

**ANALYSIS, INTERPRETATION, AND THE
CHROMATIC THIRD RELATIONS OF BEETHOVEN'S NO.8,
OP.13 PATHETIQUE SONATA, ADAGIO CANTABILE**

***Beethoven'in No.8, Op.13 Pathetique Sonat, Adagio Cantabile
Analiz, Yorum ve Kromatik Majör Üçlü Değişimleri***

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Abstract

This paper suggests a reading of Beethoven's No. 8, Op. 13 Pathetique Sonata Adagio Cantabile that provides a pedagogical model. This analysis offers ways for students to interpret the compositional organizations and engages them with what happens in the music, why it happens, how it happens, and what is achieved. Thus, the paper does not attempt to hierarchize the major constructs as a Schenkerian approach would, interpret the extra musical content, or search for a hermeneutic meaning of the music, even though music analysis is a multi-dimensional activity. Rather, the paper considers "how does it work?" (Bent, 2001) and looks at the structural functions and interprets them during the process. In doing so, it provides a pedagogical tool to show how to read a musical piece linearly and examine the structural elements to aid understanding, performance, and interpretation. This paper also shows the chromatic major third relations of the piece and how Beethoven used the borders of the tonality of his time in the light of Riemannian theories. Beethoven's chromatic major third relations, particularly (Ab- [C] – E) collections, pushed the borders of the tonality because of the contemporary tuning practices; as well, they were significant because the nineteenth century composers used these relations as a model.

Keywords: *Beethoven, Pathetique Sonata, Chromatic Third Relations, Riemannian Analysis, Hexatonic Cycle.*

Özet

Bu çalışma, Beethoven'in No.8, Op.13 Pathetique Sonata Adagio Cantabile'nin bir analizini sunmakta ve bir pedagojik örnek sağlamayı amaçlamaktadır. Pathetique sonat'ın bu ikinci Adagio Cantabile bölümünün analizi, öğrencilerin kompozisyon organizasyonunda ne oldu, neden oldu,

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nasıl oldu ve bu unsurlarla ne elde edildi sorularıyla yapısal analiz unsurlarını yorumlayabilme yolu önermektedir. Müzik analizinin çok boyutlu bir eylem olmasına rağmen, bu analiz Schenker yaklaşımlarının ifade edeceği gibi eserin hiyerarşik yapısını çıkartmak, müziğin kültür içindeki yerini araştırmak veya extra-müzikal yorumlar yapmak veya yorum bilgisel analizlere odaklanmak yerine, Ian Bent'in söylediği gibi eserin "nasıl işlediğini bulmayı" amaçlamaktadır (Bent, 2001). Bu çalışma, sadece betimsel bir analiz olmanın dışında, analiz sırasında eserin gözlemlenen yapısal unsurlarının yorumunu da içermesi açısından öğrenciler için bir örnek olma amacı taşımaktadır. Böylelikle, doğrusal analiz okumaları ve izlenen yapısal unsurlar bir müzikal eserin anlaşılmasına, icrasına ve yorumlanmasına yardımcı olacaktır. Çalışmanın bir diğer amacı ise, Beethoven'in eserde kullandığı kromatik majör üçlü ilişkilerine Riemann teorileri ışığı altında bir açıklama getirmektir. Zira, Beethoven'in ani kromatik majör üçlü ton değişimleri, özellikle (Lab- [Do] – Mi) dizgesi kendi döneminin akort sistemi dolayısıyla tonalitenin sınırlarını zorlamakta ve 19. yüzyıl bestecilerine de model oluşturması sebebiyle özel bir önem taşımaktadır.

Anahtar Kelimeler: Beethoven, Pathetique Sonat, Kromatik Üçlü İlişkileri, Riemann Analizi, Hexatonik Çember.

INTRODUCTION

Music theory as musicology follows new rules under the authority of anti-formalist approaches. The new methodology reads music through the lens of postmodern approaches like semiotics, hermeneutics, gender theory, and cultural criticism. Regardless, the structural analysis still has a significant role in music pedagogy and the new practices do not invalidate the pedagogical necessity and significance of positivist methods. On the contrary, the analytical examples prepare students for advanced methodologies.

