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Undulator and Resonator Optimization Studies for TARLA FEL

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Abstract: Free Electron Lasers (FELs) ensure superior features such as adjustable wavelength, coherence, mono-chromaticity, and high peak power connected to the high intensity and short pulse length, high flux and brightness values. FELs' wavelength region lies from infrared to X-ray region. A FEL facility, called Turkish Accelerator and Radiation Laboratory at Ankara (TARLA), is under construction with aim to scan wavelength region from 2.5 µm to 250 µm with two undulator lines supplied with 15-40 MeV electron beam out of two superconducting accelerating cavities. Analytical calculations for determination of TARLA FEL undulator and resonator parameters carried out with Mathematica and FELO code used for simulations. In this paper, laser wavelength, beam waist, normalized emittance, Rayleigh length, single-pass gain, ideal gain, mirror radius of resonators, average extracted power, output power and efficiency values for both FEL line for TARLA were presented.

Key words: TAC, TARLA, Free Electron Laser, optimization, simulation studies, analytical calculations

TARLA FEL için Salındırıcı ve Rezonatör Optimizasyon Çalışmaları

Özet: Serbest Elektron Lazerleri (SEL) ayarlanabilir dalga boyu, eş fazlılık, monokromatiklik, yüksek tekrarlama oranıyla ilişkilendirilen yüksek-pik şiddeti, yüksek akı ve parlaklık değerleri gibi önemli özelliklere sahiptirler. SEL'lerin dalga boyu aralığı, kızıl-ötesinden X-ışınlarına kadar değişir. Süper iletken hızlandırıcıdan elde edilen 15-40 MeV elektron demetlerinin iki salındırıcı hattından geçirilmesi ile 2.5 µm ile 250 µm dalga boyu aralığını taramayı hedefleyen Hızlandırıcı ve Lazer tesisinin (TARLA) kurulum çalışmaları devam etmektedir. Bu çalışmada TARLA tesisinin salındırıcı ve rezonatör optimizasyon çalışmaları yapılmıştır. TARLA SEL'in undulator ve rezonatör parametrelerinin belirlenmesi için yapılan analitik hesaplamalarda Mathematica, benzetim çalışmalarında ise FELO kullanılmıştır. Bu makalede, TARLA'daki her iki SEL hattı için lazer dalga boyu, demet beli, normalize yayınım, Rayleigh uzunluğu, tek-geçiş kazancı, rezonatör aynalarının yarıçapları, alınacak ortalama güç, çıkış gücü ve verimlik değerleri sunulmuştur.

Anahtar kelimeler: THM, TARLA, Serbest Elektron Lazeri, optimizasyon, benzetim çalışmaları, analitik hesaplamalar

1. Introduction

Free Electron Lasers (FELs) are the system for producing coherent electromagnetic radiation using a relativistic electron beam as lasing medium. When a relativistic electron beam passes the alternating magnetic field of the undulator, this field generates a velocity component transverse to the direction of the electron beam propagation. The transverse velocity couples the electron beam to the electric field component of the radiation field, thus produce an energy exchange between them [1, 2]. This exchange of energy in return produces coherent laser beam, which is known as 4th generation light sources that can operate in IR, XUV and VUV regions [3].

This widely tunable coherent radiation of FEL can be used in a wide range of research that include investigation of coherent transition effects and band-gap in semiconductor structures, resonant excitation of molecular structures, reaction dynamics studies, IR microscopy of biological samples, multi-photon dissociation experiments, surface absorption spectroscopy, cleaning of cholesterol in selected region, investigation of the dynamic of protein and tissue, nanotechnology and defense industry etc.[4].

The first FEL facility named "Turkish Accelerator and Radiation Laboratory at Ankara" (TARLA) is under construction now as an inter university collaboration of Ankara, Gazi, Boğaziçi, Doğuş, İstanbul, Uludağ, Erciyes, Osmangazi, Dumlupınar, Niğde, S. Demirel and GYTE Universities in Turkey [5].

TARLA was planned as a free electron laser center operating in oscillator mode scanning wavelengths from 2.5 to 250 μm [6]. In TARLA electron beam with energy between 15-40 MeV can be injected into undulator-1 (U25) and andundulator-2 (U90) line independently, which has a period of 2.5 and 9 cm respectively. In this paper, analytical and simulative calculations were carried out for determination and optimization of optical cavity parameters. Analytical calculations were fulfilled using Mathematica [7] and simulation studies were carried out using FELO [8]. Undulator and electron beam parameters that are used for calculation and optimization studies are given in Table 1 and Table 2 [6].

Table 1.Undulator parameters for U25 and U90

Undulator Parameters	U25	U90
Period (cm)	2.5	9
Gap (cm)	5-10	5-10
Remnant Magnetic Field [T]	1.2	1.2
Length (m)	1.5	3.6
Period Number (N)	60	40

Table2.Electron Beam Parameters

Parameters	Value
Electron beam energy [MeV]	15-40
Peak current (mA)	1
Bunch charge [pC]	77
Energy spread	%0.1
Micro pulse repetition frequency (MHz)	13
Macro pulse repetition frequency (Hz)	10
RF cavity frequency (MHz)	1300

