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Different Approaches To Ruled Surfaces

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Abstarct: In this study, surfaces are defined by using dual vectors and line transformations. A new approach is given for the transformation of parametrically surfaces. Dual curve and dual surface representational model for 3-dimensional geometric entities based on dual unit vectors are proposed. Some well-known new approaches like Blaschke approach of ruled surfaces are used. Moreover, geometric explanations of Bishop and Frenet are presented. Finally, an analytical comparison and the relation between Blaschke and Darboux approaches are represented showing the merits of our method.

Key words: Ruled surface, dual vectors, Bishop frame.

Regle Yüzeylere Farklı Yaklaşımlar

Özet: Bu çalışmada, yüzeyler dual vektörler ve doğru transformasyonları kullanılarak tanımlanmaktadır. Sonrasında parametrik yüzey transformasyonları için yeni bir yaklaşım verilmektedir. Üç boyutlu geometric öğeler için temeli dual birim vektörlere dayanan temsili dual eğri ve dual yüzey modeli ileri sürülmektedir. Burada bazı bilinen yeni yaklaşımlar kullanılmaktadır. Ayrıca, Bishop ve Frenet geometrik tanımları sunulmaktadır. Sonuç olarak, Blaschke ve Darboux yaklaşımları arasındaki analitik mukayese ve ilişki, metodumuzun doğruluğu gösterilerek belirtilmektedir.

Anahtar kelimeler: Regle yüzey, dual vektörler, Bishop çatı.

1. Introduction

Dual numbers were introduced in the 19th century by Clifford and their applications to rigid body kinematics were generalized by study in their principle of transference [16]. As it is known, the analytical tools in the study of 3-dimensional kinematics and differential geometry of ruled surfaces are based on dual vector calculus.

Important contributions to the curvature theory, frame approaches have been made by R.L. Bishop, Mc Carthy, Bottema, Blaschke, Hacısalihoglu H., Rashad A. Abdel-Baky, etc. In the study of P. Azariadis and N. Aspragathos, an alternative representational model for 3 dimensional geometric entities is expressed, which is based on dual unit vectors and dual unit quaternions. Veldkamp, Study, Bottema use the term `dual point' to express a point of a dual curve

$$\hat{x} = x(u) + \varepsilon x_0(u) \tag{1}$$

or dual surface

$$\widehat{X} = x(u,v) + \varepsilon x_0(u,v) \tag{2}$$

Dual points are used to describe line segments, curves and surfaces in E^3 as well as geometric invariant properties such as normal vectors or curvature vectors.

In the first part of this study, we briefly give basic concepts for the reader unfamiliar with dual elements, Darboux, Blaschke and Frenet frames. Necessary mathematical formulations and conventions are presented.

The next part of this study includes the analysis of unit dual spherical curves and ruled surfaces with Frenet and Blaschke frames. When we give representation of geometric entities using dual points, we use the 5th part of [7]. The representation of a dual surface using dual quaternions and dual matrices are briefly discussed.

S. G. Papageorgiou and N.A. Aspragathos expressed similarly that the 3D surface is given by unique parametric equation. A general spatial displacement of an object is equivalent to a rotation around a line and a translation along the same line. The name of motion is screw displacements around an axis by a dual angle.

We try to give the fundamental idea of replacing points by lines as basic concepts of geometry. At this time, [10] shows us that we can study a ruled surface as a curve on the dual unit sphere by using Blaschke approach. In [7], it can be seen that a dual curve can be defined as the set of dual points

In addition, we present the kinematic interpretation of dual representations by using relations between Blaschke, Frenet frames and Darboux vector. In this section, [6-7,10-11] help us to see another feature of the Frenet Frame. That is, it is adapted to the curve; the members are either tangent to or perpendicular to the curve. These studies say that the normal development of a curve in n-dimensional oriented Euclidean space is defined by a straight forward generalization of 3-dimensional case. It exists for C^2 regular curve and is a continous parametrized curve in oriented centro-euclidean space of dimension n-1, that is, it is defined up to a proper orthogonal transformation [6-7,10-11].

In addition, we investigate the development of the pitch of general line congruence by using fundamental dual elements and representations. In this connection, we study dual curve and dual surface representations which are showed by [14, 18] on these approaches of ruled surfaces.