Finding analytical examples, however, that students can comprehend and learn from has become more and more challenging because either the analysts apply such advanced models that require years for a student to comprehend, or these pieces are evaluated in the frame of postmodern methodologies; so these examples become irrelevant to the students. Or the analyses mostly follow a few analytical models and paradigms, such as Schenkerian or set theory. Although following such models is fine, when they are the only analytical models students read, a reductive approach is engaged whereby analysis often equals Schenkerian analysis. (1 For further reading: Beach, D. (2019). Schenkerian Analysis: Perspectives on

Phrase Rhythm, Motive and Form. New York: Routledge.; Cadwallader, A., & Gagné, D. (2011); Felix Salzer. (1952). Structural Hearing: Tonal Coherence in Music. New York: Charles Boni.; Forte, A., & Gilbert, S. (1982). Introduction to Schenkerian Analysis. New York, London: W. W. Norton & Company; Jonas, O. (1982). Introduction to the Theory of Heinrich Schenker: The Nature of the Musical Work of Art. New York: Schirmer Books.; Schachter, C., & Siegel, H. (Eds.). (1999). Schenker Studies. Cambridge: Cambridge University Press.; Schenker, H. (1969). Five graphic music analyses. New York: Dover Publications.; Schenker, H. (1973). Harmony. (O. Jonas, Ed., & E. M. Borgese, Trans.) Cambridge, MA: MIT Press.; Schenker, H. (1979). Free Composition. (E. Oster, & O. Jonas, Trans.) New York: Longman.; Schenker, H. (1987). Counterpoint: a translation of Kontrapunkt. (J. Rothgeb, Ed., J. Rothgeb, & J. Thym, Trans.) New York: Schirmer Books.; Schenker, H. (1990). Der Tonwille. Hildesheim: Georg Olms Verlag.

According to the Schenkerian approach, every tonal piece has an underlying fundamental structure, *uratz*, “the basic contrapuntal design” (Drabkin, 1980: 138) (or, an unfolding tonic) contains two elements: the fundamental line, *Urlinie*, and the arpeggiated bass, *Bassbrechung*. Through observing the linear coherence of the piece layer by layer, the goal, overall, is to reduce the tonal pieces to this fundamental structure. Along the way, the graphic analyses of the layers display the major organizational materials. All of that to say, in providing a pedagogical reading of the second movement of Beethoven’s Op.13 *Pathétique* Sonata, this paper aims to display an analysis and interpretation, showing how to find the organizational meaning of a piece beyond mere description.

Apart from the pedagogical purpose, the second objective of this paper is to map the tonal space and the ways in which Beethoven used the chromatic major third relationships, namely the [A \flat - C – E] complex. What Beethoven did in this movement was not the most conventional approach to using the tonal fabric; thus, it caused controversy among the music theoretical circles in the nineteenth century. Even though the section of chromatic major third relationships, and (Neo-) Riemannian theories, might be unfamiliar to most students, I included it as part of my analysis as I hope this glimpse of the terminology and microscopic jargon will encourage further investigation of the gamut of studies and make the students acquainted with a different theoretical realm.

Analysis

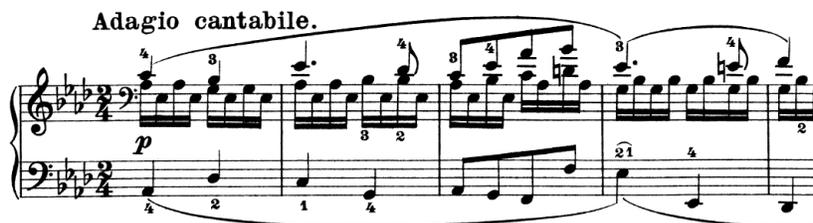
Beethoven's works are probably some of the most analyzed pieces in music history. There are multiple valid reasons for this well-earned popularity; one of the major aspects of his music is they are great examples for pedagogical purposes without being too simplistic, too dry, or too didactic; they are just right. Even though No. 8, Op. 13 *Pathétique* Sonata *Adagio Cantabile* seems like a simple Rondo, the ways Beethoven uses his compositional materials and his tonal language makes it a great example to show his mastery of manipulating the structural functions. Also, as the piece is quite well-known, most students can play it in their heads, which make it easier for them to grasp the compositional skeleton. As the linear analysis is going to describe the structural elements and the formal construction, the short interpretations / explanations will offer an (one possible) understanding of the described analytical components. Thus, by doing so, the readers/students will be encouraged to ponder of the meanings of the analytical components. This will help to develop students' habit of interpreting the analysis of the future pieces.

