C : X \rightarrow [0,1]$ *is called a fuzzy magnified translation of* μ *if* $\mu_{\beta\alpha}{}^C = \beta.\mu(x) + \alpha$ *for all* $x \in X$ (MAJUMDER & SARDAR 2008).

Definition 2.4. *Let* $A = \{(x, \mu_A(x), \nu_A(x)) / x \in X\}$ *be an intuitionistic fuzzy subset of a non-empty set* X *and* $\alpha \in [0, 1 - \sup\{\mu(x) : x \in X\}]$ *and* $\beta \in [0,1]$. *The intuitionistic fuzzy magnified translation* $A_{\beta\alpha}{}^C$ *of* A *is an object having the form*

$$A_{\beta\alpha}{}^C = \{(x, (\mu_A)_{\beta\alpha}{}^C(x), (\nu_A)_{\beta\alpha}{}^C(x)) / x \in X\}$$

where the functions $(\mu_A)_{\beta\alpha}{}^C : X \rightarrow [0,1]$ *and* $(\nu_A)_{\beta\alpha}{}^C : X \rightarrow [0,1]$ *is defined by*

$$(\mu_A)_{\beta\alpha}{}^C(x) = \beta.\mu_A(x) + \alpha \text{ and } (\nu_A)_{\beta\alpha}{}^C(x) = \beta.\nu_A(x) - \alpha$$

for all $x \in X$ (MAJUMDER & SARDAR 2008).

Definition 2.5. *Let* S *and* Γ *be two non-empty sets.* S *is called a* Γ*-semigroup if there exist a mapping* $S \times \Gamma \times S \rightarrow S$, *written as* $(a, \gamma, b) \rightarrow a\gamma b$, *satisfying*

$$(a\gamma b)\mu c = a\gamma(b\mu c)$$

for all $a, b, c \in S$ *and* $\gamma, \mu \in \Gamma$ (SEN & SAHA 1986).

Definition 2.6. *Let* S *be a* Γ*-semigroup. An intuitionistic fuzzy subset* $A = (\mu_A, \nu_A)$ *of* S *is said to be an intuitionistic fuzzy subsemigroup of* S *if*

$(i)\ \mu_A(x\gamma y) \geq \min\{\mu_A(x), \mu_A(y)\}$

$(ii)\ \nu_A(x\gamma y) \leq \max\{\nu_A(x), \nu_A(y)\}$

for all $x, y \in S$ *and for all* $\gamma \in \Gamma$ (UCKUN et al. 2007).

Definition 2.7. *Let* S *be a* Γ*-semigroup. An intuitionistic fuzzy subset* $A = (\mu_A, \nu_A)$ *of* S *is said to be an intuitionistic fuzzy left ideal of* S *if*

$(i)\ \mu_A(x\gamma y) \geq \mu_A(y)$

$(ii)\ \nu_A(x\gamma y) \leq \nu_A(y)$

for all $x, y \in S$ *and for all* $\gamma \in \Gamma$ (UCKUN et al. 2007).

Definition 2.8. *Let S be a Γ-semigroup. An intuitionistic fuzzy subset $A = (\mu_A, \nu_A)$ of S is said to be an intuitionistic fuzzy right ideal of S if*
$(i)\ \mu_A(x\gamma y) \geq \mu_A(x)$
$(ii)\ \nu_A(x\gamma y) \leq \nu_A(x)$
for all $x, y \in S$ and for all $\gamma \in \Gamma$ (UCKUN et al. 2007).

Definition 2.9. *Let S be a Γ-semigroup. An intuitionistic fuzzy subset $A = (\mu_A, \nu_A)$ of S is said to be an intuitionistic fuzzy ideal of S if it is both intuitionistic fuzzy left ideal and intuitionistic fuzzy right ideal of S* (UCKUN et al. 2007).

Definition 2.10. *Let S be a Γ-semigroup. An intuitionistic fuzzy subset $A = (\mu_A, \nu_A)$ of S is said to be an intuitionistic fuzzy interior ideal of S if*
$(i)\ \mu_A(x\beta y\gamma z) \geq \mu_A(y)$
$(ii)\ \nu_A(x\beta y\gamma z) \leq \nu_A(y)$
for all $x, y, z \in S$ and for all $\beta, \gamma \in \Gamma$ (UCKUN et al. 2007).

