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On Fuzzy Weakly Completely Prime Ideal in Γ-Semigroups

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Abstract: In this paper the notion of fuzzy weakly completely prime ideal in Γ-semigroups has been introduced. Finally, the concept of operator semigroups of a Γ-semigroup has been employed to study the relationship between their respective fuzzy weakly completely prime ideals.

Key words: Γ-semigroup, Operator semigroups, Fuzzy subsemigroup, Fuzzy weakly completely prime ideal.

Mathematics Subject Classifications [2000]: 20M12, 03F55, 08A72

Γ-Yarıgruplarda Bulanık Zayıf Tam Asal İdeal Üzerine

Özet: Bu çalışmada, Γ-yarıgruplarda bulanık zayıf tam asal ideal kavramı verilmiştir. Bir Γ-yarıgrubunun operator yarıgrupları kavramı, bunların temsili bulanık zayıf tam asal idealleri arasındaki ilişkiler ortaya konulmuştur.

Anahtar kelimeler: Γ-yarıgrup, Operatör yarıgruplar, Bulanık altyarıgruplar, Bulanık zayıf tam asal ideal.

1. Introduction

A semigroup (see [1]) is an algebraic structure consisting of a non-empty set S together with an associative binary operation. The formal study of semigroups began in the early 20th century. Semigroups are important in many areas of mathematics, for example, coding and language theory, automata theory, combinatorics and mathematical analysis. In 1981, *M.K. Sen* [2] introduced the notion of Γ-semigroup as a generalization of semigroup and ternary semigroup. We call this Γ-semigroup a both sided Γ-semigroup. *M.K. Sen* and *N.K. Saha* [3] and *N.K. Saha* [4] modified the definition of *Sen's* Γ-semigroup. This newly defined Γ-semigroup is known as one sided Γ-semigroup. Γ-semigroups have been analyzed by lot of mathematicians, for instance by *Chattopadhay* [5,6], *Dutta and Adhikari* [7,8], *Hila* [9,10], *Chinram* [11], *Saha* [4], *Sen et al* [3], *Seth* [12]. *Dutta and Adhikari* [7,8] mostly worked on both sided Γ-semigroups. They defined operator semigroups of such type of Γ-semigroups and established many results and found out many correspondences. In this paper we have considered one sided Γ-semigroup of Sen and Saha. After the introduction of fuzzy sets by *Zadeh* [13], reconsideration of the concept of classical mathematics began. As an immediate result fuzzy algebra is a well established branch of mathematics at present. Many authors have studied semigroups in terms of fuzzy sets. *Kuroki* [14,15,16] is the pioneer of this study. *Uckun et al* [17] initiated the study of Γ-semigroups in terms of intuitionistic fuzzy subsets. Motivated by *Kuroki* [14,15,16], *Uckun et al* [17], *Sardar et al* [18,19,20,21] studied Γ-semigroups in terms of fuzzy subsets. In this short communication the notion of fuzzy weakly completely prime ideal in Γ-semigroups has been introduced and some of their important properties have been observed. Various relationships between fuzzy weakly completely prime ideals of a Γ-semigroup and fuzzy weakly completely prime ideals(fuzzy subsemigroups) of its operator semigroups have been obtained. Among other results an inclusion preserving bijection between the set of all fuzzy subsemigroups of a Γ-semigroup and that of its operator semigroups has been obtained by using the inclusion preserving bijection between their set of respective fuzzy weakly completely prime ideals.

2. Preliminaries

Throughout this paper S denotes a Γ-semigroup unless or otherwise mentioned.

Let $S = \{x, y, z,.....\}$ and $\Gamma = \{\alpha, \beta, \gamma,.....\}$ be two non-empty subsets. Then S is called a Γ-semigroup if there exists a mapping $S \times \Gamma \times S \to S$ (images to be denoted by $x\alpha y$) satisfying

(1) $x\gamma y \in S$,
(2) $(x\beta y)\gamma z = x\beta(y\gamma z)$, for all $x, y, z \in S$ and for all $\beta, \gamma \in \Gamma$ (see [2]).