The lyrical second movement of the *Pathétique* Sonata is constructed as a Rondo, which reveals a strict A (A) B A C A structure. As the movement starts with an eight measure period, the main key, A \flat major, also constructs one of the core elements of the chromatic third relationship that Beethoven uses boldly here; this use caused a theoretical controversy particularly during the nineteenth century since it was analyzed under the lens of just intonation (or, pure intonation). (The tuning system based on the ratios of the frequencies). The theoretical background and further discussion will be presented during the analysis of the second episode, where the sonority oscillates from one end to another of the tonality.

In the beginning of the movement, Beethoven juxtaposes motion and stability through starting the first refrain's period with contrapuntal and harmonic progressions. This simultaneity of vertical and horizontal relations creates musical organism. In the first sentence, the two linear lines of the bass and the soprano flow contrapuntally. Vertically, the movement starts on a semi-stable root position tonic, until the half cadence in measure 4, while the outer voices display the contrapuntal 10-6-10-6-5-10-10-10-8 motion. This use of imperfect consonance patterns in m. 4 provides the framework for the motion from I to V, the linear flow from unstable to stable perfect consonance (the octave). Although the whole sentence is a

tonic prolongation, there is a strong drive to the dominant chord. Three tonic chords are in every downbeat in the first three measures, but none of them has a stable position; they are connected by the first and the third inversions of dominant seventh chords that function as passing or contrapuntal chords. From the beginning to the upbeat of bar 3, the repetition of E \flat in alto emphasizes the common-tone association between I and V. The polyphonic texture of the period shows strict contrapuntal elements, particularly the treatment of the leaps, which is one of the most important aspects of the counterpoint. Whenever the melody leaps, it is balanced by a subsequent contrary motion.

Figure 1. Measures 1-4



Kaynak: Beethoven, (n. d.: 152).

The first sentence provides three connected components: a climax, followed by stability, leading to a strong tendency for further motion. In order to achieve the climax, both harmonic organization and contrapuntal elements of the sentence interacts in m.3. To build up the melodic tension, harmonic rhythm increases and the outer voices move in contrary motion towards the resolution, where the melody reaches the highest tone of the period. To emphasize the stability and release of the tension, the treble line descends a P5 to E \flat . The alto part plays a significant role in creating the feeling of resolution because of the highlighted D natural, which functions as a leading tone as well as linking to E \flat in a stepwise motion. As the dominant chord does not have the seventh, it lacks a very strong tendency for further motion, which makes this prolonged half cadence quite stable. On the other hand, the tendency for further motion is created by a chromatic construction of the secondary dominant, the subsequent E natural.

In order to avoid monotony, Beethoven changes the motion of the bass line from the upbeat of m3 to a perfect authentic cadence

(PAC) in m.8: first, alternating M2 and P8 leaps in the first two measures, and then from m.6 changing to the cycle of fifths (C-F-Bb-Eb-Ab). The structural bass reveals an octave long stepwise motion, which is subdivided by Eb in the half cadence (HC, here on) into two. The chromatic passing tone Eb connects V and V7/iii in measure 4 and triggers the consequent phrase through the dissonant charge. By means of the secondary dominants, the chromatic elements intensify the tension of the prolongation; the 2-1 suspension, in particular, creates a strong, stable resolution feeling. In the melody, because of the gravity of the tonic, the stepwise descent from F to Ab provides complete closure on the PAC.

