Celal Bayar University Journal of Science
Volume 16, Issue 2, 2020, p 225-228
Doi: 10.18466/cbayarfbe.669390

Celal Bayar University Journal of Science

Total Domination Type Invariants of Regular Dendrimer

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Received: 02 January 2020
Accepted: 15 June 2020
DOI: 10.18466/cbayarfbe.669390

Abstract

Dendrimers are used in chemistry, textile engineering and related fields. Thus they are well studied materials. Graphs represents; structural formula of molecules, electric circuits, networks and distribution channels. Therefore they are used in many areas. Domination is a graph invariant which is used for investigation of some properties of the mentioned areas. Dendrimers are highly branched materials. Thus number of vertices form a geometric serie. We investigate some total domination type invariants of regular dendrimers. In this paper total vertex-edge domination number and total edge-vertex domination number are calculated for regular dendrimer graphs. New equations are obtained for regular dendrimers by using geometric series properties.

Keywords: total domination, total edge-vertex domination, total vertex-edge domination.

1. Introduction

Let $G=(V,E)$ be a simple connected graph whose vertex set V and the edge set E. For the open neighborhood of a vertex v in a graph G, the notation $N_G(v)$ is used as $N_G(v)=\{u|\ uv\in E(G)\}$ and the closed neighborhood of v is used as $N_G[v]=N_G(v)\cup\{v\}$. For a set $S\subseteq V$, the open neighbourhood of S is $N(S)=\bigcup_{v\in S}N(v)$ and the closed neighborhood of S is $N[S]=N(S)\cup S$.

A subset $D\subseteq V$ is a dominating set, if every vertex in G either is an element of D or is adjacent to at least one vertex in D. The domination number of a graph G is denoted by $\gamma(G)$ and it is equal to the minimum cardinality of a dominating set in G. Moreover a subset $D\subseteq V$ is a total dominating set if every vertex of G has a neighbor in D. The total domination number of a graph G is denoted by $\gamma_t(G)$ and it is equal to the minimum cardinality of a total dominating set in G. Fundamental notions of domination theory are outlined in the books [1] and [2].

A vertex v ve-dominates an edge e which is incident to v, as well as every edge adjacent to e. A set $S\subseteq V$ is a ve-dominating set if every edges of a graph G is ve-dominated by at least one vertex of S ([3,4]). The minimum cardinality of a ve-dominating set is called ve-domination number and denoted by $\gamma_{ve}(G)$.

An edge e ev-dominates a vertex v which is a vertex of e, as well as every vertex is adjacent to v ([5,6]). A subset $D\subseteq E$ is an edge-vertex dominating set (simply, ev-dominating set) of G, if every vertex of G is ev-dominated by at least one edge of D. The minimum cardinality of an ev-dominating set is called ev-domination number and denoted by $\gamma_{ev}(G)$.

Total version of the vertex-edge domination was introduced and studied by Boutrig and Chellali [7]. A subset $D\subseteq V$ is a total vertex-edge dominating set (simply, total ve-dominating set) of G, if D is a ve-dominating set and every vertex of D has a neighbour in D ([7]) . The total ve-domination number of a graph G is denoted by $\gamma_{ve}^t(G)$ and it is equal to the minimum cardinality of a total ve-dominating set.

Total edge-vertex domination was introduced in [8] as follows, a subset $D\subseteq E$ is a total edge-vertex dominating set (simply, total ev-dominating set) of G, if D is an ev-dominating set and every edge of D has a neighbor in D. The total ev-domination number of a graph G is denoted by $\gamma_{ev}^t(G)$ and it is equal to the minimum cardinality of a total ev-dominating set.

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The number of papers about domination number of chemical graphs is limited. For example, number of dominating sets of cactus chains is determined in [9] and domination number of some classes of benzenoid chains is studied in [10], [11], [12] and [13]. Moreover, domination number and total domination of regular dendrimer is studied in [14] and in [15].

Domination invariant is used in analysis of secondary RNA structure [16], electric power networks [17] and drug chemistry [18].

In this paper, we attain two total domination type invariants for regular dendrimers. This can be useful in some applications of this materials.

2. Materials and Methods

We give some important properties of regular dendrimers in this section.

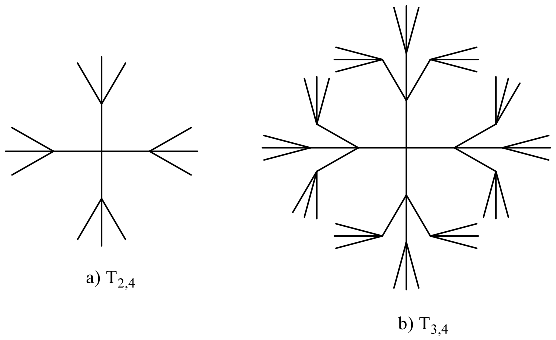

Figure 1. Dendrimers $T_{2,4}$ and $T_{3,4}$.