Example 1. Let S be the set of all negative rational numbers. Let $\Gamma = \{-\frac{1}{p} : p$ is prime$\}$. Let $a, b, c \in S$ and $\alpha, \beta \in \Gamma$. Now if $a\alpha b$ is equal to the usual product of rational numbers a, α, b, then $a\alpha b \in S$ and $(a\alpha b)\beta c = a\alpha(b\beta c)$. Hence S is a Γ-semigroup.

Let S is a Γ-semigroup. By a left(right) ideal of S we mean a non-empty subset A of S such that $S\Gamma A \subseteq A (A\Gamma S \subseteq A)$ (see [7]). By a two sided ideal or simply an ideal, we mean a non-empty subset A of S which is both a left ideal and a right ideal of we mean a non-empty subset A of S (see [7]). An ideal P of S is said to be prime if, for any two ideals A and B of S, $A\Gamma B \subseteq P$ implies $A \subseteq P$ or $B \subseteq P$ (see [8]).

A function μ from a non-empty set S to the unit interval $[0,1]$ is called a fuzzy subset of S (see [13]).

A non-empty fuzzy subset μ of a Γ-semigroup S is called a fuzzy left ideal of S if $\mu(x\gamma y) \geq \mu(y) \forall x, y \in S, \forall \gamma \in \Gamma$ (see [18]).

A non-empty fuzzy subset μ of a Γ-semigroup S is called a fuzzy right ideal of S if $\mu(x\gamma y) \geq \mu(x) \forall x, y \in S, \forall \gamma \in \Gamma$ (see [18]).

A non-empty fuzzy subset μ of a Γ-semigroup S is called a fuzzy ideal of S if μ is a fuzzy left ideal and a fuzzy right ideal of S (see [18]).
Let μ be a fuzzy subset of a set S. Then for $t \in [0,1]$, the set $\mu_t = \{x \in S : \mu(x) \geq t\}$ is called the t-level subset or simply the level subset of μ (see [18]).

3. Fuzzy Weakly Completely Prime Ideal

Definition 3.1. A fuzzy ideal μ of a Γ-semigroup S is called a fuzzy weakly completely prime ideal of S if $\mu(x) \geq \mu(x\gamma y)$ or $\mu(y) \geq \mu(x\gamma y)$ $\forall x, y \in S$ and $\forall \gamma \in \Gamma$.

Example 2. Let S be the set all 1×2 matrices over GF_2 (the finite field with two elements) and Γ be the set of all 2×1 matrices over GF_2. Then S is a Γ-semigroup where $a\alpha b$ and $\alpha a \beta (a, b \in S, \alpha, \beta \in \Gamma)$ denote usual matrix product. Let $\mu : S \to [0,1]$ be defined by $\mu(x) = 0.3$, if $x = (0,0)$ and 0.4, otherwise. Then μ is a fuzzy weakly completely prime ideal of S.

Definition 3.2. A fuzzy ideal μ of a Γ-semigroup S is called a fuzzy prime ideal of S if $\inf_{\gamma \in \Gamma} \mu(x\gamma y) = \max\{\mu(x), \mu(y)\} \forall x, y \in S$ (see [19]).

Remark 1. Every fuzzy prime ideal of a Γ-semigroup S is a fuzzy weakly completely prime ideal of S. The converse is not always true which is clear from the following example.

Example 3. Let $S = \{e, a, b\}$ and $\Gamma = \{\gamma\}$, where γ is defined on S with the following caley table:

$$\begin{bmatrix} \gamma & e & a & b \\ e & e & e & e \\ a & e & a & e \\ b & e & e & b \end{bmatrix}$$

Then S is a Γ-semigroup . We define the fuzzy subset $\mu : S \to [0,1]$ as $\mu(x) = 0.5$, if $x = e$ and 0.5 if $x = a, b$. Then μ is a fuzzy weakly completely prime ideal of S but it is not a fuzzy prime ideal of S.

Theorem 3.3. Let μ be a non-empty fuzzy subset of a Γ-semigroup S. Then $1 - \mu$ is a fuzzy subsemigroup of S if and only if μ is a fuzzy weakly completely prime ideal of S.