In order to consolidate and reinforce the refrain, the whole period repeats an octave higher in a richer texture, which is initiated by an arpeggiated triplet tonic *aufackt*. This triplet figure plants the seeds of the rhythmic structure of the second part of the movement that starts with ab minor episode in m.37. In terms of melodic and rhythmic motion, the added tenor line imitates the alto line; these inner parts, in m.11, move with the bass and the soprano. The tenor and bass together descend stepwise to the dominant chord, which is contrary to soprano and alto parts with parallel octaves. In measure 12, the bass line of half cadence (HC) changes from the previous quarter notes to linear sixteenth arpeggiation. This linear rhythmic and intervallic pattern deteriorates the feeling of stability and supports continuity.

The first episode starts with an anacrusis in f minor at measure 17. The first melodic motive demarcates the melodic ambitus of the f minor part of the episode; the melody starts with C on the anacrusis and leaps an octave while the LH provides a pulse on C, which is the same pitch of the anacrusis. Although the descending line of Ab-G-F motive has a tendency to continue to E, it leaps to C instead; however, E natural shows itself in the bass line. Thus, Beethoven satisfies the auditory expectation through transferring the pitch two octaves lower by using a V6 chord, creating a musical space as well as a looping feeling by using two different register C's around the descending motive.

Beethoven then builds up a sense of excitement through a very intentional progression. From the upbeat of m.18 to the upbeat of m.21, the half cadence of the bass line has ascending progression. Although V34, in measure 19, has the tendency to resolve to i, it

resolves to i6 instead to maintain the ascending line; through this resolution, he establishes anticipation. The extended use of C highlights the dominant prolongation and common tone association. After three repetitions, the motive gives an impression as if it is trapped. In order to break that loop, Beethoven uses a secondary dominant, V7/VII; then he intensifies the chord's instability by a chromatic alteration that makes the upbeat of m.20 a full diminished chord. The soprano is elaborated by a turn, which creates a new sonority and pushes the directed motion to V in m.21. With the bass of the V, the arch is completed through transferring the C of the anacrusis from the melody to the bass in m.17. Although the harmonic rhythm is not fast, the consistent sixteenth note left hand figurations create a continuum. The descending A \flat -G-F motive recalls the main theme of the refrain at the structural level.

The section from the upbeat of m.21 gives hint of the subsequent E \flat major part, particularly the elaborated melodic line of the V7 chord in m.22. The whole elaboration carries the line to the temporary goal, E \flat tonic in m.23. The measure continues with a descending tonic arpeggiation and a contrapuntally functioned V7 chord; until the second refrain, the section is an E \flat tonic prolongation (or dominant in A \flat major) that sets up A \flat major (or tonic). Although the alternated tonic and dominant chords have a particular linear motion to maintain the prolongation, the tonic has the construction of an arpeggio, which repeats an octave lower in m.25. The dominant seventh chord has descending chromatic figuration that highlights the gravity of tonic; the second repetition of the elaborated V7 is prepared by the upbeat of the tonic at the same measure. Through this elaborative alternation, Beethoven creates fluidity. In m. 27 the tonic chord uses the motion of chromatic descent of the dominant, rather than the arpeggiation, to destroy stability and to charge the chord with chromatic dissonant properties; particularly the second repetition in m.28 transforms its function to a dominant seventh chord of A \flat to make the resolution much stronger. To balance the gravity of the subsequent tonic, the dynamic of the falling chromatic figuration changes into *pp* with a decrescendo.

Although the refrain repeats the first part of three voiced texture, the initial tonic chord has two extra voices to re-establish and highlight the A \flat major key. The bass has an octave lower addition and the *divisi* alto 2 imitates the soprano, which is significant because the

added C is the resolution of the Db of the descending chromatic line; thus, it underlines stronger gravity. Unlike the first refrain, the theme repeats only once.

The second episode begins after the cadence with an anacrusis in Ab minor. *pp* dynamics is used to balance the sixteenth triplet right hand accompaniment, which creates rhythmic dissonance, as well as a pseudo acceleration feeling. The melody is reminiscent of the first episode's minor tonality in terms of interval content, and descending chromatic figuration. Triplets are first introduced at the upbeat of the refrain; here, they became the main characteristic. Until measure 41, harmonic rhythm, unlike the lively accompaniment, is slow. Just like the first episode's repetitive four note motive, there is a recurrent four note idea. The descending chromatic figuration in the bass recalls the F minor episode's tenor figuration, which constructs a *motivic parallelism* but an extended and rhythmically transformed version; it creates a concealed imitative relationship among these parts. In measure 41, a *crescendo* changes the dynamics of the four note motive from *pp*, which indicates a forthcoming movement and increases the momentum. In m.42 the section modulates to E major, the dynamic accents emphasizing the chords (V-I) and highlighting the new tonality.