2. Material and Method

For TARLA undulators, rare earth cobalt permanent magnet materials Sm_2Co_{17} are used. In a permanent magnet undulator on-axis peak magnetic field per period is given by Halbach formula [9];

$$B_u[T] = 2 \cdot B_r \cdot e^{-\pi g/\lambda_u} \cdot \frac{\sin(n\pi/M)}{(n\pi/M)} \left(1 - e^{-2\pi h/\lambda_u}\right)$$

where B_r is remanent magnetic field, n is package factor, g is undulator gap, M is block number per period, h is block height and λ_u is the undulator period. There are 4 blocks per period ($M = 4$), the package factor (n) is equal to one and block height is equal to $\lambda_u/2$. In this case magnetic field becomes;

$$B_u[T] = 1.723 \cdot B_r \cdot e^{-\pi g/\lambda_u}$$

A dimensionless undulator parameter K is determined by N undulator period number, λ_u undulator period and B_u undulator magnetic field and it is expressed as;

$$K = \frac{eB_u}{m_e c k_u} = \frac{eB_u \lambda_u}{2\pi m_e c} = 0.934 \, . \, B_u[T] \, . \, \lambda_u[cm]$$

where e is electron charge, m_e is electron mass, c is speed of light and $k_u = 2\pi/\lambda_u$ is undulator period number [9]. For an undulator with a period larger than 1 cm, the wavelength λ of the first harmonic of the FEL radiation related to the period λ_u of a planar undulator is given by;

$$\lambda = \frac{\lambda_u}{2\gamma^2}(1 + K^2)$$

where γ is Lorentz constant and given by $\gamma = E/m_e c^2 + 1$, E is electron beam energy and K is effective undulator parameter as mentioned before. Equation above, which shows radiation wavelength, exhibits two main advantages of the free-electron laser. The first one is the tunability of the wavelength that can be adjusted by changing E and K and the second one is the very short wavelengths can be obtained by using ultra-relativistic electrons [10].

In the FEL, electrons oscillate in an undulator and emit synchrotron radiation, which are phase matched. This process allows energy exchange between electrons and electromagnetic radiation when the resonance condition is satisfied. This energy exchange causes electrons to bunch at the resonant wavelength and subsequently emitting coherent radiation at this wavelength and in its harmonics. This energy exchange is directly related to an important characteristic of a FEL called single pass gain which can be described as;

$$g_u = \frac{16\pi}{\gamma} \lambda_R L_u \frac{J_e N^2}{1.7\times10^4} \xi [J_0(\xi) - J_1(\xi)]^2$$

where $\xi = \frac{K^2}{2(1+K^2)}$, K is the undulator parameter, N is the number of periods, L_u is the length of undulator, λ is the resonance wavelength and J_e is the density current of electrons [11].

According to Dattoli ideal gain can be obtained by multiplying single pass gain with other attenuation factors.

$$G_p^* = F_f[0.85(g_0\alpha) - 0.19(g_0\alpha)^2 + 4.12\times10^{-3}(g_0\alpha)^3]$$

where the coefficient $\alpha = F_{inh}F_s$ in expression is an attenuation factor for the gain coefficient g_0. It depends on the inhomogeneous broadening factor F_{inh}:

$$F_{inh} = \frac{1}{1+1.7\mu_\varepsilon^2}\frac{2}{2+\mu_r^2}$$

with $\mu_\varepsilon = 4N_u(\sigma_\gamma/\gamma)$ depending on the relative RMS (σ_γ/γ) energy spread of the electron beam, $\mu_r = \frac{N_u\sqrt{2}}{\lambda_u}\frac{K}{1+K^2/2}\varepsilon_{nor}$ corresponding to the normalized emittance $\varepsilon_{nor} = 4\pi\gamma\sigma\sigma'$ along vertical axis (y).The longitudinal mode-coupling factor F_s is:

$$F_s = \frac{1}{1+\mu_c/3}$$

where $\mu_c = N_u\lambda_R/c\sigma_e$, is the ratio between slippage length $N_u\lambda_R$ and electron bunch length $c\sigma_e$ with σ_e the RMS time duration of electron bunch.

In free electron lasers, "electron efficiency" can be expressed as percentage of energy the electron beam transferred to the optical laser mode during the FEL interaction and is given by:

$$\eta = \frac{\Delta W_e}{W_e}$$

where $W_e = Q(\gamma mc^2/e)$ is an electron bunch energy with Q which is a charge of electron bunch and ΔW_e is an amount of energy produced in FEL interaction by each electron micro-bunch [11]. In order to fit the measurements of electron efficiency with theory, we can write another expression for η:

$$\eta = \frac{1}{4N_u}(\alpha + 0.22g_0\alpha^2 + 4.85\cdot10^{-3}\cdot g_0^2\alpha^3)(1-e^{-h})F_f\frac{\sum_0\cdot\Delta t_0}{\sum_0\cdot\Delta t_e}$$

where g_0 is the gain coefficient for linearly polarized undulators. The last part of expression involves the size of the laser pulse: $\Sigma_0 = \pi\sigma_{0x}\sigma_{0y}$ and $\Delta t_0 = \sqrt{\pi}\sigma_0$ which are respectively the cross-section and time duration of intensity, with σ_0 the RMS time duration of the laser mode amplitude. The size of the electron bunch is :$\Sigma_e = \pi\sigma_{ex}\sigma_{ey}$

for the cross-section and $\Delta t_e = \sqrt{\pi}\sigma_e$ for the time duration with σ_e. The "filling factor" depending on the transverse cross-sections: $F_f \cong 1/(1 + \Sigma_0/\Sigma_e)$ and h coefficient depends on the Small Signal Gain of power G_p^i and on the optical losses L.

$$h = [1.8/(1 + G_p^i)] \cdot [(1 - L)G_p^i - L]/L$$

Average extracted power in the FEL is given by;

$$\langle P_x \rangle = W_e \eta \Delta T_{sat} \frac{T_x}{L} f_\mu f_M$$

Where f_M and f_μ are respectively the repetition rates of macro-pulses and micro-pulses, ΔT_{sat} is the time duration of the FEL saturation in the macro-pulse. The ratio T_x/L is the extraction ratio between losses by hole coupling T_x and the total losses L of the laser cavity [11].