Proof. Let $1-\mu$ be a fuzzy subsemigroup of S. Let $x, y \in S$ and $\gamma \in \Gamma$. Then $1-\mu(x\gamma y) \geq \min\{1-\mu(x), 1-\mu(y)\} \Leftrightarrow 1-\mu(x\gamma y) \geq 1-\max\{\mu(x), \mu(y)\}$

$$\Leftrightarrow \max\{\mu(x), \mu(y)\} \geq \mu(x\gamma y)$$
$$\Leftrightarrow \mu(x) \geq \mu(x\gamma y) \text{ or } \mu(y) \geq \mu(x\gamma y).$$

Hence μ is a fuzzy weakly completely prime ideal of S.

Theorem 3.4. Let $\{\mu_i : i \in I\}$ be a family of fuzzy weakly completely prime ideals of a Γ-semigroup S. Then $\bigcap_{i\in I} \mu_i$ is a fuzzy weakly completely prime ideal of S.

Proof. By hypothesis, $\mu_i(x) \geq \mu_i(x\gamma y)$ or $\mu_i(y) \geq \mu_i(x\gamma y)$ $\forall x, y \in S$, $\forall \gamma \in \Gamma$ and $\forall i \in I$. Then

$$\bigcap_{i\in I} \mu_i(x\gamma y) = \inf\{\mu_i(x\gamma y) : i \in I\} \leq \inf\{\mu_i(x) : i \in I\}$$

or

$$\inf\{\mu_i(y) : i \in I\}.$$

This implies that

$$\bigcap_{i\in I} \mu_i(x\gamma y) \leq \bigcap_{i\in I} \mu_i(x)$$

or

$$\bigcap_{i\in I} \mu_i(x\gamma y) \leq \bigcap_{i\in I} \mu_i(y).$$

Hence $\bigcap_{i\in I} \mu_i$ is a fuzzy weakly completely prime ideal of S.

Theorem 3.5. Let S be a Γ-semigroup and μ be a non-empty fuzzy subset of S. Then the following are equivalent: (1) μ is a fuzzy weakly completely prime ideal of S,(2) for any, $t \in [0,1]$, μ_t (if it is non-empty) is a prime ideal S.

Proof. Let μ be a fuzzy weakly completely prime ideal of S. Let $t \in [0,1]$ be such that μ_t is non-empty. Let $x, y \in S, x\Gamma y \subseteq \mu_t$. Then $\mu(x\gamma y) \geq t \forall \gamma \in \Gamma$. Since μ is a fuzzy weakly completely prime ideal of S, so we have $\mu(x) \geq \mu(x\gamma y)$ or $\mu(y) \geq \mu(x\gamma y)$. Then $\mu(x) \geq t$ or $\mu(y) \geq t$ which implies that $x \in \mu_t$ or $y \in \mu_t$. Hence μ_t is a prime ideal of S.
Conversely, let us suppose that μ_t is a prime ideal of S. Let $\mu(x\gamma y) = t$ (we note here that since $\mu(x\gamma y) \in [0,1] \forall \gamma \in \Gamma, \mu(x\gamma y)$ exists). Then $\mu(x\gamma y) \geq t \forall \gamma \in \Gamma$. Hence μ_t is non-empty and $x\Gamma y \subseteq \mu_t$. Since μ_t is a prime ideal of S, so we have $x \in \mu_t$ or $y \in \mu_t$. Then $\mu(x) \geq t$ or $\mu(y) \geq t$ which implies that $\mu(x) \geq \mu(x\gamma y)$ or $\mu(y) \geq \mu(x\gamma y)$. Hence μ is a fuzzy weakly completely prime ideal of S.

Theorem 3.6. Let A be a non-empty subset of a Γ-semigroup S and μ_A be the characteristic function of A. Then A is a left ideal(right ideal, ideal) of S if and only if μ_A is a fuzzy left ideal(fuzzy right ideal, fuzzy ideal) of S (see [18]).

