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Introducing a Global Optical Model Approach for Analysing $^{16}O+^{16}O$ Elastic Scattering at 5-10MeV/nucleon Region

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Abstract: In this paper, the experimental data on elastic scattering of the $^{16}O+^{16}O$ reaction for the energy range 5-10 MeV/nucleon have been analyzed within the optical model (OM) formalism by using the phenomenological potential forms in Fresco code. When developing the shape of the nuclear potential for the calculations, we have used the Woods-Saxon (WS) or Woods-Saxon squared (WS2) potentials for the imaginary part together with a WS2 type real part. Although most of the previous OM analyses using phenomenological potentials have provided reasonably good fits with the experimental measurements, none of them could completely relate the behavior of the imaginary potential to the energy of the projectile yet. However, we have managed to introduce two analyses that can keep the real potential parameters almost constant and suggest a linear expression for the depth of the imaginary part of the nuclear potential depending on the incidence energy. Thus, $^{16}O+^{16}O$ system within this wide energy range has been described globally by the optical potentials having a deep, attractive real potential part and a weaker, energy dependent absorptive imaginary potential part. It has been also shown that, our calculations with these potential forms can reproduce the experimental elastic scattering angular distributions successfully and the maxima and minima are predicted correctly for most of the energies.

Key words: $^{16}O+^{16}O$ reaction, optical model, elastic scattering, cross-section, phenomenological potentials.

5-10MeV/nükleon Bölgesinde $^{16}O+^{16}O$ Esnek Saçılmasının Analizi için Global bir Optik Model Yaklaşımının Tanıtılması

Özet: Bu çalışmada 5-10 MeV/nükleon enerji aralığında $^{16}O+^{16}O$ reaksiyonunun deneysel esnek saçılma verileri, optik model (OM) formalizmi altında, Fresco kodunda fenomenolojik potansiyel formları kullanılarak analiz edilmektedir. Hesaplamalar için nükleer potansiyelin şekli oluşturulurken gerçel kısım için Woods-Saxon kare (WS2) tipinde bir potansiyel ile birlikte sanal kısım için ya Woods-Saxon (WS) ya da WS2 formundaki potansiyeller kullanılmıştır. Fenomenolojik potansiyelleri kullanan daha önceki OM analizlerinin çoğu deneysel ölçümlerle yeterince uyumlu sonuçlar üretse bile bu güne dek hiçbir analiz sanal potansiyelin davranışını mermi enerjisi ile bütünlüklü bir biçimde ilişkilendirememiştir. Bununla birlikte çalışmamızda, gerçel potansiyel parametrelerini neredeyse sabitleyen ve nükleer potansiyelin sanal bileşeninin potansiyel derinliği için gelme enerjisine bağlı olarak lineer bir ifade öne süren iki analiz başarıyla oluşturulmuştur. Böylelikle, derin, çekici bir gerçel potansiyel ve daha zayıf, enerji bağımlı soğurucu formdaki bir sanal potansiyelden oluşan optik potansiyellerle $^{16}O+^{16}O$ sistemi bu geniş enerji aralığında global bir biçimde tanımlanmıştır. Ayrıca, bu potansiyel formlarıyla yapılan hesaplamaların deneysel esnek saçılma açısal dağılımlarını başarılı bir biçimde üretebildiği ve enerjilerin çoğu için maksimum ve minimumların doğru biçimde tahmin edilebildiği gösterilmiştir.

Anahtar kelimeler: $^{16}O+^{16}O$ reaksiyonu, optik model, esnek saçılma, tesir-kesiti, fenomenolojik potansiyeller.