FEL radiation is always coherent. In other words FEL radiation can always be focused on to a spot whose size is defined totally by diffraction effects. Focusing the electron beam downstream of the beam line could even give smaller spot sizes [12, 13]. A distance along the beam propagation direction from the waist where the beam spot radius increases by a factor $\sqrt{2}$ $(\sqrt{2}w_0)$ is known as Rayleigh length. Spot radius or size can be given as follows where R_1 and R_2 are mirror radiuses, L is cavity length and λ is radiation wavelength.

$$w_0^4 = \left(\frac{\lambda}{\pi}\right)^2 \frac{L(R_1 - L)(R_2 - L)(R_1 + R_2 - L)}{(R_1 + R_2 - 2L)^2}$$

If $R_1 = R_2 = R$, then w_0 is given by;

$$w_0^2 = \frac{\lambda}{2\pi}[L(2R - L)]^{1/2}$$

and Rayleigh length according to spot radius and radiation wavelength is given as;

$$Z_R = \frac{\pi w_0^2}{\lambda}$$

or equally from the radius of curvature of mirrors when two mirror radius are equal Rayleigh length is given as [14];

$$Z_R = \frac{1}{2}[L(2R - L)]^{1/2}$$

An area of the ellipse enclosing the beam in phase space is known as emittance of the beam. However, when particles are accelerated, emittance decreases inversely proportional to the momentum, because longitudinal component of momentum is increased where as transverse momentum vector remains invariant. So we can define a new quantity as a normalized emittance as $\varepsilon_N = \gamma\beta\varepsilon$ which stays constant during acceleration [15].

3. Results and Discussion

Using values in Table 1 and Table 2, K parameter, magnetic field, output extracted power, efficiency, single pass gain, ideal gain, w_0 spot radius, z_R Rayleigh length, normalized emittance and mirror radius were calculated for U25 and U90 according to various gap using these formulas above.

Magnetic field and K parameter are calculated for both undulators and are shown in Figure 1. The peak field for undulators changes slightly between 0.0382 T and 0.281 T and 0.065 T and 0.358 T for U25 and U90, respectively. Also, the effective *K*-parameter varies slightly with gap for U25 from 0.061 to 0.653 and for U90 from 0.554 to 3.048.

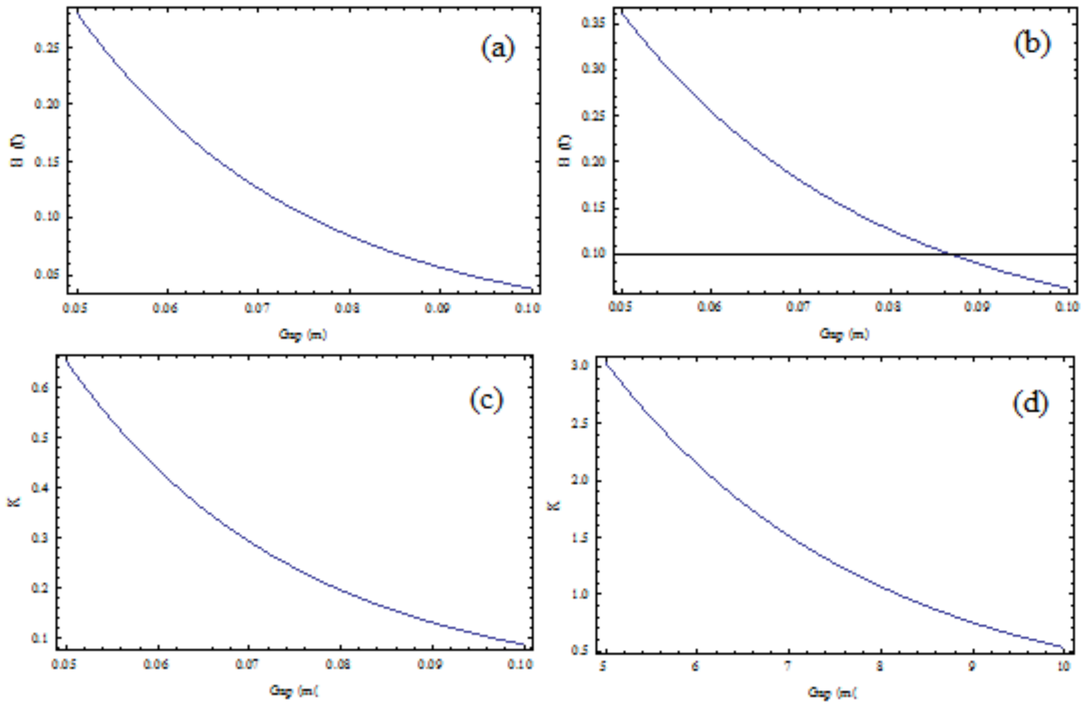

Figure 1. For different gap values; magnetic field calculated (a) for U25, (b) for U90, effective K parameter variation (c) for U25, (d) for U90.

The Figure 2 shows the FEL radiation wavelength calculated analytically using wavelength formula. As seen from Figure 2, calculated radiation wavelength includes radiation wavelength between 2.5 and 250 μm as planned for TARLA.

