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Diffusion Approximation with Henyey-Greenstein Phase Function Using U_N Method

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Abstract: U_1 and P_1 approximations are applied to one-dimensional neutron transport equation and Henyey-Greenstein phase function is used for calculating diffusion length. Numerical results obtained from U_1 and P_1 approximations are compared with each other for different collision parameters and t parameters.

Key words: Diffusion approximation, Henyey-Greenstein phase function, U_N method

U_N Metodu Kullanarak Henyey-Greenstein Faz Fonksiyonu ile Difüzyon Yaklaşımı

Özet: U_1 ve P_1 yaklaşımları tek boyutlu nötron transport denklemine uygulandı ve difüzyon uzunluğu hesabı için Henyey-Greenstein faz fonksiyonu kullanıldı. Farklı çarpışma parametreleri ve t parametresi için U_1 ve P_1 yaklaşımlarından elde edilen nümerik sonuçlar birbiri arasında kıyaslandı.

Anahtar kelimeler: Difüzyon yaklaşımı, Henyey-Greenstein faz fonksiyonu, U_N metodu

1. Introduction

As well known, designing a nuclear reactor is difficult since the neutrons move complicated paths through the system. As the neutrons repeated nuclear collisions in the system, this problem become a more difficult case. To get a satisfactory solution, this problem can be solved with diffusion approximation. Diffusion approximation has the great advantage to predict many of the properties of nuclear reactors for example neutron transport and energy spectrum and also it is widely used first estimates of reactor properties [1,2].

Diffusion equation is based on Fick's law which was originally used to account for chemical diffusion and this law relating to current to the gradient of the flux. Fick's law expresses the net number of neutrons which pass per unit time through a unit area perpendicular to the *x*-direction in one-dimensional case [1].

Many methods have been proposed and applied to variety of transport problems. Among them, the spherical harmonics method (P_N) is most commonly used one by many scientists. However, Chebyshev polynomials of the second kind have been used in some recent studies for calculating critical thickness, diffusion length by certain scientists [3-5].

In this study, diffusion equation is solved with Chebyshev polynomial expansion using Henyey-Greenstein phase function. Henyey-Greenstein phase function (HG) is used in several studies to describe stellar light propagation throughout an atmosphere and light scattering in the sea-water [6-9]. HG phase function is also used in bio-medical applications by some researchers [10,11]. Up to now, HG phase function is not applied to diffusion equation using U_N method. In this study, we use HG phase function to solve diffusion equation then calculate the diffusion lengths for different values of collision scattering parameters. The results obtained from U_1 and P_1 approximations are given in the tables for comparison.

2. Theory

The steady-state neutron transport equation without sources is given as

$$\mu\frac{\partial\psi(x,\mu)}{\partial x}+\sigma_T\psi(x,\mu)=\int_0^{2\pi}\int_{-1}^{1}\psi(x,\mu')\sigma_S(\mu_0)\,\mathrm{d}\mu'\,\mathrm{d}\varphi',\qquad -a\le x\le a,\ \ -1\le\mu\le 1. \quad (1)$$

where $\psi(x,\mu)$ is the angular flux or flux density of neutrons at position x traveling in direction μ, σ_T and σ_S are macroscopic total and scattering differential cross section, respectively [12]. It is aimed to solve this equation with HG phase function in this study. To do this, we use σ_S in terms of HG phase function and it is given as [6],

$$\sigma_S^{HG}(\mu_0)=\frac{\sigma_S(1-t^2)}{4\pi\left(1-2\mu_0 t+t^2\right)^{3/2}} \quad (2)$$

where σ_S is any non-negative coefficient, the parameter t is in the range of $0\le t\le 1$ and $\mu_0=\mathbf{\Omega}\cdot\mathbf{\Omega}'$ is the cosine of the scattering angel,

$$\mu_0=\mu\mu'+\sqrt{1-\mu^2}\sqrt{1-\mu'^2}\cos(\varphi-\varphi'). \quad (3)$$

The steady state transport equation for one-dimensional case can be written when the HG phase function given in Eq. (2) is inserted on the right hand side of Eq. (1),

$$\mu\frac{\partial\psi(x,\mu)}{\partial x}+\sigma_T\,\psi(x,\mu)=\int_{-1}^{1}\psi(x,\mu')\mathrm{d}\mu'\int_0^{2\pi}\frac{\sigma_S(1-t^2)}{4\pi\left(1-2\mu_0 t+t^2\right)^{3/2}}\mathrm{d}\varphi' \quad (4)$$

The integral of the HG phase function appeared on the right hand side of Eq. (4) over $\mathrm{d}\varphi'$ can be obtained using the addition theorem of the Legendre polynomials [13],

$$\int_0^{2\pi}\sigma_S^{HG}(\mu_0)\,\mathrm{d}\varphi'=\frac{\sigma_S}{2}\sum_{n=0}^{\infty}(2n+1)t^n P_n(\mu)P_n(\mu'), \quad (5)$$

$$\mu\frac{\partial\psi(x,\mu)}{\partial x}+\nu\psi(x,\mu)=\frac{\nu c}{2}\sum_{n=0}^{N}(2n+1)t^n P_n(\mu)\int_{-1}^{+1}\psi(x,\mu')P_n(\mu')\,\mathrm{d}\mu'. \quad (6)$$