2. Basic Concepts

Now we give basic concepts on classical differential geometry of space curves. References [1-5,8-9,12] contain basic concepts about the dual elements and one to one correspondence between ruled surface and one parameter spherical motions.

2.1 Frenet Frame

We assume that the curve α is parametrized by arclength. Then, $\alpha'(s)$ is the unit tangent vector to the curve, which we denote by $T(s)$ Since T has constant length, $T'(s)$ will be orthogonal to $T(s)$. If $T'(s) \neq 0$ then we define principal normal vector

$$N(s)=\frac{T'(s)}{\|T'(s)\|} \tag{3}$$

and the curvature

$$\kappa(s)=\|T'(s)\| \tag{4}$$

So far, we have

$$T'(s)=\kappa(s)N(s). \tag{5}$$

If $\kappa(s)=0,$ the principal normal vector is not defined. If $\kappa(s)\neq 0$ then the binormal vector $B(s)$ is given by

$$B(s)=T(s)\times N(s) \tag{6}$$

Then $\{T(s),N(s),B(s)\}$ form a right-handed orthonormal basis for $\mathbb{R}^3$. In summary, Frenet formulas can be given as

$$\begin{aligned} T'(s)&=\kappa(s)N(s)\\ N'(s)&=-\kappa(s)T(s)+\tau(s)B(s)\\ B'(s)&=-\tau(s)N(s). \end{aligned} \tag{7}$$

2.2 Blaschke Frame

Let $M(s,u)$ be the ruled surface and $A(s)$ be the dual spherical curve in D^3;

$$A(s)=a(s)+\varepsilon a^{*}(s) \tag{8}$$

We now define an orthonormal moving frame along this dual curve as follows:

$$\begin{aligned} A_1&=A(s),\\ A_2&=\frac{A_1'}{\|A_1'\|},\\ A_3&=A_1\times A_2. \end{aligned} \tag{9}$$

2.3 Bishop Frame

Suppose α is an arc-length parametrized C^2 curve. Suppose we have C^1 unit vector fields N_1 and $N_2=T\times N_1$ along α so that

$$TN_1 = TN_2 = N_1 N_2 = 0; \tag{10}$$

T, N_1, N_2 will be smoothly varying right-handed orthonormal frame as we move along the curve. We want to impose the extra condition that

$$N_1' N_2 = 0 \tag{11}$$

We say the unit normal vector field N_1 is parallel along α; this means that the only change of N_1 is in the direction of T. In this event, T, N_1, N_2 is called a Bishop Frame for α. There is a big relation between Bishop Frame and Frenet Frame.

2.4 Mannheim Partner Curves in 3-space

Let E^3 be the 3-dimensional Euclidean space with the standard inner product $\langle , \rangle$. If there exists a corresponding relationship between the space curves Γ and Γ_1 such that, at the corresponding points of the curves, the principal normal lines coincide with the binormal lines of Γ_1, then Γ is called a Mannheim curve, and Γ_1 a Mannheim partner curve of Γ. The pair $\{\Gamma, \Gamma_1\}$ is said to be a Mannheim pair. From the elementary differential geometry we have the well-known characterizations of Bertrand pair. But there are rather few works on Mannheim pair. It is just known that a space curve in E^3 is a Mannheim curve if and only if its curvature κ and torsion τ satisfy the formula $\kappa = \lambda(\kappa^2 + \tau^2)$, where λ is a nonzero constant.

Here and further, we denote by the derivative with respect to the arc length parameter of a curve.

2.5 Mannheim Partner Curves in Dual Space

Let ID^3 be the dual space with the standard inner product $\langle , \rangle$. If there exists a corresponding relationship between the dual space curves $\widehat{\alpha}$ and $\widehat{\beta}$ such that, at the corresponding points of the dual curves, the principal normal lines of $\widehat{\alpha}$ coincide with the binormal lines of $\widehat{\beta}$, then $\widehat{\alpha}$ is called a dual Mannheim curve, and $\widehat{\beta}$ a dual Mannheim partner curve of $\widehat{\alpha}$. The pair $\left\{\widehat{\alpha}, \widehat{\beta}\right\}$ is said to be a dual Mannheim pair.