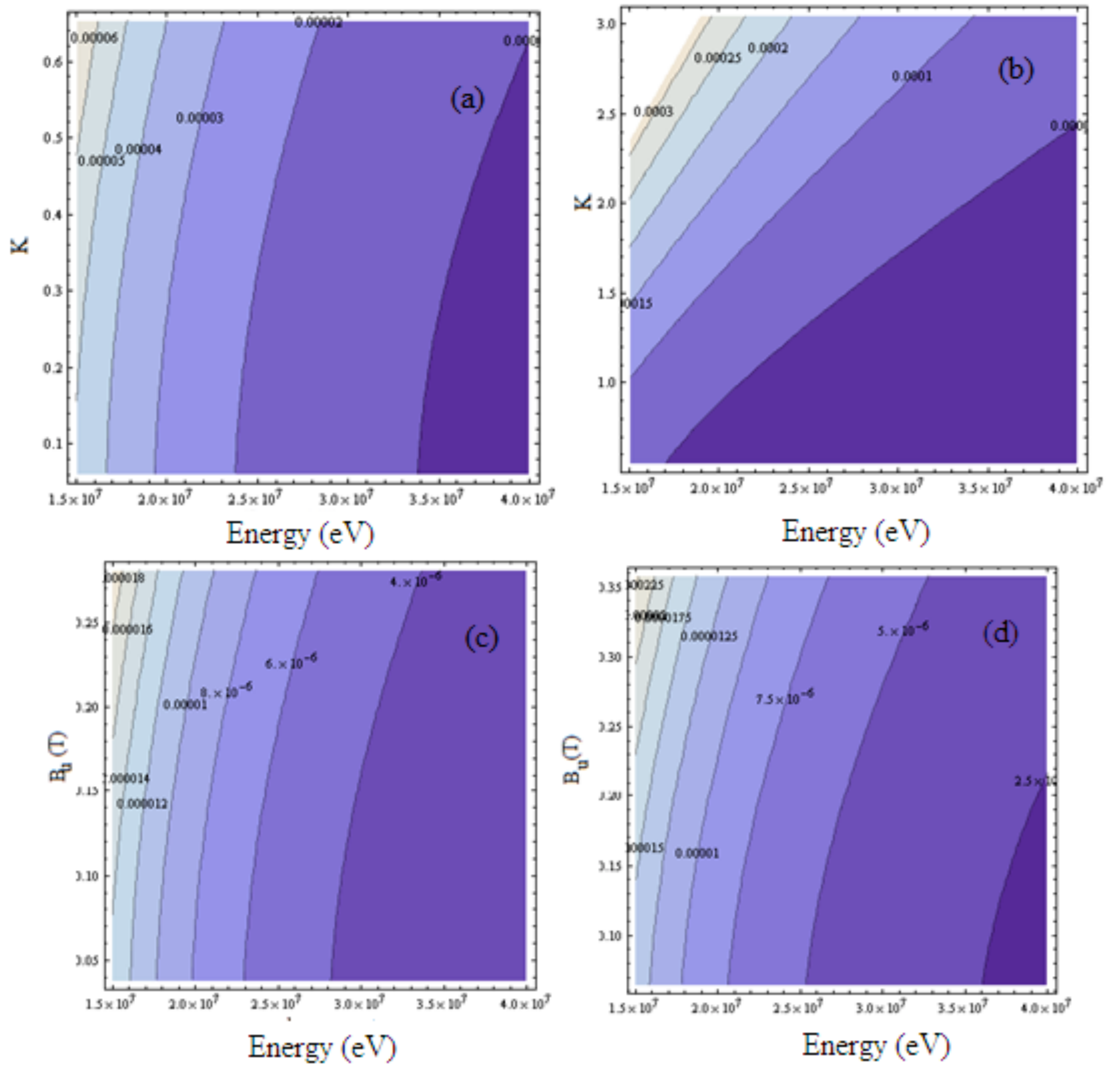

Figure 2. **Radiation wavelength variation with beam energy and K parameter for different gaps (a) for U25, (b) for U90; Radiation wavelength variation regarding beam energy and B_u peak magnetic field for different gaps (c) for U25,(d) for U90.**

In the Figure 3, variations of single pass gain versus wavelength are shown in energy-gap graphic for both undulators. It can be seen that, single pass gain takes values in the range of 0.07-0.18 (7% -18%) for U25 and 0.15-0.50 (15%-50%) for U90.

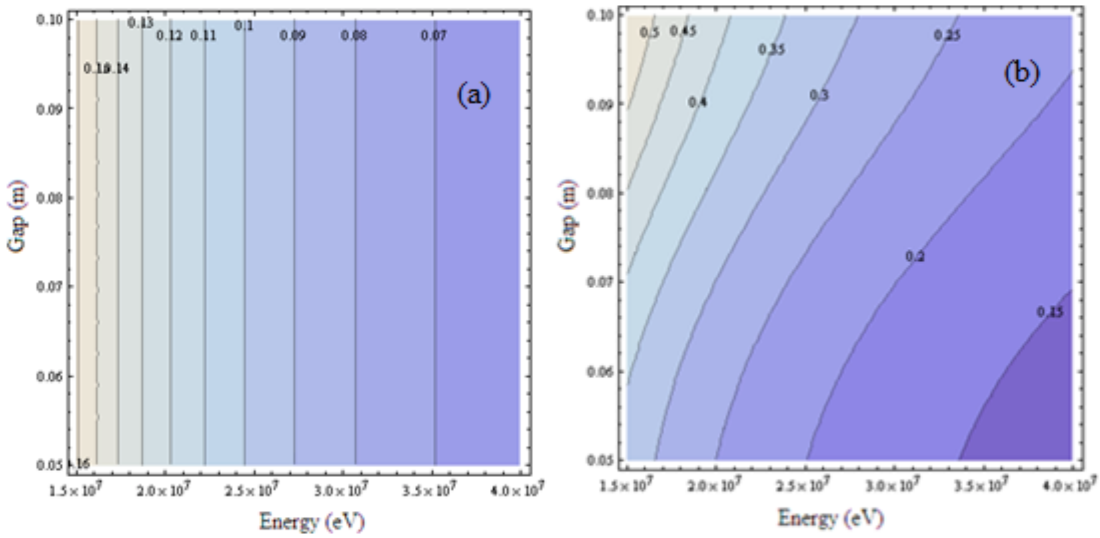

Figure 3.Variation of gain versus wavelength in energy-gap graph (a) forU25, (b) forU90.

In the Figure 4, variations of ideal gain according to Dattoli are shown in energy-bunch length graph for U25 and for U90. An ideal gain value with attenuation factor decreases with increasing energy and bunch length. As seen from figures ideal gain takes values between 0.0016 (%1.6) and 0.0030 (%3) for U25 whereas it takes values between 0.0024 (%2.4) and 0.0069 (%6.9) for U90 for different bunch length and beam energy.