To simplify the derivation of the equations, here a dimensionless space variable such that $\sigma_T x/\nu\to x$ is defined and ν is the eigenvalues. In order to solve Eq.(6), it is well known that in the U_N approximation the angular flux is expanded in terms of the Chebyshev polynomial of second kind as,

$$\psi(x,\mu)=\frac{2}{\pi}\sqrt{1-\mu^2}\sum_{n=0}^{N}\Phi_n(x)U_n(\mu),\quad -a\le x\le a,\ -1\le\mu\le 1. \tag{7}$$

If the neutron angular flux $\psi(x,\mu)$ given in Eq. (7) is inserted into Eq. (6), and the resulting equation is multiplied by $U_n(\mu)$ and integrated over $\mu\in[-1,1]$ using the orthogonality and the recurrence relations of the Chebyshev polynomials [13],

$$\int_{-1}^{1}U_n(\mu)U_m(\mu)\sqrt{1-\mu^2}\,d\mu=\begin{cases}\pi/2, & n=m\\ 0, & n\ne m\end{cases} \tag{8a}$$

$$2\mu U_n(\mu)=U_{n+1}(\mu)+U_{n-1}(\mu),\qquad -1\le\mu\le 1. \tag{8b}$$

One can obtain the U_N moments of the angular flux for n = 0 and n = 1, respectively;

$$\frac{d\Phi_1(x)}{dx}+2v\Phi_0(x)=2vc\Phi_0(x)\,, \tag{9}$$

$$\frac{d\Phi_2(x)}{dx}+\frac{d\Phi_0(x)}{dx}+2v\Phi_1(x)=2vct\,\Phi_1(x) \tag{10}$$

Eqs. (9) and (10) are U_1 equations of the present method for the neutron transport equation and the condition for n = 1 stated in Eq. (10) is equivalent to diffusion approximation as in spherical harmonics (P_N) method by setting $d\Phi_{N+1}/dx=0$ [12]. In the case of U_1 approximation, a familiar equation known as Fick's law is obtained by taking $d\Phi_2/dx=0$ in Eq. (10),

$$\Phi_1(x)=-\frac{1}{2v(1-ct)}\frac{d\Phi_0(x)}{dx} \tag{11}$$

Then Equation (11) is inserted into Eq.(9) to obtain the diffusion equation;

$$\frac{d^2\Phi_0(x)}{dx^2}-4v^2(1-c)(1-ct)\Phi_0(x)=0 \tag{12}$$

From Eq.(12), the diffusion length (L) in U_1 approximation can be given,

$$L^{HG}=\frac{1}{2v\sqrt{(1-c)(1-ct)}}\,. \tag{13}$$

3. Results and Discussion

In this work, we applied the U_N approximation to diffusion theory using HG phase function in slab geometry. HG phase function is used as the phase function which plays an important role for the accurate solution of the transport equation. The diffusion lengths are calculated for different values of *c* and *t* parameters. Numerical results obtained from the present method are compared with ones obtained from P_N method and it can be summarized from the tables that the U_1 approximation gives coherent results with P_1 approximation in slab geometry.

Table 1. Diffusion lengths *L* (cm) as calculated by P_1 and U_1 approximations for $c = 0.99$, 0.98 and 0.95 with HG phase function

t	$c = 0.99$		$c = 0.98$		$c = 0.95$	
	U_1	P_1	U_1	P_1	U_1	P_1
0.00	5.0000	5.7735	3.5355	4.0825	2.2361	2.5820
0.25	5.7639	6.6556	4.0689	4.6984	2.5607	2.9569
0.50	7.0360	8.1244	4.9507	5.7166	3.0861	3.5635
0.70	9.0240	10.4201	6.3094	7.2855	3.8633	4.4610
0.85	12.5590	14.5019	8.6516	9.9900	5.0965	5.8849
1.00	50.0000	57.7350	25.0000	28.8675	10.0000	11.5470

Table 2. Diffusion lengths *L* (cm) as calculated by P_1 and U_1 approximations for $c = 0.89$, 0.85 and 0.80 with HG phase function

t	$c = 0.89$		$c = 0.85$		$c = 0.80$	
	U_1	P_1	U_1	P_1	U_1	P_1
0.00	1.5076	1.7408	1.2910	1.4907	1.1180	1.2910
0.25	1.7097	1.9742	1.4548	1.6798	1.2500	1.4434
0.50	2.0236	2.3367	1.7025	1.9659	1.4434	1.6667
0.70	2.4453	2.8351	2.0286	2.3424	1.6855	1.9462
0.85	3.0551	3.5277	2.4507	2.8298	1.9764	2.2822
1.00	4.5455	5.2486	3.3333	3.8490	2.5000	2.8868

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