Definition 2.11. *Let S be a Γ-semigroup. An intuitionistic fuzzy subset $A = (\mu_A, \nu_A)$ of S is said to be an intuitionistic fuzzy bi-ideal of S if*
$(i)\ \mu_A(x\beta s\gamma y) \geq \min\{\mu_A(x), \mu_A(y)\}$
$(ii)\ \nu_A(x\beta s\gamma y) \leq \max\{\nu_A(x), \nu_A(y)\}$
for all $x, s, y \in S$ and for all $\beta, \gamma \in \Gamma$ (UCKUN et al. 2007).

Definition 2.12. *A Γ-semigroup S is called regular if, for each element x in S, there exist $s \in S$ and $\beta, \gamma \in \Gamma$ such that $x = x\beta s\gamma x$* (UCKUN et al. 2007).

Definition 2.13. *A Γ-semigroup S is called left zero if, for each element x in S, there exist $y \in S$ and $\gamma \in \Gamma$ such that $x = x\gamma y$* (UCKUN et al. 2007).

3. PROPERTIES OF INTUITIONISTIC FUZZY MAGNIFIED TRANSLATION

Theorem 3.1. *If $A = (\mu_A, \nu_A)$ is an intuitionistic fuzzy subsemigroup of a Γ-semigroup S, then $A_{\beta\alpha}{}^C = ((\mu_A)_{\beta\alpha}{}^C, (\nu_A)_{\beta\alpha}{}^C)$ is an intuitionistic fuzzy subsemigroup of S.*

Proof. Let $A = (\mu_A, \nu_A)$ be an intuitionistic fuzzy subsemigroup of S and $x, y \in S, \gamma \in \Gamma$. Then

$$\begin{aligned}(\mu_A)_{\beta\alpha}{}^C(x\gamma y) &= \beta\mu_A(x\gamma y) + \alpha \\ &\geq \beta.\min\{\mu_A(x), \mu_A(y)\} + \alpha \\ &= \min\{\beta\mu_A(x) + \alpha, \beta\mu_A(y) + \alpha\}\end{aligned}$$

$$= \min\{(\mu_A)_{\beta\alpha}{}^C(x), (\mu_A)_{\beta\alpha}{}^C(y)\}$$

and

$$\begin{aligned}(\nu_A)_{\beta\alpha}{}^C(x\gamma y) &= \beta.\nu_A(x\gamma y) - \alpha \\ &\leq \beta.\max\{\nu_A(x), \nu_A(y)\} - \alpha \\ &= \max\{\beta\nu_A(x) - \alpha, \beta\nu_A(y) - \alpha\} \\ &= \max\{(\nu_A)_{\beta\alpha}{}^C(x), (\nu_A)_{\beta\alpha}{}^C(y)\}.\end{aligned}$$

Hence $A_{\beta\alpha}{}^C = ((\mu_A)_{\beta\alpha}{}^C, (\nu_A)_{\beta\alpha}{}^C)$ is an intuitionistic fuzzy subsemigroup of S.

Theorem 3.2. *If $A = (\mu_A, \nu_A)$ is an intuitionistic fuzzy left ideal (right ideal, ideal) of a Γ-semigroup S, then $A_{\beta\alpha}{}^C = ((\mu_A)_{\beta\alpha}{}^C, (\nu_A)_{\beta\alpha}{}^C)$ is an intuitionistic fuzzy left ideal (resp. right ideal, ideal) of S.*

Proof. Let $A = (\mu_A, \nu_A)$ be an intuitionistic fuzzy left ideal of S and $x, y \in S, \gamma \in \Gamma$. Then

$$(\mu_A)_{\beta\alpha}{}^C(x\gamma y) = \beta.\mu_A(x\gamma y) + \alpha \geq \beta.\mu_A(y) + \alpha = (\mu_A)_{\beta\alpha}{}^C(y)$$

and

$$(\nu_A)_{\beta\alpha}{}^C(x\gamma y) = \beta.\nu_A(x\gamma y) - \alpha \leq \beta.\nu_A(y) - \alpha = (\nu_A)_{\beta\alpha}{}^C(y).$$

Hence $A_{\beta\alpha}{}^C = ((\mu_A)_{\beta\alpha}{}^C, (\nu_A)_{\beta\alpha}{}^C)$ is an intuitionistic fuzzy left ideal of S. In a similar way we can prove the other cases also.