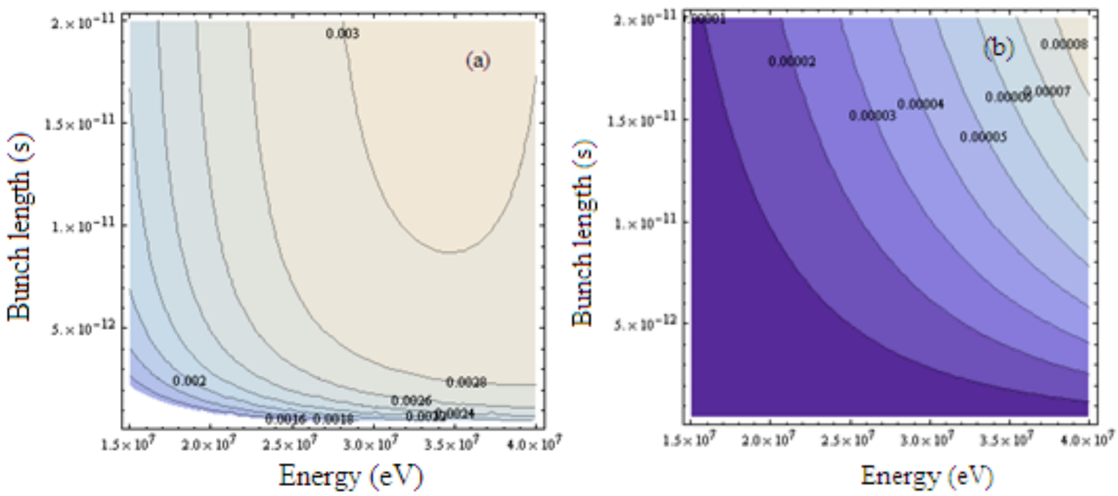

Figure 4.Ideal gain with attenuation factor (a) for U25 (gap=5 cm), (b) for U90 (gap=7 cm)

High output power and high efficiency could be obtained with high electron beam energy. In the lower electron energy, efficiency gradually decreases while beam energy decreases. In Figure 5, variation of electron efficiency is represented for U25 (gap=5 cm) and for U90 (gap=7 cm) in bunch length versus beam energy graph. In these graphs calculated efficiency values depending on bunch length changes between 0.00005

(%0.005) and 0.00025 (%0.25) for U25 and it takes values between 0.00005 (%0.005) and 0.00034 (%0.34) for U90.

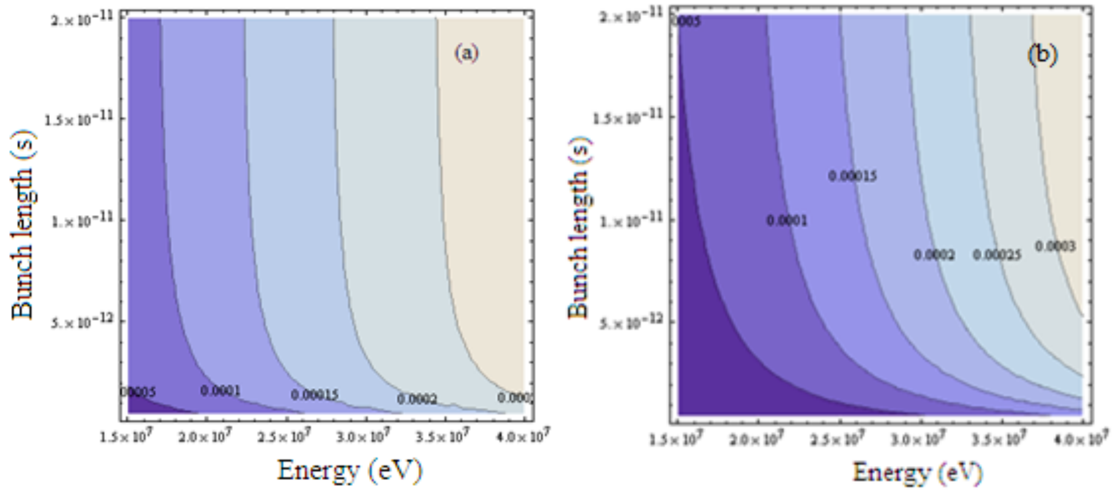

Figure 5. Electron efficiency values in beam energy versus bunch length variation (a) for U25 (gap=5 cm), (b) for U90 (gap=7 cm).

The average extracted power (energy per second) depends on the intra-cavity laser mode, and it is obtained here by numerical simulation. The average extracted power is proportional to efficiency and therefore proportional to laser gain. Variation of average extracted power increases exponentially with increasing energy and bunch length as can be seen from Figure 6 for U25 and U90. Calculated average extracted power values depending on bunch length changes between 0.389 W and 50.83 W for U25 and it takes values between 0.388 W and 69.12 W for U90.

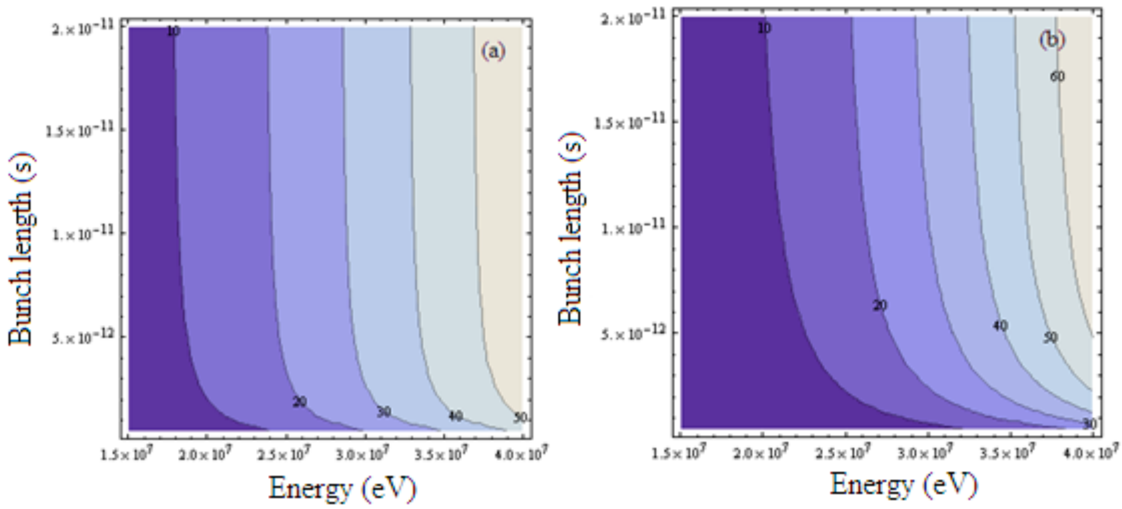

Figure 6.Variation of average extracted power in different beam energies and bunch lengths (a) for U25, (b) for U90.