1. Introduction

Light heavy-ion scattering is a very sensitive and useful tool that can reveal more information about the nucleus than the other nuclear interaction mechanisms. Thus, the investigation of the elastic and inelastic interactions of light-heavy nuclei has been one of the most popular study subjects in heavy ion (HI) physics. In theoretical basis, there are some simplified models exist for studying HI interactions such as optical model [1-4], distorted-wave Born approximation [1, 3] and folding model [3-5]. Although there has been a large volume of experimental data for the elastic and inelastic scattering of the light heavy-ions over the last fifty years, there has been no single model developed to explain simultaneously all the experimental scattering data of a specific reaction over a wide energy range [4, 5]. However, the standard OM approach, which deals with the scattering in a general way by reducing the complicated many-body problem (originated from the interactions of the nucleons of a projectile and the nucleons of a target) to a much simpler problem of two particles interacting through a potential, has been widely used for analyzing the heavy-ion scattering data in terms of empirical parameterizations of the nuclear potential.

Determination of the shape of the nuclear potential between a projectile and a target is an outstanding problem in nuclear physics. Elastic scattering, which has been extensively studied phenomenon in light heavy-ion reactions, can provide valuable information about the interaction potential between the colliding nuclei [3]. As a HI reaction, the $^{16}O+^{16}O$ scattering, has been studied experimentally and many precise elastic differential cross-section measurements have been performed for this system [3, 6-11]. These experimental data can be used for examining the validity of an existing theoretical model. In the literature, various analyses with different potential forms can be found for the $^{16}O+^{16}O$ system in the refractive regime where E_{LAB}>5MeV/nucleon [4, 6, 7, 10-25]. Since the nuclear potential has a complex form in one-channel OM formalism ($V+iW$, i.e. the real part corresponds to elastic scattering and the imaginary part represents the absorption), conventional WS form or the square of this form, WS2 can be used conveniently for the construction of the phenomenological nuclear potentials [9-11, 13-16]. Some of those studies have tried to find a global analysis for explaining the formation of the nuclear potential with phenomenological potentials in the interested energy region. It was achieved to keep the depth of the real part of the nuclear potential between 410 and 420 MeV, but the dynamical and geometry parameters could not be fixed for the imaginary part [10, 16]. On the other hand, $^{16}O+^{16}O$ elastic scattering can also be studied with microscopic potentials, i.e. double-folding (DF) and α-α double folding cluster (DFC) potentials. In DF approach, the single channel OM formalism is related to a fundamental nucleon-nucleon (NN) interaction and the most suitable normalization constant is sought [15-18]. In the α-α DFC case, the optical potential is obtained by considering α-particle density distribution and α-α interaction in a similar way to DF approach [16, 17]. All these studies concerning the $^{16}O+^{16}O$ system have provided a better understanding of the shape of the nuclear potential that have a deep real part and a weaker imaginary part.

The aim of this study is to introduce two global OM analyses for the $^{16}O+^{16}O$ system by using phenomenological potentials where the parameters of the real part of the nuclear potential have been frozen and the potential depths of the imaginary parts have been related to the incident energy in 5-10 MeV/nucleon energy region. The potentials, according to the OM approach, used in this work are introduced briefly in the following section. In the third section, the elastic scattering differential cross-section analyses of $^{16}O+^{16}O$ reaction at the laboratory energies 75.0, 80.6, 87.2, 92.4, 94.8, 98.6, 103.1, 115.9, 124.0 and 145.0MeV are given by using two phenomenological potential sets. The outcomes of the calculations are compared to the experimental elastic scattering angular distribution data. The last section is devoted to our conclusion.

2. Model Potential

In the $^{16}O+^{16}O$ scattering problem, according to the OM, the form of the interaction potential can be conveniently given by

$$V(r)=V_C(r)+V_N(r)+V_l(r). \tag{1}$$

The terms in Eq. (1) are the Coulomb potential, the nuclear (or central) potential and centrifugal potential respectively. The Coulomb potential and the centrifugal potential are well-defined potentials. V_C, corresponds to the scattering of charged particles and is treated as the potential of a uniformly charged sphere with a radius R_C given by

$$V_C(r)=\begin{cases}\dfrac{Z_pZ_te^2}{2R_C}\left(3-\dfrac{r^2}{R_C{}^2}\right) & r<R_C\\[2ex] \dfrac{Z_pZ_te^2}{r} & r\geq R_C\end{cases} \tag{2}$$

with the Coulomb radius $R_C=r_C\left(A_p^{1/3}+A_t^{1/3}\right)$, where A_p and A_t are the masses of projectile and target nuclei, and r_C is *1.2 fm* for the $^{16}O+^{16}O$ system [4]. Z_p and Z_t represent the charge numbers of the projectile and the target respectively.