Theorem 3.3. *If $A = (\mu_A, \nu_A)$ is an intuitionistic fuzzy interior ideal of a Γ-semigroup S, then $A_{\beta\alpha}{}^C = ((\mu_A)_{\beta\alpha}{}^C, (\nu_A)_{\beta\alpha}{}^C)$ is an intuitionistic fuzzy interior ideal of S.*

Proof. Let $A = (\mu_A, \nu_A)$ be an intuitionistic fuzzy interior ideal of S and $x, y, z \in S, \delta, \gamma \in \Gamma$. Then

$$(\mu_A)_{\beta\alpha}{}^C(x\delta y\gamma z) = \beta.\mu_A(x\delta y\gamma z) + \alpha \geq \beta.\mu_A(y) + \alpha = (\mu_A)_{\beta\alpha}{}^C(y)$$

and

$$(\nu_A)_{\beta\alpha}{}^C(x\delta y\gamma z) = \beta.\nu_A(x\delta y\gamma z) - \alpha \leq \beta.\nu_A(y) - \alpha = (\nu_A)_{\beta\alpha}{}^C(y).$$

Hence $A_{\beta\alpha}{}^C = ((\mu_A)_{\beta\alpha}{}^C, (\nu_A)_{\beta\alpha}{}^C)$ is an intuitionistic fuzzy interior ideal of S.

Theorem 3.4. *If $A = (\mu_A, \nu_A)$ is an intuitionistic fuzzy bi-ideal of a Γ-semigroup S, then $A_{\beta\alpha}{}^C = ((\mu_A)_{\beta\alpha}{}^C, (\nu_A)_{\beta\alpha}{}^C)$ is an intuitionistic fuzzy bi-ideal of S.*

Proof. Let $A = (\mu_A, \nu_A)$ be an intuitionistic fuzzy bi-ideal of S and $x, s, y \in S, \delta, \gamma \in \Gamma$. Then

$$\begin{aligned}(\mu_A)_{\beta\alpha}{}^C(x\delta s\gamma y) = \beta\mu_A(x\delta s\gamma y) + \alpha &\geq \beta.\min\{\mu_A(x), \mu_A(y)\} + \alpha \\ &= \min\{\beta\mu_A(x) + \alpha, \beta\mu_A(y) + \alpha\}\end{aligned}$$

$$= \min\{(\mu_A)_{\beta\alpha}{}^C(x), (\mu_A)_{\beta\alpha}{}^C(y)\}$$

and

$$(\nu_A)_{\beta\alpha}{}^C(x\delta s\gamma y) = \beta\nu_A(x\delta s\gamma y) - \alpha \le \beta.\max\{\nu_A(x), \nu_A(y)\} - \alpha$$
$$= \max\{\beta\nu_A(x) - \alpha, \beta\nu_A(y) - \alpha\}$$
$$= \max\{(\nu_A)_{\beta\alpha}{}^C(x), (\nu_A)_{\beta\alpha}{}^C(y)\}.$$

Hence $A_{\beta\alpha}{}^C = ((\mu_A)_{\beta\alpha}{}^C, (\nu_A)_{\beta\alpha}{}^C)$ is an intuitionistic fuzzy bi-ideal of S.

Theorem 3.5. *If $A = (\mu_A, \nu_A)$ is an intuitionistic fuzzy interior ideal of a regular Γ-semigroup S, then $A_{\beta\alpha}{}^C = ((\mu_A)_{\beta\alpha}{}^C, (\nu_A)_{\beta\alpha}{}^C)$ is an intuitionistic fuzzy left ideal of S.*

Proof. Let S be regular and $x, y \in S, \eta \in \Gamma$, then there exist $s \in S$ and $\delta, \gamma \in \Gamma$ such that $y = y\delta s\gamma y$. Now, taking into consideration regularly of S,

$$(\mu_A)_{\beta\alpha}{}^C(x\eta y) = \beta.\mu_A(x\eta y) + \alpha$$
$$= \beta.\mu_A(x\eta(y\delta s\gamma y)) + \alpha$$
$$= \beta\mu_A(x\eta y\delta(s\gamma y)) + \alpha$$
$$\ge \beta\mu_A(y) + \alpha = (\mu_A)_{\beta\alpha}{}^C(y)$$

and

$$(\nu_A)_{\beta\alpha}{}^C(x\eta y) = \beta\nu_A(x\eta y) - \alpha$$
$$= \beta\nu_A(x\eta(y\delta s\gamma y)) - \alpha$$
$$= \beta\nu_A(x\eta y\delta(s\gamma y)) - \alpha$$
$$\le \beta\nu_A(y) - \alpha = (\nu_A)_{\beta\alpha}{}^C(y).$$

Hence $A_{\beta\alpha}{}^C = ((\mu_A)_{\beta\alpha}{}^C, (\nu_A)_{\beta\alpha}{}^C)$ is an intuitionistic fuzzy left ideal of S.