The Figures 7 and 8 show the spot radius and Rayleigh length of U25 and U90, respectively. Spot radius changes between 1.14×10^{-4} m and 2.97×10^{-4} m for U25 when the gap chosen as 5 cm, whereas it changes between 1.90×10^{-2} m and 4.97×10^{-2} m for U90 when the gap chosen as 7 cm. The Rayleigh length increase exponentially with increasing spot radius but decrease inversely with radiation wavelength and were calculated for both undulators as 2.04×10^{-2} m for U25 and 1.86×10^{-2} m for U90 for chosen gap sizes.

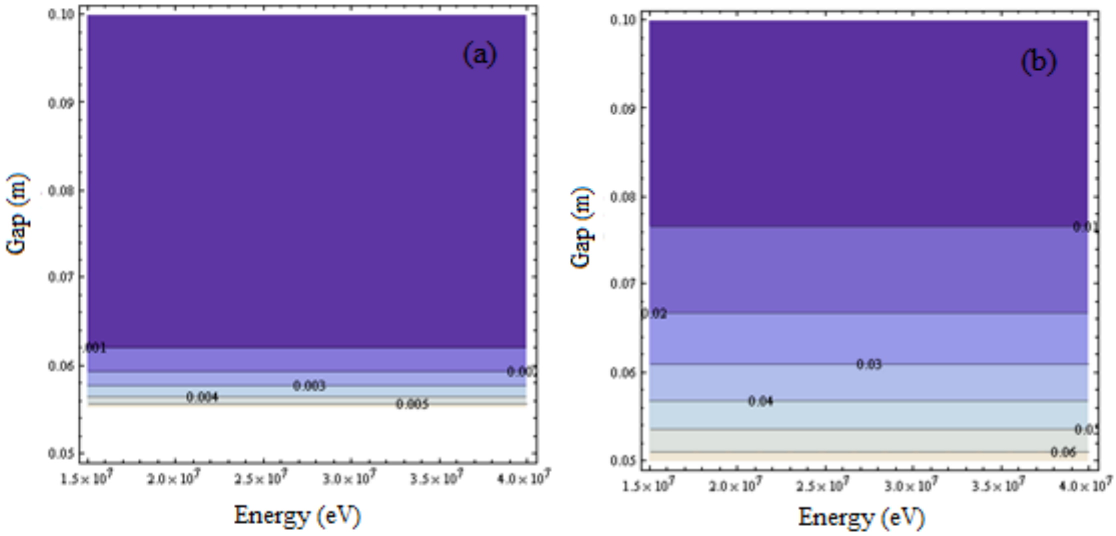

Figure 7.Variation of spot radius in different beam energies and gaps (a) for U25, (b) for U90.

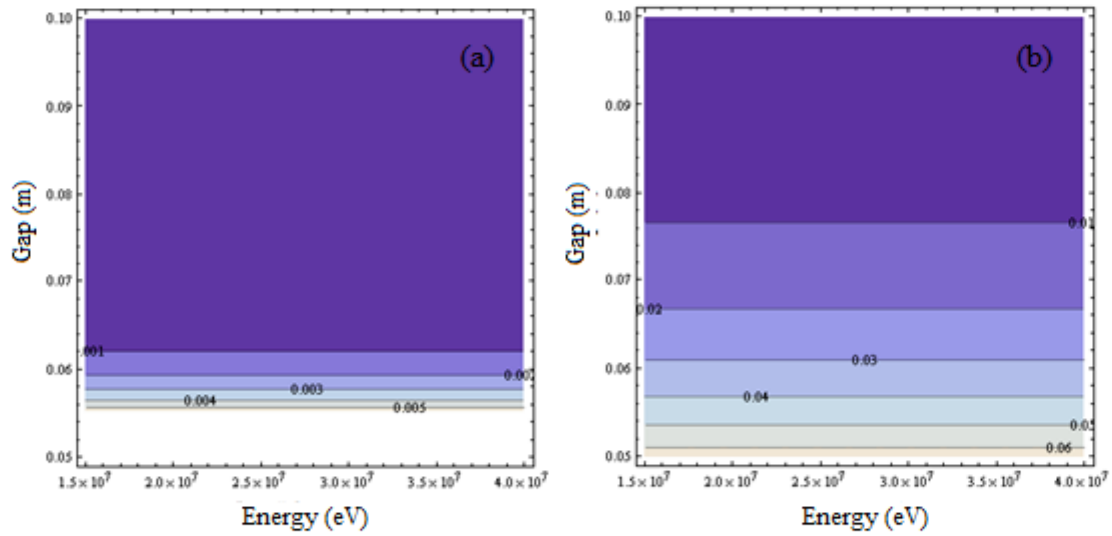

Figure 8.Variation of Rayleigh lengths in different beam energies and gaps (a) for U25, (b) for U90.

The mirror radius for U25 with value of 5.92 m is shown in Figure 9a and for U90 with a value of 5.7653 m is shown in Figure 9b. When two mirror radius are equal this means a concentric or spherical resonator [14, 15].