The centrifugal potential is represented by

$$V_l(r)=\frac{l\,(l+1)\hbar^2}{2\mu r^2}. \tag{3}$$

Here, ℓ is the angular momentum quantum number and μ is the reduced mass of the projectile and the target for the $^{16}O+^{16}O$ system.

When studying a HI scattering with phenomenological OM potentials, the challenge originates from defining the shape of the nuclear potential appropriately. Previous studies have shown that the optical potential for the $^{16}O+^{16}O$ system can be generally constructed by a nuclear potential which is a combination of a relatively weak, WS2 or WS volume type imaginary potential part plus a deep, attractive, WS2 type real potential part [4, 10, 11, 14-18]. Therefore, for our calculations, the form of the nuclear potential was chosen as

$$V_N(r)=\frac{-V_0}{\left[1+\exp\left(\dfrac{r-R_0}{a_0}\right)\right]^2}+i\frac{-W_V}{\left[1+\exp\left(\dfrac{r-R_V}{a_V}\right)\right]^n}. \tag{4}$$

In Eq. (4), $R_0 = r_0\left(A_p^{1/3} + A_t^{1/3}\right)$ and $R_V = r_V\left(A_p^{1/3} + A_t^{1/3}\right)$ where r_0 and r_V are the radius parameters of real and imaginary parts of the nuclear potential, respectively. For the imaginary part, WS type imaginary potential is obtained when n=1, and n is set to 2 for WS2 shape. In Figure 1, the shape of the interaction potential according to distance is shown together with the real and imaginary parts of the nuclear potential for E_{LAB}=145 MeV. In general, when proper parameter sets are used, analyses with WS2 and WS type imaginary potentials have displayed similar behaviour with each other for the $^{16}O+^{16}O$ elastic scattering at the interested energy region (the similarity of WS and of WS2 type imaginary potentials are demonstrated in the small box in Figure 1. The imaginary volume integral results given in the Table 1 (J_W values) also support this similarity). The parameter values used for the analyses are presented in Table 1.