3. Dual Curves and Ruled Surfaces

3.1 Unit Dual Spherical Curves and Ruled Surfaces

In this section, we can investigate unit dual spherical curves with ruled surfaces in these kinds:

At first, $A(s) = a(s) + \varepsilon a^*(s)$ is the unit dual curve that corresponds to ruled surface

$$\Phi(s,u)=\alpha(s)+ua(s) \tag{12}$$

where $\alpha(s)$ is a base curve and $a(s)$ is a rulling of ruled surface.

Secondly, $A(s)=a(s)+\varepsilon a^*(s)$ is the unit dual curve that corresponds to ruled surface

$$\Phi(s,u)=\alpha(s)+ua(s) \tag{13}$$

where

$$a^*(s)=\alpha(s)\wedge a(s) \tag{14}$$

Example 1. $A(s)=a(s)+\varepsilon a^*(s)$ is a unit dual curve where $a(s)=(0,0,1)$ and $a^*(s)=(\sin s,-\cos s,0)$. $A(s)$ corresponds to ruled surface $\Phi(s,u)$:

$$\begin{aligned}\Phi(s,u)&=a(s)\wedge a^*(s)+ua(s)\\&=(\cos s,\sin s,0)+u(0,0,1)\end{aligned} \tag{15}$$

This shows that we can get a cylinder. On the contrary, if we take the ruled surface $\Phi(s,u)$ as follows

$$\Phi(s,u)=(\cos s,\sin s,0)+u(0,0,1) \tag{16}$$

then $A(s)=a(s)+\varepsilon a^*(s)$ is a unit dual curve where

$$\begin{aligned}a(s)&=(0,0,1)\\a^*(s)&=\alpha(s)\wedge a(s)\\&=(\sin s,-\cos s,0).\end{aligned} \tag{17}$$

3.2. Dual Curves and Ruled Surfaces With Frenet and Blaschke Frames

Let

$$\begin{aligned}\alpha:I&\to E^3\\s&\mapsto\alpha(s)\end{aligned} \tag{18}$$

be unit speed curve and $\{T,N,B\}$ be the Frenet frame of α. T,N and B are the unit tangent, principal normal and binormal vectors, respectively. With the help of α, we define a dual curve in D^3. So, let us have a closed spherical dual curve $\widehat{\alpha}$ of class C^1 on a unit dual sphere S_D^2 in D^3. The curve $\widehat{\alpha}$ describes a closed dual spherical motion.

Here we can easily say that the curve $\widehat{\alpha}$ on unit dual sphere is corresponds to a ruled surface in E^3.

On the other hand, we can give the ruled surfaces of α produced by T, N, B as follows:

$$\Phi_T(s,v) = \alpha(s) + vT(s) \tag{19}$$

with dual curve representation

$$\widehat{\alpha}(s) = \widehat{T}(s) = T(s) + \varepsilon\alpha \wedge T(s). \tag{20}$$

and for N with dual curve representation

$$\begin{aligned} \Phi_N(s,v) &= \alpha(s) + vN(s), \\ \widehat{N}(s) &= N(s) + \varepsilon\alpha \wedge N(s), \end{aligned} \tag{21}$$

and then for B with dual curve representation

$$\begin{aligned} \Phi_B(s,v) &= \alpha(s) + vB(s), \\ \widehat{B}(s) &= B(s) + \varepsilon\alpha \wedge B(s). \end{aligned} \tag{22}$$

In [7], it can be seen that a dual curve can be defined as the set of dual points. So they choose $\{T, N\}$ as points. According to all of these, now we define an orthonormal moving frame along dual curve in D^3; the tangent indicatrice of $\widehat{\alpha}$ is

$$A_1(s) = \widehat{T}. \tag{23}$$

The principal normal indicatrice of $\widehat{\alpha}$ is

$$A_2(s) = \widehat{N} = \frac{\frac{d\widehat{T}}{ds}}{\left\| \frac{d\widehat{T}}{ds} \right\|}, \tag{24}$$

The binormal indicatrice of $\widehat{\alpha}$ is

$$A_3(s) = \widehat{B} = \widehat{T} \times \widehat{N} \tag{25}$$

Thus, we get a frame on dual sphere and the following result can be given.