Dendrimers are highly branched trees ([19]). A regular dendrimer $T_{k,d}$ is a tree with a central vertex v. Every non-pendant vertex of $T_{k,d}$ is of degree $d \geq 2$ and the radius is k, distance from v to each pendant vertex. Dendrimers $T_{2,4}$ and $T_{3,4}$ are demonstrated in Figure 1. Some properties of regular dendrimers are denoted in the following lemma ([20]).

Lemma 2.1. If $T_{k,d}$ is a tree with central vertex v, then

i) The order of $T_{k,d}$ is $1+\frac{d[(d-1)^k-1]}{d-2}$,

ii) $T_{k,d}$ has d branches,

iii) Each branch of $T_{k,d}$ has $\frac{(d-1)^k-1}{d-2}$ vertices,

iv) Each branch of $T_{k,d}$ has $(d-1)^{k-1}$ pendant vertices,

v) Each branch of $T_{k,d}$ has $\frac{(d-1)^{k-1}-1}{d-2}$ non pendant vertices,

vi) The number of vertices on radius k is $d(d-1)^{k-1}$.

Lemma 2.2. ([5]) Let P_n path and C_n cycle with n vertices, then

i) $\gamma(P_n)=\gamma(C_n)=\left\lceil \frac{n}{3} \right\rceil$,

ii) $\gamma_{ve}(P_n)=\gamma_{ev}(P_n)=\left\lfloor \frac{n+2}{4} \right\rfloor$,

iii) $\gamma_{ve}(C_n)=\gamma_{ev}(C_n)=\left\lfloor \frac{n+3}{4} \right\rfloor$.

3. Results and Discussion

In this section we determine the total vertex-edge domination number and total edge-vertex domination number of regular dendrimer graphs.

Theorem 3.1. If $T_{k,d}$ be a regular dendrimer, then total ve-domination number of $T_{k,d}$ is

$$\gamma_{ve}^{t}(T_{k,d}) = \begin{cases} d^2(d-1)\dfrac{(d-1)^k-1}{(d-1)^5-1}, & k \equiv 0\ (mod5) \\ 2+d^2(d-1)^2\dfrac{(d-1)^{k-1}-1}{(d-1)^5-1}, & k \equiv 1\ (mod5) \\ 2+d^2(d-1)^3\dfrac{(d-1)^{k-2}-1}{(d-1)^5-1}, & k \equiv 2\ (mod5) \\ 1+d+d^2(d-1)^4\dfrac{(d-1)^{k-3}-1}{(d-1)^5-1}, & k \equiv 3\ (mod5) \\ d^2\dfrac{(d-1)^{k+1}-1}{(d-1)^5-1}, & k \equiv 4\ (mod5) \end{cases}$$

Proof. We use a general approach to construct the total ve-dominating set of a regular dendrimer with the minimum cardinality. Let R_m be the set of vertices $T_{k,d}$ whose distance to v is equal to m. Let S be a total ve-dominating set of $T_{k,d}$. If a vertex in S is also in R_k or R_{k-1}, then it can be replaced by its ancestor in R_{k-2}. Then since S is a total ve-dominating set, every vertex in R_{k-3} should appear in S. Hence vertices in S which belong to R_{k-4} or R_{k-5} can be replaced by their ancestors in R_{k-6}. Therefore, it is not hard to see that

$$\gamma_{ve}^{t}(T_{k,d}) = \gamma_{ve}^{t}(T_{k-5,d}) + d(d-1)^{k-4} + d(d-1)^{k-3}$$

and hence, by induction on k (initial cases are $k=1,2,3,4,5$) we obtain the following results.

Let k be a multiple of five. In this case the total ve-dominating set of $T_{k,d}$ is consisted of vertices on radius $i = 2,3,7,8,12,13,\ldots,k-3,k-2$. Summation of all vertices is,

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$$
\begin{aligned}
\gamma_{ve}^{t}(T_{k,d}) &= d(d-1) + d(d-1)^2 + d(d-1)^6 \\
&\quad + d(d-1)^7 \\
&\quad + \cdots + d(d-1)^{k-4} + d(d-1)^{k-3} \\
&= d(d-1)[1 + (d-1)^5 + \cdots + (d-1)^{k-5}] \\
&\quad + d(d-1)^2[1 + (d-1)^5 + \cdots + (d-1)^{k-5}] \\
&= (d(d-1) + d(d-1)^2)[1 + (d-1)^5 + \cdots \\
&\quad + (d-1)^{k-5}] \\
&= d^2(d-1)\frac{(d-1)^k - 1}{(d-1)^5 - 1} \qquad (\mathbf{3.1})
\end{aligned}
$$