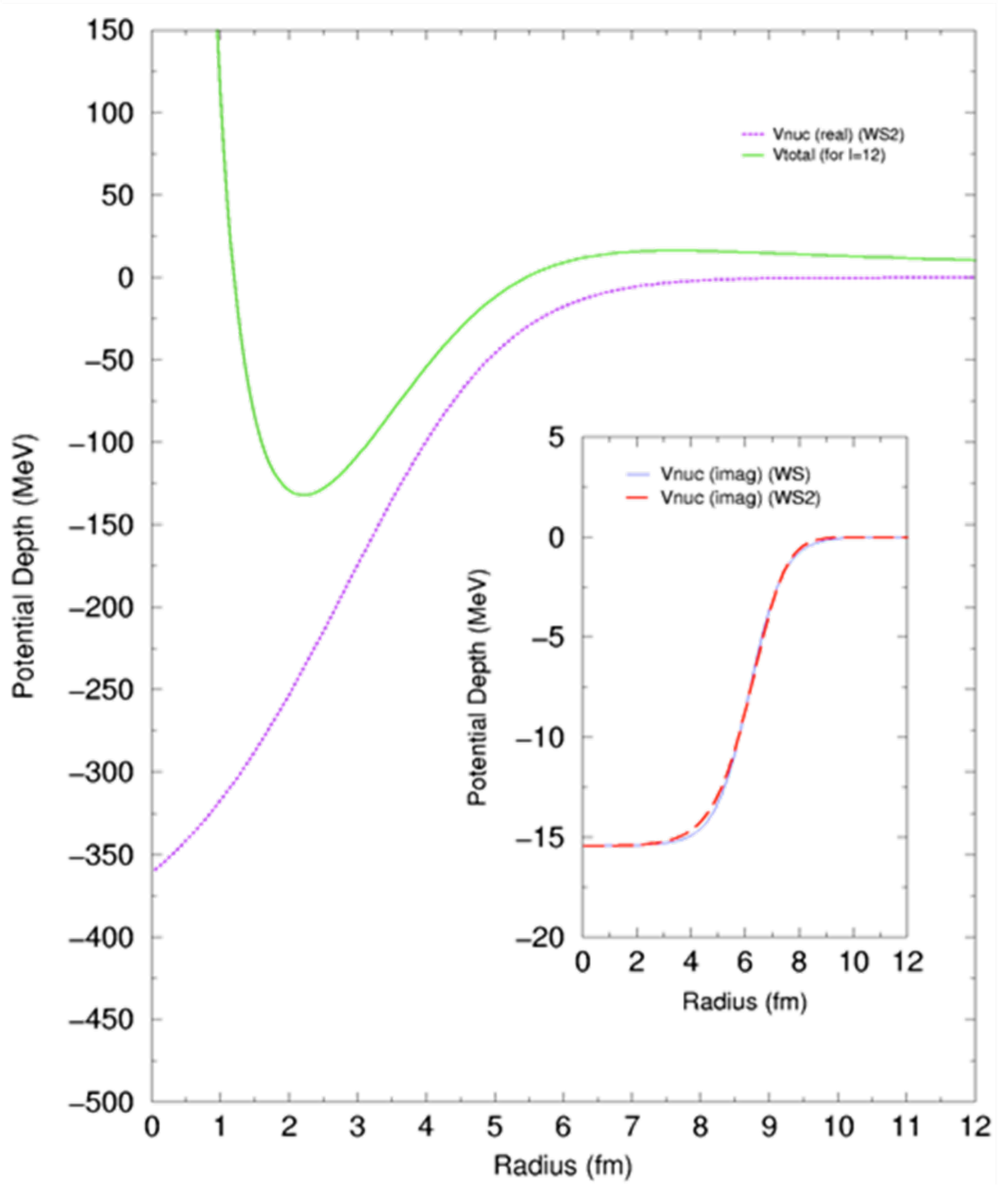

Figure 1. Radial shape of the real (purple) and imaginary parts (grey represents WS form and red stands for WS2 form) of the nuclear potential and the interaction potential (green) according to the distance, r for the $^{16}O+^{16}O$ system at E_{LAB}=145 MeV (parameters were taken from Table 1).

Table 1. The dynamical and geometry parameters of the phenomenological potentials used in the code Fresco [26] for the OM analyses of $^{16}O+^{16}O$ elastic scattering at the energies between E_{LAB}=75.0 and 145.0MeV (V_0, r_0 and a_0 represents the real part parameters, and the parameters used for WS or WS2 type imaginary parts are shown by W_V, r_V and a_V). χ^2 values of these analyses and the volume integral calculations of the real and imaginary potentials are also given.