Result: $\{\widehat{T}, \widehat{N}, \widehat{B}\}$ is a Blaschke frame. Indeed,

$$A_1 = \widehat{T},\qquad A_2 = \widehat{N} = \frac{\dfrac{d\widehat{T}}{ds}}{\left\|\dfrac{d\widehat{T}}{ds}\right\|},\qquad A_3 = \widehat{B} = A_1 \times A_2 \tag{26}$$

Now, we investigate the motion of this Blaschke frame:

$$\begin{bmatrix} \widehat{T}' \\ \widehat{N}' \\ \widehat{B}' \end{bmatrix} = \begin{bmatrix} 0 & \kappa & 0 \\ -\kappa & 0 & \tau+\varepsilon \\ 0 & -\tau-\varepsilon & 0 \end{bmatrix} \begin{bmatrix} \widehat{T} \\ \widehat{N} \\ \widehat{B} \end{bmatrix} \tag{27}$$

Subsequently, as [14], we can define Darboux screw as follows:

$$\widehat{T}' = \widehat{W} \wedge \widehat{T},\qquad \widehat{N}' = \widehat{W} \wedge \widehat{N},\qquad \widehat{B}' = \widehat{W} \wedge \widehat{B} \tag{28}$$

Then, we get

$$d_F X = \widehat{W} \wedge X \tag{29}$$

from [3]. And with all of these, we give the Darboux screw as:

$$\widehat{W} = (\tau+\varepsilon)\widehat{T}(t) + \kappa \widehat{B}(t). \tag{30}$$

in [14].

On the other hand, if we take $A_1 = \widehat{T}$, $A_2 = \widehat{N}$, $A_3 = \widehat{B}$, then the Blaschke's invariants of the dual curve $\widehat{T}(t)$ is given by

$$P = p + \varepsilon p^*,\qquad Q = q + \varepsilon q^* = \frac{\det\left\|\widehat{T},\ \widehat{T}',\ \widehat{T}''\right\|}{P^2} \tag{31}$$

where

$$\begin{aligned} \kappa &= P \\ \tau + \varepsilon &= Q \end{aligned} \tag{32}$$

Thus, the distribution parameters of the ruled surfaces A_1, A_2, A_3, respectively, can be given as:

$$\begin{aligned} \lambda_1 &= \frac{p^*}{p} \\ \lambda_2 &= \frac{pp^* + qq^*}{p^2 + q^2} \\ \lambda_3 &= \frac{q^*}{q} \end{aligned} \tag{33}$$

where

$$\begin{aligned} &p = \kappa, \quad p^* = 0; \\ &q = \tau, \quad q^* = 1. \end{aligned} \tag{34}$$

In this situation, we can give $\lambda_1, \lambda_2, \lambda_3$ as follows:

$$\begin{aligned} P_{A_1} &= \lambda_1 = 0 \\ P_{A_2} &= \lambda_2 = \frac{0 + \tau}{\kappa^2 + \tau^2} = \frac{\tau}{\kappa^2 + \tau^2} \\ P_{A_3} &= \lambda_3 = \frac{1}{\tau} \end{aligned} \tag{35}$$

and

$$\begin{aligned} P_T &= \frac{\det(T, T, T')}{\left\|A_1'\right\|^2} = 0 \\ P_N &= \frac{\det(T, N, N')}{\left\|N'\right\|^2} = \frac{\det(T, N, -\kappa T + \tau B)}{\kappa^2 + \tau^2} \\ &= \frac{\tau}{\kappa^2 + \tau^2} \\ P_B &= \frac{\det(T, B, B')}{\left\|B'\right\|^2} = \frac{1}{\tau} \end{aligned} \tag{36}$$

The values P_T, P_N and P_B, respectively, are distributions of the ruled surfaces $\Phi_T(s,v), \Phi_N(s,v)$ and $\Phi_B(s,v)$.

Theorem 1. If the curve $\alpha(s)$ is a cylindirical helix in E^3, then

$$\widehat{\alpha}(s) = \int \widehat{T} ds \tag{37}$$

is a dual Mannheim curve.

Proof. Assume that the curve $\alpha(s)$ is a cylindirical helix in E^3. Hence, κ and τ are constant and according to this

$$\frac{\kappa}{\kappa^2 + (\tau + \varepsilon)^2} = \frac{\kappa}{(\kappa^2 + \tau^2) + 2\varepsilon\tau} \tag{38}$$

is constant. From [9], $\widehat{\alpha}(s)$ is a Mannheim curve.