Theorem 3.7. Let S be a Γ-semigroup and A be a non-empty subset of S. Then following are equivalent: (1) A is a prime ideal of S, (2) the characteristic function μ_A of A is a fuzzy weakly completely prime ideal of S.

Proof. Let A be a prime ideal of S and μ_A be the characteristic function of A. Since $A \neq \varphi$, so μ_A is non-empty. Let $x, y \in S$. Suppose $x\Gamma y \subseteq A$. Then $\mu_A(x\gamma y) = 1$ for $\gamma \in \Gamma$. Since A is a prime ideal of S, so $x \in A$ or $y \in A$ which implies that $\mu_A(x) = 1$ or $\mu_A(y) = 1$. Hence $\mu_A(x) \geq \mu_A(x\gamma y)$ or $\mu_A(y) \geq \mu_A(x\gamma y)$. Suppose $x\Gamma y \not\subseteq A$. Then $\mu_A(x\gamma y) = 0$ for $\gamma \in \Gamma$. Since A is a prime ideal of S, so $x \notin A$ or $y \notin A$ which implies that $\mu_A(x) = 0$ or $\mu_A(y) = 0$. Hence $\mu_A(x) \geq \mu_A(x\gamma y)$ or $\mu_A(y) \geq \mu_A(x\gamma y)$. Consequently, μ_A is a fuzzy weakly completely prime ideal of S.
Conversely, let μ_A is a fuzzy weakly completely prime ideal of S. Then μ_A is a fuzzy ideal of S. By Theorem 3.6, A is an ideal of S. Let $x, y \in S$ be such that $x\Gamma y \subseteq A$. Then $\mu_A(x\gamma y) = 1$. Let if possible $x \notin A$ and $y \notin A$. Then $\mu_A(x) = \mu_A(y) = 0$ which implies $\mu_A(x) < \mu_A(x\gamma y)$ and $\mu_A(y) < \mu_A(x\gamma y)$. This contradicts our assumption that μ_A is a fuzzy weakly completely prime ideal of S. Hence A is a prime ideal of S.

Remark 2. Theorem 3.5 and 3.7 are true in case of semigroup also.

4. Corresponding Fuzzy Weakly Completely Prime Ideal

Unless or otherwise stated, throughout this section S denotes a Γ-semigroup and $L.R$ be its left and right operator semigroups respectively.

Definition 4.1. Let S be a Γ-semigroup. Let us define a relation ρ on $S \times \Gamma$ as follows: $(x,\alpha)\rho(y,\beta)$ if and only if $x\alpha s = y\beta s$ for all $s \in S$ and $\gamma x\alpha = \gamma y\beta$ for all $\gamma \in \Gamma$. Then ρ is an equivalence relation. Let $[x,\alpha]$ denote the equivalence class containing (x,α). Let $L = \{[x,\alpha] : x \in S, \alpha \in \Gamma\}$. Then L is a semigroup with respect to the multiplication defined by $[x,\alpha][y,\beta] = [x\alpha y,\beta]$. This semigroup L is called the left operator semigroup of the Γ-semigroup S. Dually the right operator semigroup R of the Γ-semigroup S is defined where the multiplication is defined by $[\alpha,a][\beta,b] = [\alpha a\beta,b]$ (see [7]).

Definition 4.2. For a fuzzy subset μ of R we define a fuzzy subset μ^* of S by $\mu^*(a) = \inf_{\gamma\in\Gamma} \mu([\gamma,a])$, where $a \in S$. For a fuzzy subset σ of S we define a fuzzy subset

$\sigma^{*'}$ of R by $\sigma^{*'}([\alpha,a]) = \inf_{s\in S}\sigma(s\alpha a)$, where $[\alpha,a]\in R$. For a fuzzy subset δ of L, we define a fuzzy subset δ^{+} of S by $\delta^{+}(a) = \inf_{\gamma\in\Gamma}\mu([a,\gamma])$, where $a\in S$. For a fuzzy subset η of S we define a fuzzy subset $\eta^{+'}$ of L by $\eta^{+'}([a,\alpha]) = \inf_{s\in S}\sigma(a\alpha s)$, where $[a,\alpha]\in L$.

Now, we recall the following propositions (see [18]).