E_{LAB} [MeV]	Real Part	V_0 [MeV]	r_0 [fm]	a_0 [fm]	J_V [MeVfm^3]	Imaginary Part	Wv [MeV]	r_v [fm]	a_v [fm]	J_W [MeVfm^3]	χ^2
145.0				1.54	331.3	WS2	15.53	1.363	0.78	63.6	15.5
						WS		1.220	0.62	64.9	16.1
124.0				1.58	335.5	WS2	15.06	1.402	0.89	65.9	27.1
						WS		1.239	0.73	68.1	29.1
115.9				1.54	331.3	WS2	14.83	1.362	0.80	60.3	26.5
						WS		1.225	0.61	62.6	31.4
103.1				1.59	336.6	WS2	14.55	1.380	0.84	61.2	36.2
						WS		1.231	0.65	62.9	41.4
98.6				1.57	334.5	WS2	14.44	1.324	1.04	51.6	30.3
	WS2	420	0.777			WS		1.129	0.83	52.7	30.6
94.8				1.57	334.5	WS2	14.35	1.351	0.88	56.0	31.2
						WS		1.186	0.72	57.3	35.6
92.4				1.57	334.5	WS2	14.33	1.438	0.53	74.0	42.3
						WS		1.361	0.39	78.1	45.0
87.2				1.58	335.5	WS2	14.17	1.303	1.23	47.3	45.4
						WS		1.057	1.02	47.7	45.8
80.6				1.59	336.6	WS2	14.01	1.217	1.32	37.8	22.4
						WS		0.958	1.04	37.6	22.2
75.0				1.54	331.3	WS2	13.86	1.170	1.26	33.2	37.0
						WS		0.932	0.94	32.8	36.5

3. Results and Discussion

In order to obtain a global OM approach, the $^{16}O+^{16}O$ system at the incident energies between E_{LAB}=75.0 and 145.0MeV have been reanalyzed by using two phenomenological potential sets and the elastic scattering angular distribution results were compared to experimental measurements [7, 10]. In the calculations, WS2 shape has been used for the real part of the nuclear potential together with an imaginary part of either WS or WS2 form. The new parameters of the phenomenological optical potentials are given in Table 1.

In this study, we have performed two analyses that keep the real part of the nuclear potential depth constant (V_0=420 MeV) and the radius of the real part of the nuclear potential, r_0 is set to 0.777 *fm* for all the energies. Diffuseness parameter of the real potential, a_0 is almost constant that changes slightly between 1.54 and 1.59 *fm* (Table 1). Furthermore, the imaginary part of the nuclear potential has a depth between 13.94 and 15.45 MeV depending on the incident energy by a linear relation

$$W = 12.115 + 0.0236 E_{LAB}. \tag{5}$$

In order to obtain the best fit to the experimental data, WS or WS2 type imaginary volume terms have been used with the parameters r_V and a_V, which can vary from 1.220 to 1.438 *fm* and from 0.39 to 1.32 *fm* respectively (Table 1).

The agreement between the theoretical and the experimental data has been determined by the χ^2 error calculation given as

$$\chi^2 = \frac{1}{N_\sigma} \sum_{i=1}^{N_\sigma} \frac{(\sigma_{th} - \sigma_{ex})^2}{(\Delta\sigma_{ex})^2}. \tag{6}$$

Here, σ_{th}, σ_{ex} and $\Delta\sigma_{ex}$ are the theoretical cross-section, the experimental cross-section and the error variation of the experimental cross-section respectively. N_σ represents the total number of the angles measured. The χ^2 values (see Table 1) have been calculated by assuming the constant experimental error value of 10% at all data points. The χ^2 values obtained from our calculations are in good agreement with the earlier studies [7, 10, 15].

In Figures 2a and 2b, the results of OM calculations are shown in comparison with the experimental angular distributions of the $^{16}O+^{16}O$ system for ten energies between E_{LAB}=75.0 and 145.0MeV. As seen from these figures, the places of the maxima and minima in experimental data have been reproduced successfully by our phenomenological potential sets. Although a good agreement between the theoretical calculations and experimental data has been established within the framework of OM formalism, the out of phase problem (especially for 92.4 and 98.6 MeV energies) still remains around 90° as reported by the previous works [4, 10, 15].