Now assume that $k \equiv 1\ (mod5)$. In this case the total ve-dominating set of $T_{k,d}$ is consisted of central vertex v, a vertex from first radius and vertices on radius $i = 3,4,8,9,\dots,k-3,k-2$. Thus,

$$
\begin{aligned}
\gamma_{ve}^{t}(T_{k,d}) &= 1 + 1 + d(d-1)^2 + d(d-1)^3 \\
&\quad + d(d-1)^7 + d(d-1)^8 \\
&\quad + \cdots + d(d-1)^{k-4} + d(d-1)^{k-3} \\
&= 2 + d(d-1)^2[1 + (d-1)^5 + \cdots + (d-1)^{k-6}] \\
&\quad + d(d-1)^3[1 + (d-1)^5 + \cdots + (d-1)^{k-6}] \\
&= 2 + (d(d-1)^2 + d(d-1)^3)[1 + (d-1)^5 + \cdots \\
&\quad + (d-1)^{k-6}] \\
&= 2 + d^2(d-1)^2\frac{(d-1)^{k-1} - 1}{(d-1)^5 - 1} \qquad (\mathbf{3.2})
\end{aligned}
$$

Now assume that $k \equiv 2\ (mod5)$. In this case the total ve-dominating set of $T_{k,d}$ is consisted of central vertex v, a vertex from first radius and vertices on radius $i = 4,5,9,10,\dots,k-3,k-2$. Thus,

$$
\begin{aligned}
\gamma_{ve}^{t}(T_{k,d}) &= 1 + 1 + d(d-1)^3 + d(d-1)^4 \\
&\quad + d(d-1)^8 + d(d-1)^9 \\
&\quad + \cdots + d(d-1)^{k-4} + d(d-1)^{k-3} \\
&= 2 + d(d-1)^3[1 + (d-1)^5 + \cdots + (d-1)^{k-7}] \\
&\quad + d(d-1)^4[1 + (d-1)^5 + \cdots + (d-1)^{k-7}] \\
&= 2 + (d(d-1)^3 + d(d-1)^4)[1 + (d-1)^5 + \cdots \\
&\quad + (d-1)^{k-7}] \\
&= 2 + d^2(d-1)^3\frac{(d-1)^{k-2} - 1}{(d-1)^5 - 1} \qquad (\mathbf{3.3})
\end{aligned}
$$

Now assume that $k \equiv 3\ (mod5)$. In this case the total ve-dominating set of $T_{k,d}$ is consisted of central vertex v and vertices on Radius $i = 1,5,6,10,11,\dots,k-3,k-2$. Therefore,

$$
\begin{aligned}
\gamma_{ve}^{t}(T_{k,d}) &= 1 + d + d(d-1)^4 + d(d-1)^5 \\
&\quad + d(d-1)^9 + d(d-1)^{10} \\
&\quad + \cdots + d(d-1)^{k-4} + d(d-1)^{k-3} \\
&= 1 + d + d(d-1)^4[1 + (d-1)^5 + \cdots \\
&\quad + (d-1)^{k-8}] \\
&\quad + d(d-1)^5[1 + (d-1)^5 + \cdots + (d-1)^{k-8}] \\
&= 1 + d + (d(d-1)^4 + d(d-1)^5)[1 + (d-1)^5 \\
&\quad + \cdots + (d-1)^{k-8}] \\
&= 1 + d + d^2(d-1)^4\frac{(d-1)^{k-3} - 1}{(d-1)^5 - 1} \qquad (\mathbf{3.4})
\end{aligned}
$$

Finally, we assume that $k \equiv 4\ (mod5)$. For this situation the total ve-dominating set of $T_{k,d}$ is consisted vertices on radius $i = 1,2,6,7,11,12,\dots,k-3,k-2$. Therefore,

$$
\begin{aligned}
\gamma_{ve}^{t}(T_{k,d}) &= d + d(d-1) + d(d-1)^5 + d(d-1)^6 \\
&\quad + d(d-1)^{10} + d(d-1)^{11} \\
&\quad + \cdots + d(d-1)^{k-4} + d(d-1)^{k-3} \\
&= d[1 + (d-1)^5 + \cdots + (d-1)^{k-4}] \\
&\quad + d(d-1)[1 + (d-1)^5 + \cdots + (d-1)^{k-4}] \\
&= (d + d(d-1))[1 + (d-1)^5 + \cdots + (d-1)^{k-4}] \\
&= d^2\frac{(d-1)^{k+1} - 1}{(d-1)^5 - 1} \qquad (\mathbf{3.5})
\end{aligned}
$$