3.2.1 Kinematic Interpretation

In this section, from [11], we can give the kinematic interpretation of the moving Blaschke frame which is provided by the Blaschke invariants κ and $\tau + \varepsilon$. Similarly, here we show the Frenet curvatures of the curve α with $\kappa(s)$ and $\tau(s)$; Bishop curvatures of the curve α with $k_1(s)$ and $k_2(s)$

$$\widetilde{W} = a + \varepsilon a^* = (\tau + \varepsilon)A_1 + \kappa A_3 \tag{39}$$

known as the Darboux vector, the dual angular velocity vector of the Blaschke frame with respect to itself has a component $(\tau + \varepsilon)$ about A_1 and κ about A_3.

$$\left\|\widetilde{W}\right\| = \sqrt{\kappa^2 + (\tau + \varepsilon)^2} = \widetilde{W} = w + \varepsilon w^* \tag{40}$$

is the angular speed of A_1 about $\widetilde{W}$;

$$\begin{aligned} w &= \sqrt{\kappa^2 + \tau^2} \\ w^* &= \frac{\tau}{\sqrt{\kappa^2 + \tau^2}} \end{aligned} \tag{41}$$

are the rotational angular speed and translational angular speed of A_1, respectively. The pitch of $\widehat{T}$ or A_1 along $\widetilde{W}$ is w^* / w which is equal to the distribution parameter of A_2. So,

$$\frac{w^*}{w} = \frac{\tau}{\kappa^2 + \tau^2} \tag{42}$$

We can give the same results from [18]. If we define an instantaneous screw axis (I.S.A) of the motion of A_1 in the Blaschke frame with H, we can obtain the dual angle between the I.S.A and the ruled surface A_1.

$$H(t) = h(t) + \varepsilon h^*(t) = \frac{\widetilde{W}}{\left\|\widetilde{W}\right\|} = \frac{(\tau+\varepsilon)A_1 + \kappa A_3}{\sqrt{\kappa^2 + (\tau+\varepsilon)^2}} \tag{43}$$

Let

$$\Psi = \psi + \varepsilon\psi^* \tag{44}$$

be the dual angle between the I.S.A and the ruled surface A_1; Then we have

$$H = \cos\psi A_1 + \sin\psi A_3 \tag{45}$$

where

$$\cot\Psi = \frac{\tau+\varepsilon}{\kappa} \tag{46}$$

Thus, taking similiar calculations from [11], the trigonometric function $\cot\Psi = \frac{\tau+\varepsilon}{\kappa}$ can be written as

$$\cot\Psi = \cot\psi - \varepsilon\psi^*(1+\cot^2\psi) = \frac{\tau+\varepsilon}{\kappa} = \frac{q+\varepsilon q^*}{p+\varepsilon p^*} \tag{47}$$

From this equation, we get:

$$\psi^* = \frac{p^* q - q^* p}{p^2 + q^2} \tag{48}$$

is the minimal distance from I.S.A to the ruled surface A_1. This distance is measured along the central normal A_2 and is seen to the combination of the invariants of ruled surface A_1.

Theorem 2. The curve α is a Mannheim curve if and only if the minimal distance from I.S.A to the ruled surface A_1 is a constant.

Proof. If the curve α is Mannheim curve, $\frac{\kappa}{\kappa^2+\tau^2}$ equals to c (constant). Therefore,

$$\begin{aligned}\psi^* &= -\frac{\kappa}{\kappa^2+\tau^2} \\ &= -c\end{aligned} \tag{49}$$

Theorem 3. If the curve α is a cylindirical helix in E^3, then $\|\widetilde{W}\|$ is constant.