Additionally, the volume integrals both real and imaginary parts of the proposed potential sets were investigated for ensuring the consistency of our analyses (see Table 1 and Figure 3). For a complex nuclear potential having the form of $V_N(r) = V + iW$ in OM formalism, the volume integrals [11, 14, 17] can be conveniently expressed as

$$J_{\mathrm{V,W}}(E) = -\frac{4\pi}{A_p A_t} \int V, W(r,E) r^2 dr. \tag{7}$$

The energy dependence of the volume integrals for the phenomenological potentials considered here (for the potential sets given by Eq. (4) at the incidence energies between E_{LAB}=75.0 and 145.0MeV) are displayed in Figure 3 in comparison with the dispersion curves introduced by Gonzales and Brandan [14], and with the results of higher energies performed by former studies [9, 11, 14, 19] for $^{16}O+^{16}O$ system. Our volume integrals comply with the dispersion relation curve and are followed by the higher energy results in a compatible manner.

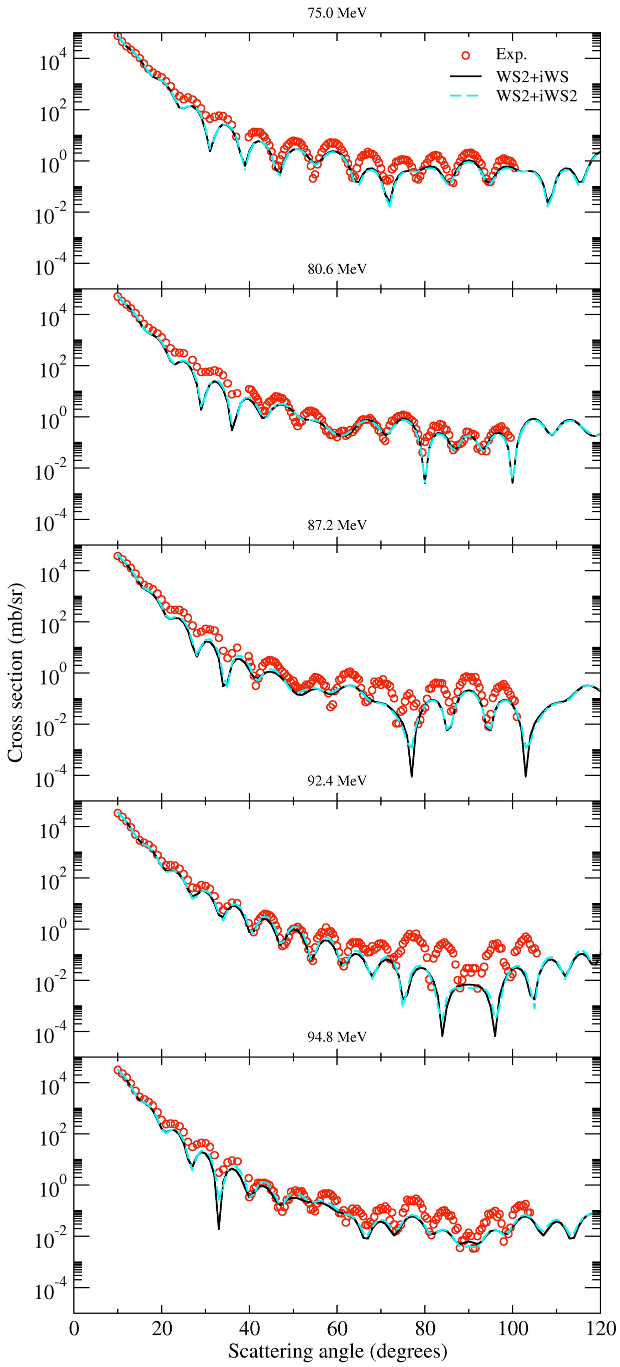

Figure 2 a. Experimental elastic angular distributions of the $^{16}O+^{16}O$ reaction (circles) and cross-section calculations (lines) for the energies 75.0, 80.6, 87.2, 92.4 and 94.8 MeV. Analyses with WS (black straight lines) and WS2 (blue dashed lines) type imaginary potentials have exhibited almost the same results with each other for the potential depths and the parameter values given in Table 1. In the graphs, x-axes represent the scattering angles in the CM (Center of Mass) frame and y-axes show the Rutherford differential cross-sections in logarithmic scale.