Proposition 4.3. Let S be a Γ-semigroup and L be its left operator semigroup. If P is a prime ideal of L then P^{+} is a prime ideal of S (see [8]).

Proposition 4.4. Let S be a Γ-semigroup and L be its left operator semigroup. If Q is a prime ideal of S then $Q^{+'}$ is a prime ideal of L (see [8]).

Proposition 4.5. Let S be a Γ-semigroup and R be its right operator semigroup. If P is a prime ideal of R then P^{*} is a prime ideal of S (see [8]).

Proposition 4.6. Let S be a Γ-semigroup and R be its right operator semigroup. If Q is a prime ideal of R then $Q^{*'}$ is a prime ideal of S (see [8]).

For convenience of the readers, we may note that for a Γ-semigroup S and its left and right operator semigroups L,R respectively four mappings namely $()^{+},()^{+'},()^{*},()^{*'}$ occur. They are defined as follows :

$$(i)\ \text{For } I\subseteq R, I^{*} = \{s\in S, [\alpha,s]\in I\forall\alpha\in\Gamma\};$$
$$(ii)\ \text{For } P\subseteq S, P^{*'} = \{[\alpha,x]\in R : s\alpha x\in P\forall s\in S\};$$
$$(iii)\ \text{For } J\subseteq L, J^{+} = \{s\in S, [s,\alpha]\in J\forall\alpha\in\Gamma\};$$
$$(iv)\ \text{For } Q\subseteq S, Q^{+'} = \{[x,\alpha]\in L : x\alpha s\in Q\forall s\in S\}.$$

Proposition 4.7. Let μ be a fuzzy subset of R (the right operator semigroup of a Γ-semigroup S). Then $(\mu^{*})_{t} = (\mu_{t})^{*}$, for all $t\in[0,1]$ such that the sets are non-empty (see [18]).

Proposition 4.8. Let σ be a fuzzy subset of a Γ-semigroup S. Then $(\sigma^{*'})_{t} = (\sigma_{t})^{*'}$, for all $t\in[0,1]$ such that the sets are non-empty (see [18]).

Proposition 4.9. If μ is a fuzzy weakly completely prime ideal of R then $1-\mu^{*}$ is a fuzzy subsemigroup of S.

Proof. Let μ be a fuzzy weakly completely prime ideal of R. Then μ_t is a prime ideal of R (*cf.* Remark 2). Hence $(\mu_t)^*$ is a prime ideal of S (*cf.* Proposition 4.5). Since $(\mu_t)^*$ and $(\mu^*)_t$ are non-empty, so by Proposition 4.7, we have $(\mu_t)^* = (\mu^*)_t$. Hence $(\mu^*)_t$ is a prime ideal of S. Consequently, μ^* is a fuzzy weakly completely prime ideal S (*cf.* Theorem 3.5). Hence $1-\mu^*$ is a fuzzy subsemigroup of S (*cf.* Theorem 3.3).

Theorem 4.10. Let μ be a non-empty fuzzy subset of a semigroup S. Then $1-\mu$ is a fuzzy subsemigroup of S if and only if μ is a fuzzy weakly completely prime ideal of S (see [22]).

Proposition 4.11. If σ is a fuzzy weakly completely prime ideal of S then $1-\sigma^{*'}$ is a fuzzy subsemigroup of R.

Proof. Let σ be a fuzzy weakly completely prime ideal of S. Then σ_t is a prime ideal of S (*cf.* Theorem 3.5). Hence $(\sigma_t)^{*'}$ is a prime ideal of R (*cf.* Proposition 4.6). Since $(\sigma_t)^{*'}$ and $(\sigma^{*'})_t$ are non-empty, so by Proposition 4.8, we have $(\sigma_t)^{*'} = (\sigma^{*'})_t$. Hence $(\sigma^{*'})_t$ is a prime ideal of R. Consequently, $\sigma^{*'}$ is a fuzzy weakly completely prime ideal R (*cf.* Remark 2). Hence $1-\sigma^{*'}$ is a fuzzy subsemigroup of R.

Remark 3. The left operator analogues of the above two propositions are true.