*Proof***.** The curve α is cylindirical helix in E^3 so that $\kappa(s)$ and $\tau(s)$ are constant. Then

$$\begin{aligned}\|\widetilde{W}\| &= \sqrt{\kappa^2+\tau^2} + \varepsilon\frac{\tau}{\sqrt{\kappa^2+\tau^2}} \\ &= \text{constant}\end{aligned} \tag{50}$$

4. Bishop Approach to Ruled Surface

Let $\{O,\widehat{T},\widehat{N},\widehat{B}\}$ and $\{O,\widehat{e_1},\widehat{e_2},\widehat{e_3}\}$ be two orthonormal coordinate systems of S moving unit dual sphere and S' fixed unit dual sphere with the same O origin, and let us represent one parameter dual spherical motion (dual rotation) between S and S' dual spheres with S/S'. During the spherical motion S/S', $\Psi=\psi+\varepsilon\psi^*$ is the dual rotation Pfaffian vector of S/S'.

In this section, on this motion, it can be seen that dual orthonormal matrix occurs and at this time, it can be seen that all dual matrices correspond to the motions on real space. This motion makes only angular rotation.

Let the frame $\{T,N_1,N_2\}$ be Bishop Frame of the curve α and if we take

$$\begin{aligned}\widehat{T} &= T+\varepsilon\alpha\wedge T \\ \widehat{N}_1 &= N_1+\varepsilon\alpha\wedge N_1 \\ \widehat{B}_2 &= N_2+\varepsilon\alpha\wedge N_2\end{aligned} \tag{51}$$

then we can give the variation of the frame $\{\widehat{T},\widehat{N}_1,\widehat{N}_2\}$ as follows:

$$\begin{bmatrix}\widehat{T}' \\ \widehat{N}_1' \\ \widehat{N}_2'\end{bmatrix} = \begin{bmatrix}0 & k_1 & k_2 \\ -k_1 & 0 & \varepsilon \\ -k_2 & -\varepsilon & 0\end{bmatrix}\begin{bmatrix}\widehat{T} \\ \widehat{N}_1 \\ \widehat{N}_2\end{bmatrix} \tag{52}$$

On the other hand, Darboux vector of the moving frame can occur as follows:

$$\widetilde{W} = \varepsilon\widehat{T} - k_2\widehat{N}_1 + k_1\widehat{N}_2 \tag{53}$$

with these calculations:

$$\begin{aligned}\widetilde{W}\Lambda\widehat{T} &= k_2\widehat{N}_2 + k_1\widehat{N}_1 = \widehat{T}' \\ \widetilde{W}\Lambda\widehat{N}_1 &= \varepsilon\widehat{N}_2 - k_1\widehat{T} = \widehat{N}_1' \\ \widetilde{W}\Lambda\widehat{N}_2 &= -\varepsilon\widehat{N}_1 - k_2\widehat{T} = \widehat{N}_2'\end{aligned} \tag{54}$$

where

$$\left\|\widetilde{W}\right\| = w + \varepsilon w^* \tag{55}$$

and

$$\left\langle \widetilde{W}, \widetilde{W} \right\rangle = k_1^2 + k_2^2 \text{ and } \left\|\widetilde{W}\right\| = \sqrt{k_1^2 + k_2^2} = \kappa \tag{56}$$

According to kinematic interpretation and approaches, it can be seen that for all t there is a rotation and if $w^* = 0,$ then there is a rotation by $\sqrt{k_1^2 + k_2^2}.$

Result: The real part and dual part of Ψ correspond to the rotation motions and the translation motions. In order to leave out the pure translation motions we will suppose that $\psi \neq 0.$

On the other hand, if $\psi \neq 0$ and $\psi^* = 0,$ then the motion will be the pure rotation (spherical motion) around the instantaneous Pfaffian vector [4].

Theorem 4. If the curve α is Salkowski curve with $\kappa(s)$ is constant, then $\left\|\widetilde{W}\right\| = \kappa(s)$ is constant such that $w^* = 0.$

Proof. If the curve α is Salkowski curve such that $\kappa(s)$ is constant then

$$\begin{aligned}\left\|\widetilde{W}\right\| &= \sqrt{k_1^2 + k_2^2},\ k_1^2 + k_2^2 = \kappa^2(s) \\ &= \kappa(s) \\ &= \text{constant}\end{aligned} \tag{57}$$

Conclusions

In this study, at first, by investigating unit dual spherical curves with ruled surfaces in two kinds, we define an orthonormal moving frame along dual curve with its tangent, principal normal and binormal indicatrices. Then, the distribution parameters of the ruled surfaces are calculated. Finally, we have seen that all dual matrices correspond to the motions on real spaces and these motions make only angular rotations.

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