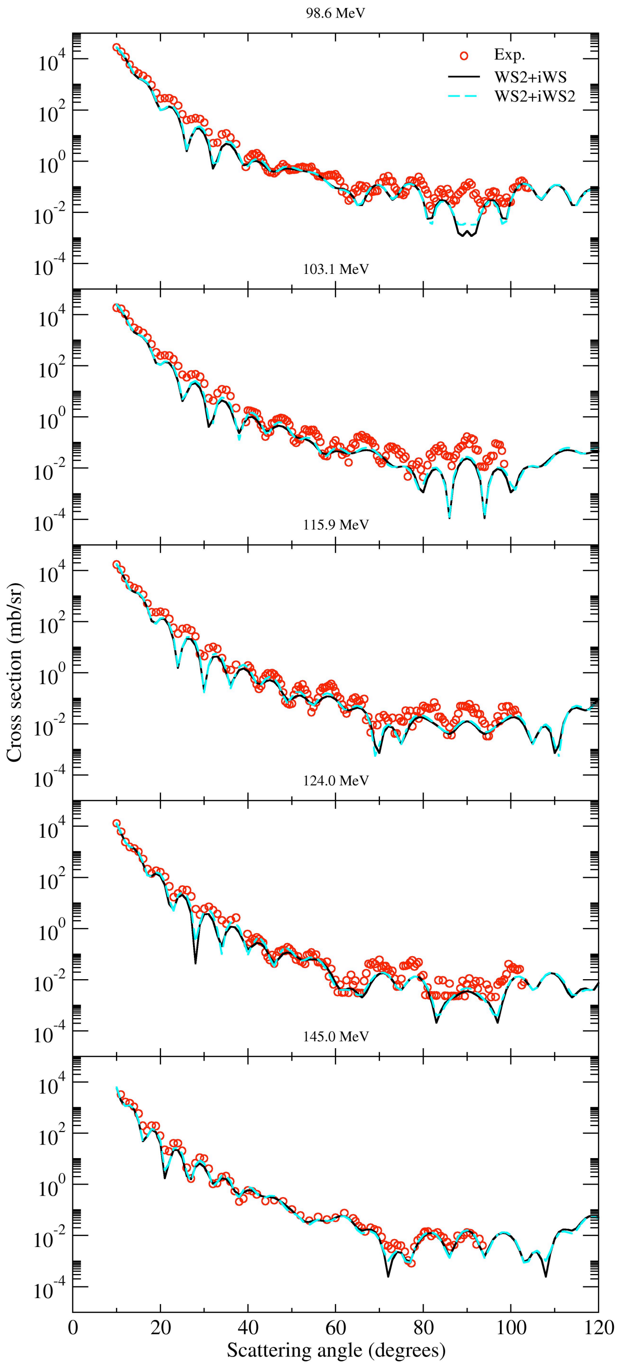

Figure 2 b. Same as Figure 2 a, but for the energies 98.6, 103.1, 115.9, 124.0 and 145.0MeV.