Theorem 3.6. *If $A = (\mu_A, \nu_A)$ is an intuitionistic fuzzy interior ideal of a regular Γ-semigroup S, then $A_{\beta\alpha}{}^C = ((\mu_A)_{\beta\alpha}{}^C, (\nu_A)_{\beta\alpha}{}^C)$ is an intuitionistic fuzzy right ideal of S.*

Proof. Let S be regular and $x, y \in S, \eta \in \Gamma$, then there exist $s \in S$ and $\delta, \gamma \in \Gamma$ such that $x = x\delta s\gamma x$. Now, taking into consideration regularly of S,

$$(\mu_A)_{\beta\alpha}{}^C(x\eta y) = \beta\mu_A(x\eta y) + \alpha$$
$$= \beta\mu_A((x\delta s\gamma x)\eta y) + \alpha$$
$$= \beta\mu_A((x\delta s)\gamma x\eta y)) + \alpha$$
$$\ge \beta\mu_A(x) + \alpha = (\mu_A)_{\beta\alpha}{}^C(x)$$

and

$$(\nu_A)_{\beta\alpha}{}^C(x\eta y) = \beta\nu_A(x\eta y) - \alpha$$
$$= \beta\nu_A((x\delta s\gamma x)\eta y) - \alpha$$

$$= \beta \nu_A((x\delta s)\gamma x\eta y) - \alpha$$
$$\leq \beta \nu_A(x) - \alpha = (\nu_A)_{\beta\alpha}{}^C(x).$$

Hence $A_{\beta\alpha}{}^C = ((\mu_A)_{\beta\alpha}{}^C, (\nu_A)_{\beta\alpha}{}^C)$ is an intuitionistic fuzzy right ideal of S.
In view of Theorem 3.5 and Theorem 3.6 we can have the following theorem.

Theorem 3.7. *If $A = (\mu_A, \nu_A)$ is an intuitionistic fuzzy interior ideal of a regular Γ-semigroup S, then $A_{\beta\alpha}{}^C = ((\mu_A)_{\beta\alpha}{}^C, (\nu_A)_{\beta\alpha}{}^C)$ is an intuitionistic fuzzy ideal of S.*

Theorem 3.8. *If $A = (\mu_A, \nu_A)$ is an intuitionistic fuzzy left ideal of a left zero Γ-semigroup S then, $A_{\beta\alpha}{}^C = ((\mu_A)_{\beta\alpha}{}^C, (\nu_A)_{\beta\alpha}{}^C)$ is a constant function.*

Proof. Let S be a left zero Γ-semigroup and $x \in S$, then there exist $y \in S$ and $\gamma \in \Gamma$ such that $x = x\gamma y$. Now,

$$\begin{aligned}(\mu_A)_{\beta\alpha}{}^C(x) &= \beta\mu_A(x) + \alpha \\ &= \beta\mu_A(x\gamma y) + \alpha \quad \text{(since } S \text{ is a left zero } \Gamma\text{-semigroup)} \\ &\geq \beta\mu_A(y) + \alpha \quad \text{(since } A \text{ is an intuitionistic fuzzy left ideal of } S\text{)} \\ &= (\mu_A)_{\beta\alpha}{}^C(y).\end{aligned}$$

Again,

$$\begin{aligned}(\mu_A)_{\beta\alpha}{}^C(y) &= \beta\mu_A(y) + \alpha \\ &= \beta\mu_A(y\gamma x) + \alpha \quad \text{(since } S \text{ is a left zero } \Gamma\text{-semigroup)} \\ &\geq \beta\mu_A(x) + \alpha \quad \text{(since } A \text{ is an intuitionistic fuzzy left ideal of S)} \\ &= (\mu_A)_{\beta\alpha}{}^C(x).\end{aligned}$$

Hence $(\mu_A)_{\beta\alpha}{}^C(x) = (\mu_A)_{\beta\alpha}{}^C(y)$ for all $x, y \in S$. Similarly we can show that

$$(\nu_A)_{\beta\alpha}{}^C(x) = (\nu_A)_{\beta\alpha}{}^C(y)$$

for all $x, y \in S$. Consequently, $A_{\beta\alpha}{}^C(x) = A_{\beta\alpha}{}^C(y)$ for all $x, y \in S$. Hence $A_{\beta\alpha}{}^C = ((\mu_A)_{\beta\alpha}{}^C, (\nu_A)_{\beta\alpha}{}^C)$ is a constant function.

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