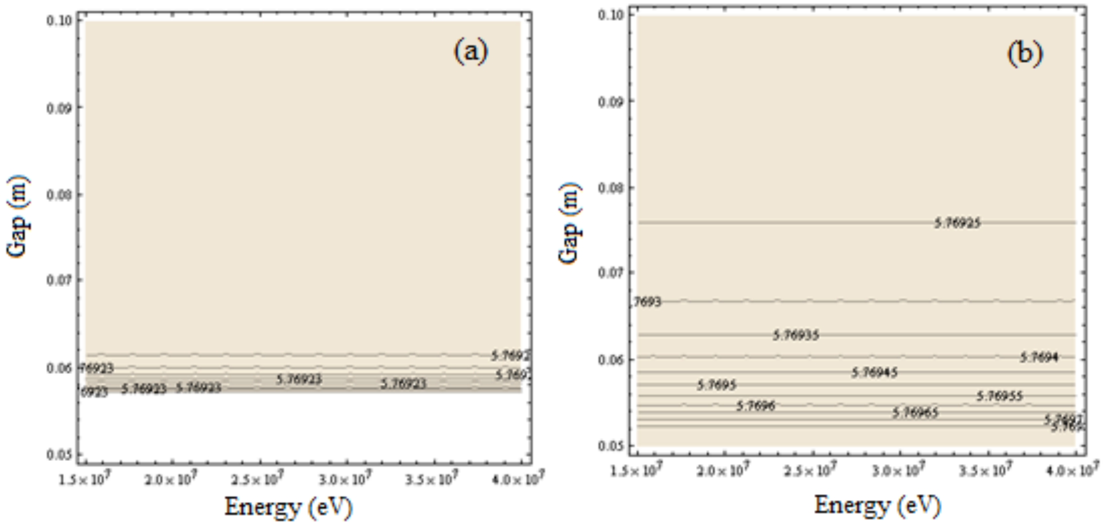

Figure 9. Variation of mirror radius in different beam energies and gaps (a) for U25 (b) for U90.

In Figure 10, variation of normalized emittance for U25 and U90 are shown in beam energy-gap graph respectively. Beam emittance scales inversely with the particle momentum.

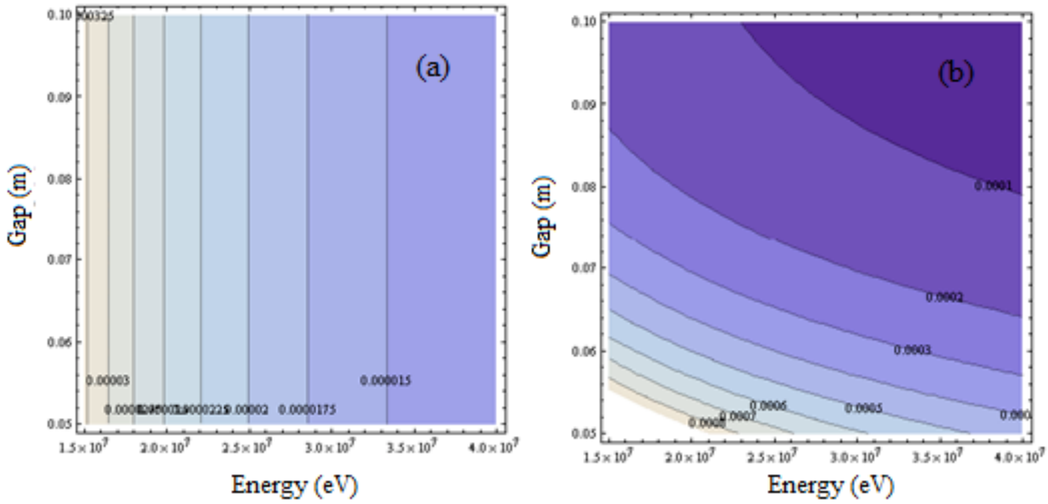

Figure 10. Variation of normalized emittance (m-rad) in different beam energies and gaps (a) forU25, (b) for U90.

As comparisons, analytical calculations for both undulators were carried out for 2.5-250 μm wavelengths and 15-40 MeV energy range using the parameter given in Table 1 and Table 2 and results are shown in Figure 11.

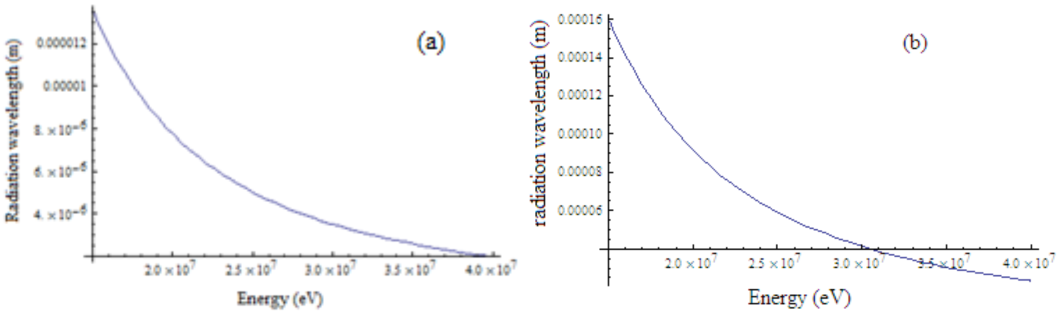

Figure 11. Variation of radiation wavelength in different beam energies (a) for U25, (b) for U90.

For simulation studies with FELO, jitter and noise effects were not taken into account. The simulated output powers versus pass number against scaled distance are shown in Figure 12 and Figure 13 for U25 and U90, respectively. The maximum output power was calculated to be higher than 0.3 MW for 5×10^{3} µm 1×10^{4} µm z scaled distance when cavity length detuning was equal to 8.6 µm as shown in Figure 12a for U25 using the simulation results obtained by FELO for 15 MeV electron beam energy. Similar calculations were carried out using Figure 13a for U90 and output power determined as 0.14 MW for 1×10^{4} µm 1.5×10^{4} z-scaled distance when cavity length detuning was equal to 5 µm. For 40 MeV electron beam energy, output power takes a value of 80 MW between 400 µm and 600 µm z scaled distance for U25 when cavity length detuning was 0.1 µm as shown in Figure 12c. This power value was 2 MW between 1000 µm and 1500 µm z scaled distance for U90 when cavity length detuning was 0.06 µm as shown in Figure 13c