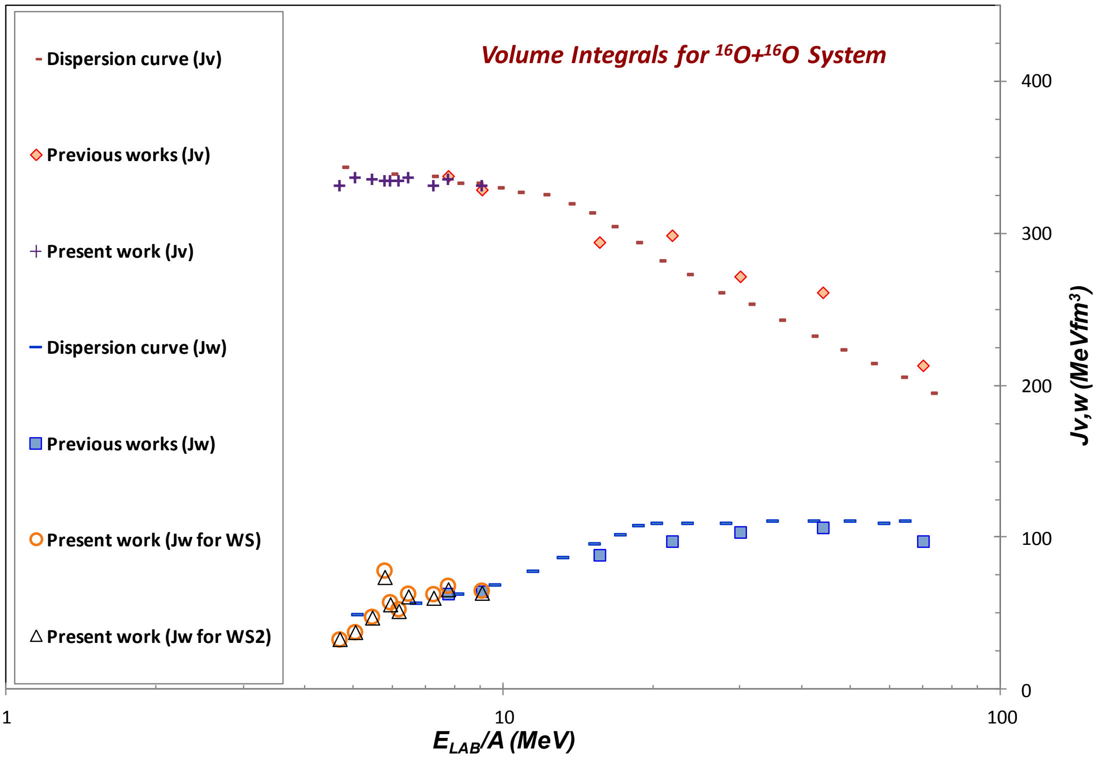

Figure 3. The volume integrals of real and imaginary parts of the nuclear potential sets (plus signs stand for the volume integrals of the WS2 type real potentials, empty circles and triangles represent the imaginary volume integrals of the WS and WS2 shape potentials respectively. The numerical values of our volume integrals, which are calculated according to Eq. (7), are given in Table 1). The dispersion relation curves of J_V and J_W (short and long dashed lines respectively) are adapted from Ref. [14]. Filled symbols represent the earlier calculations of J_V and J_W in the literature (for the energies higher than E_{LAB}=115.9MeV).

4. Conclusions

$^{16}O+^{16}O$ reaction has received considerable interest in HI physics. Although there are many detailed analyses concerning the elastic scattering angular distributions of this system studied in OM with various potential forms, just a few of them make an effort to evolve a systematization for the phenomenological potential parameters. However, we have reanalyzed $^{16}O+^{16}O$ elastic scattering in 5 to 10MeV/n energy region with conventional 6-parameter nuclear potential sets having WS2+iWS2 and WS2+iWS forms. New analyses with these nuclear potential sets have improved the previous global OM calculation attempts to a further level by introducing fixed parameters for WS2 type real potentials and suggesting an energy dependent linear expression for the imaginary potential depth, which has a constant part close to Coulomb barrier of the $^{16}O+^{16}O$ system, for all the incidence energies studied in this region. When the same real potential parameters and the same imaginary potential depths are used with the imaginary geometry parameters given in this work, OM analyses with these two phenomenological potential sets have provided almost identical results. It is found that, calculations with those potential forms can reproduce the experimental elastic angular distributions successfully. The consistency of our findings has been verified by means of χ^2 and volume integral calculations.

Within the limitations of this study, we conclude that, $^{16}O+^{16}O$ elastic scattering can be estimated for any incidence energy in the energy range between 5 and 10MeV/n with our parameterization.

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