Theorem 3.2. If $T_{k,d}$ be a regular dendrimer, then total ev-domination number of $T_{k,d}$ is

$$
\gamma_{ev}^{t}(T_{k,d}) =
\begin{cases}
2 + d(d-1)^2\dfrac{(d-1)^k - 1}{(d-1)^4 - 1} & ,k \equiv 0\ (mod4) \\
2 + d(d-1)^3\dfrac{(d-1)^{k-1} - 1}{(d-1)^4 - 1} & ,k \equiv 1\ (mod4) \\
d\dfrac{(d-1)^{k+2} - 1}{(d-1)^4 - 1} & ,k \equiv 2\ (mod4) \\
d(d-1)\dfrac{(d-1)^{k+1} - 1}{(d-1)^4 - 1} & ,k \equiv 3\ (mod4)
\end{cases}
$$

Proof. In order to construct the total ev-dominating set of a regular dendrimer with the minimum cardinality, we use the following recursive way. Let R_m be the set of the edges of $T_{k,d}$ which are lying between the distances $m-1$ and m to the central vertex v. Let S be a total ev-dominating set of $T_{k,d}$. If an edge in S is also in R_k , then it can be replaced by its ancestor in R_{k-1}. Then these edges are adjacent and the totality of S is attained. Hence vertices in S which belong to R_{k-3} can be replaced by their ancestors in R_{k-4}. Thus, it is obtained that

$$\gamma_{ev}^{t}(T_{k,d}) = \gamma_{ev}^{t}(T_{k-4,d}) + d(d-1)^{k-2}$$

and hence, by induction on k (initial cases are $k = 1,2,3,4$) we obtain the following results.

Let k be a multiple of four. In this case, the total ev-dominating set of $T_{k,d}$ is consisted of two edges incident to the central vertex v, edges lying between the radiuses $i = 2$ and $i = 3$, $i = 6$ and $i = 7,\dots,i = k-2$ and $i = k-1$. Thus,

$$
\begin{aligned}
\gamma_{ev}^{t}(T_{k,d}) &= 2 + d(d-1)^2 + \\
&\quad d(d-1)^6 + \ldots + d(d-1)^{k-2} \\
&= 2 + d(d-1)^2[1 + (d-1)^4 + \cdots + (d-1)^{k-4}] \\
&= 2 + d(d-1)^2\frac{(d-1)^k - 1}{(d-1)^4 - 1} \qquad (\mathbf{3.6})
\end{aligned}
$$

Now let $k \equiv 1\ (mod4)$. In this case, the total ev-dominating set of $T_{k,d}$ is consisted of two edges incident to the central vertex v, edges lying between the radiuses $i = 3$ and $i = 4$, $i = 7$ and $i = 8, \dots, i = k-2$ and $i = k-1$. Thus,

$$\gamma_{ev}^{t}(T_{k,d}) = 2 + d(d-1)^3 + d(d-1)^7 + \cdots + d(d-1)^{k-2}$$
$$= 2 + d(d-1)^3[1 + (d-1)^4 + \cdots + (d-1)^{k-4}]$$
$$= 2 + d(d-1)^3 \frac{(d-1)^{k-1} - 1}{(d-1)^4 - 1} \qquad \mathbf{(3.7)}$$

Now we assume that $k \equiv 2\ (mod4)$. In this case, the total ev-dominating set of $T_{k,d}$ is consisted of edges incident to the central vertex v, edges lying between the radiuses $i = 4$ and $i = 5$, $i = 8$ and $i = 9, \dots, i = k-2$ and $i = k-1$. Thus,

$$\gamma_{ev}^{t}(T_{k,d}) = d + d(d-1)^4 + d(d-1)^8 + \cdots + d(d-1)^{k-2}$$
$$= d[1 + (d-1)^4 + \cdots + (d-1)^{k-2}]$$
$$= d\frac{(d-1)^{k+2} - 1}{(d-1)^4 - 1} \qquad \mathbf{(3.8)}$$

Finally we assume that $k \equiv 3\ (mod4)$. For the last case, the total ev-dominating set of $T_{k,d}$ is consisted of edges lying between the radiuses $i = 1$ and $i = 2$, $i = 5$ and $i = 6, \dots, i = k-2$ and $i = k-1$. Thus,

$$\gamma_{ev}^{t}(T_{k,d}) = d(d-1) + d(d-1)^5 + d(d-1)^9 + \cdots + d(d-1)^{k-2}$$
$$= d(d-1)[1 + (d-1)^4 + \cdots + (d-1)^{k-3}]$$
$$= d(d-1)\frac{(d-1)^{k+1} - 1}{(d-1)^4 - 1} \qquad \mathbf{(3.9)}$$

Author's Contributions

Ümmü Gülsüm Şener: Made investigations for her master thesis and obtained the results.

Bünyamin Şahin: Arranged the equations and wrote the paper.

Ethics

There are no ethical issues after the publication of this manuscript.

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