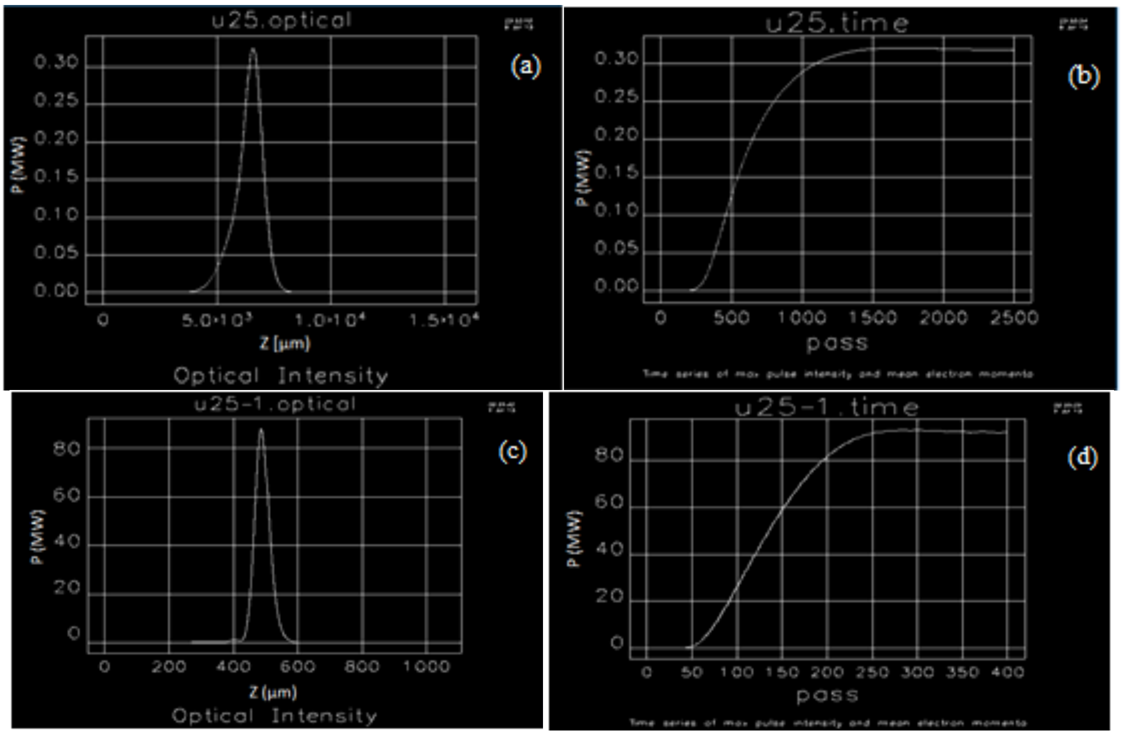

Figure 12. For U25, Variation of output power at 15 MeV (a) against scaled distance (b) against cavity pass number; Variation of output power at 40 MeV (c) against scaled distance, (d) against cavity pass number.

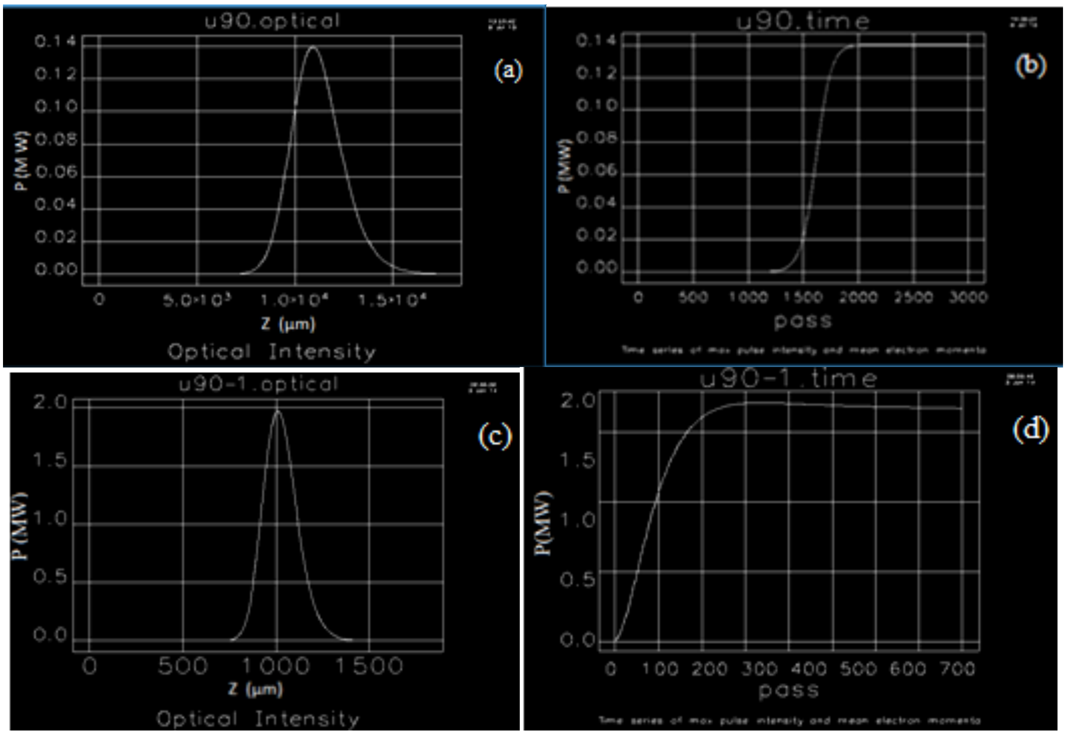

Figure 13. For U90, Variation of output power at 15 MeV (a) against scaled distance (b) against cavity pass number; Variation of output power at 40 MeV (c) against scaled distance, (d) against cavity pass number.

4. Conclusions

In this study, undulator and optical cavity parameters for obtaining desired laser parameters for TARLA were determined and parameters such as laser wavelength, beam waist, normalized emittance, Rayleigh length, single-pass gain, ideal gain, mirror radius of resonators, average extracted power, output power and efficiency were analytically calculated and simulation results were plotted.

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