Theorem 4.12. Let S be a Γ-semigroup and R be its right operator semigroup. Then there exists an inclusion preserving bijection $\mu \mapsto \mu^{*'}$ between the set of all fuzzy weakly completely prime ideals of R and the set of all fuzzy weakly completely prime ideals of S, where μ is a fuzzy weakly completely prime ideal of R.

Proof. Let $x \in S$. Then

$$(\mu^{*'})^*(x) = \inf_{\alpha\in\Gamma} \mu^{*'}([\alpha,x]) = \inf_{\alpha\in\Gamma}\inf_{s\in S} \mu(s\alpha x) \geq \mu(x)$$

(since μ is a fuzzy ideal). Consequently, $\mu \subseteq (\mu^{*'})^*$. Again for $x \in S$,

$$(\mu^{*'})^*(x) = \inf_{\alpha\in\Gamma} \mu^{*'}([\alpha,x]) = \inf_{\alpha\in\Gamma}\inf_{s\in S} \mu(s\alpha x) \leq \mu(x)$$

(since μ is a fuzzy weakly completely prime ideal). Consequently, $\mu \supseteq (\mu^{*'})^*$. Hence $\mu = (\mu^{*'})^*$ and consequently the mapping is one-one. Now for $[\alpha,x] \in R$,

$$(\mu^*)^{*'}([\alpha,x]) = \inf_{s\in S} \mu^*(s\alpha x) = \inf_{s\in S}\inf_{\beta\in\Gamma} \mu([\beta,s\alpha x]) = \inf_{s\in S}\inf_{\beta\in\Gamma} \mu([\beta,s][\alpha,x]) \geq \mu([\alpha,x]).$$

Consequently, $\mu \subseteq (\mu^*)^{*'}$. Again, since μ is a fuzzy weakly completely prime ideal, so we have

$$\mu([\beta,s][\alpha,x]) \leq \mu([\beta,s])$$

or

$$\mu([\beta,s][\alpha,x]) \leq \mu([\alpha,x])$$

for all $s \in S$ and for all $\beta \in \Gamma$. Hence for $s = x$ and $\beta = \alpha$ we have

$$\mu([\beta,s][\alpha,x]) \leq \mu([\alpha,x]).$$

This together with the relation

$$(\mu^*)^{*'}([\alpha,x]) = \inf_{s\in S}\inf_{\beta\in\Gamma}\mu([\beta,s][\alpha,x])$$

gives

$$(\mu^*)^{*'}([\alpha,x]) \leq \mu([\alpha,x]).$$

Consequently, $(\mu^*)^{*'} \subseteq \mu \forall [\alpha,x] \in R$. Hence $\mu = (\mu^*)^{*'}$. This proves that the mapping is onto. Let μ_1 and μ_2 are fuzzy ideals of S such that $\mu_1 \subseteq \mu_2$. Then for all $[\alpha,x] \in R$,

$$(\mu_1)^{*'}([\alpha,x]) = \inf_{s\in S}\mu_1(s\alpha x) \leq \inf_{s\in S}\mu_2(s\alpha x) = (\mu_2)^{*'}([\alpha,x]).$$

Hence $(\mu_1)^{*'} \subseteq (\mu_2)^{*'}$. Similarly we can show that if $\sigma_1 \subseteq \sigma_2$ where σ_1 and σ_2 are fuzzy ideals of R, then $(\sigma_1)^* \subseteq (\sigma_2)^*$. Hence $\mu \mapsto \mu^{*'}$ is an inclusion preserving bijection.

Remark 4. Similar result holds for the Γ-semigroup S and the left operator semigroup L of S.

In view of Theorem 4.10, Theorem 3.3 and Theorem 4.12 we can have the following theorem.

Theorem 4.13. Let S be a Γ-semigroup and R be its right operator semigroup. Then there exists an inclusion preserving bijection $1-\mu \mapsto 1-\mu^{*'}$ between the set of all fuzzy subsemigroups of R and the set of all fuzzy subsemigroups of S, where $1-\mu$ is a fuzzy subsemigroup of R.

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