As indicated earlier, the tonal oscillation of the movement was quite controversial during the nineteenth century because of Beethoven's tonal inclinations. The Ab major following Ab minor then suddenly shifting to E major was theoretically not a conventional progression. According to Alexander Rehding, this tonal shift was so abrupt that one of the most significant music theorists of the nineteenth century, Hugo Riemann, was baffled by the composer's harmonic preferences in this movement (Rehding, 2011: 115). Riemann debated if Beethoven's key should be different: whether it should have been written in G# major, an enharmonic equivalent of Ab major.

Riemann was a major representative of the harmonic dualism, one of the two main music theoretical approaches in Austro-German traditions with the thorough-bass theory. (According to dualist theory, minor and major triads are different but equal structures; also, these triads were mirror images to one another (Klumpenhouwer, 2011). This approach was developed under the guidance of two Prussian physicists, Hermann von Helmholtz, Arthur von Oettingen, and a German music theorist, Moritz Hauptmann. Now, dualism and

thorough-bass derived music theory are associated with harmonic dualism and scale-degree theory respectively (Klumpenhouwer, 2002). Even though in contemporary music-theoretical practices dualist theory has lost its role because of a paradigm shift when harmony is not considered in terms of acoustics anymore, the Neo-Riemannian theories developed as a solution to the chromatic music of the nineteenth century: triadic, yet highly chromatic (Cohn, 1998). While this theory was designed to explain the chromatic music of the nineteenth century, chromatic music of the Vienna classics and early romantics' music is sometimes analyzed through the lens of the Neo-Riemannian theories. Because Beethoven's tonal fluctuation was not conventional for his time, the next section provides a Neo-Riemannian explanation.

Today as we are totally acclimatized to the milieu of equal temperament, this debate might be irrelevant for some audiences; however, it is crucial to remember that Riemann did this analysis under the service of just intonation. Despite the piece being written for the piano and thus impossible to hear the difference between G# major and Ab major, the contrast is analytically quite significant because of the harmonic theory based on just intonation (Rehding, 2011). Riemann's table (1918-19) shows the spatial difference between G# major and Ab major; in his words:

with this move, however, he [Beethoven] enters not into the key of the lower third, but into that of the second upper third instead (Ab+ [C+] E+) and the modulation will be not a descent into the depths of the flat keys but an ascent into the light region of the sharp keys. Aesthetically speaking, this is an entirely different matter (Rehding, 2011).

Figure 2. Example 4. Alexander Rehding's Functional Interpretations

gis	dis
e	h
c	g
as	es
fes	ces

Kaynak: Rehding, (2011: 116).

As mentioned, the motion from A \flat minor/major to E major (In Neo-Riemannian terms, it's two LP progressions in a row) represents the edges of the tonal context of just intonation because any keys having more flats and sharps were too dissonant. In the chromatic major third relationships (A \flat + [C+] E+), (To indicate major and minor, + and - will be used respectively to be consistent with the Riemannian terminology) the harmonic motion follows the parsimonious voice leading (Cohn, 1997; Cohn, 2012) a method that aims for minimum motion (half a step in this case) between the chords. This progression generates a hexatonic cycle (A \flat +, A \flat -, [C+, C-] E+, [E-]) through the following succession: A \flat + (A \flat , C, E \flat) becomes A \flat - (minor) by descending C half a step C \flat , and then to E+ through ascending E \flat to E \sharp (E \sharp , G \sharp {enharmonic equivalent of A \flat }, B \sharp {enharmonic equivalent of C \flat }). The whole hexatonic cycle is completed through half step motions. In Riemannian terms, it is a *Leittonwechsel* (LW) (A compound leading tone operation that requires to reverse the tonal polarity) transformation from A \flat minor to E major as the tonal polarity flips, which provides the minimal melodic distance. Other than the keys of the first episode [F+, E \flat +], the rest of the movement's key areas belong to the same hexatonic cycle (A \flat +, A \flat -, [C+, C-] E+, {E-}). The melody line has successions of dynamically accented octaves on the strong beats and horizontally expressed chords on the weak; the chords are connected by common notes. In the first two bars, the accented octaves have a P4 pitch space between them.

Figure 3. Measures 40-42



Kaynak: Beethoven, (n. d.: 153).

With this episode, the mood flips dramatically. The downbeat of m.43 reaches the climax of the E major section; the secondary dominant and its rich chromatic content as well as the ambitus of the chord intensifies the effect. This prolonged dominant reaches to the tonic chord in m.44 that is accented by *fp*; at that point, E major tonality is fully established. After the downbeat, the sixteenth triplet

motion on G#, B and *decrescendo* dynamics weaken the stability of the tonic and increase the expectation of “something” coming. The whole theme of the A \flat minor section transposes into E major—the dark, gloomy theme becomes a bright and joyful one.

The descending chromatic figure destabilizes the melody and the key of E major and carries it to the second repetition of the theme; the tonic tries to re-establish the stability but the subsequent measures with a D fully diminished chord increases the tension and the tendency to resolve. The left hand linearizes the same chord by an ascending line, which also contributes to building the tension; staccato articulation intensifies the feeling of instability. This left hand figuration, with the D natural on the soprano, provides the melodic motion, since the repetitive chords are not melodically active but only function as a rhythmic (and harmonic) tension source. In measure 50, the previous measures’ full diminished chord becomes a half diminished chord that leads to a V7 of A \flat major to prepare for the upcoming refrain with a *crescendo* to increase the tendency for further motion. The grouping of the left hand accompaniment creates the feeling of the bass part widening. The dominant seventh chord smoothly resolves to the tonic chord of the A flat major in m.51.

This new refrain, unlike the previous ones, has a variation character because of the continuation of the staccato triplet sixteenth notes in the inner voice, but this modified version, sixteenth triplets in the alto part, creates a rhythmic dissonance, which changes the articulation and deteriorates the *cantabile* quality of the theme. Significantly, in m.57, the rhythmic groupings of the melody on the dominant chord of PAC are modified from sixteenth note grouping to sixteenth triplets. The second repetition of the refrain also has the added tenor part with sixteenth triplets, which consolidates the variation character through superimposition of simple and compound rhythms, or rhythmically consonant and dissonant expressions. The repetitious construction of the triplets gives a faltering feeling.

The coda section begins in m.66 as tonic-dominant alternation. The motive of the coda is introduced by the refrain in m.4 with a shorter note value. The fifth of the tonic, E \flat , accompanies to the motive. With the motive, harmonic rhythm gets slower. In the right hand, descending figuration is accompanied by a prolonged dominant seventh chord and it alternates with the tonic. In the second repetition, an octave imitation is added to the right hand. The dominant chords, unlike tonics, are dynamically accented and

prolonged in order to reinforce the tendency of resolving the tonic. The gradually descending second motive is introduced by a dominant chord with *rf* accent. The prolonged dominant seventh chord raises the expectation that resolves in the last measure with stable tonic chords.

CONCLUSION

This analytical example showed the reader how Beethoven used the chromatic major third relationships, which became distinguished harmonic practice during the nineteenth century (Brittner Stull, 2006). The tonal structure of the piece demonstrates a concise case for Neo-Riemannian operations and the parsimonious voice-leading transformations. This structure presents three cycles of major-third-related tonal areas that can create a distinct sonic atmosphere. The most important aspect of these tonal centers is they have either one (in the case of of $A^b - C$) or two common tones ($C - E$). Even though this approach might be foreign to most students, there is immense research on the topic. It is my hope that this taste of a Neo-Riemannian approach will make students want to know more.

As teachers, we expect students are able to identify the major compositional materials and procedures in a short period of time so they can move on to the next level of musical analysis, but unfortunately, particularly in certain regions or areas because of the pedagogical traditions, undergraduate students are never able to reach progressive levels. Even though theory courses try to focus on different aspects of the musical materials, most of the times the most visible aspects like Roman numeral analysis or formal analysis receive most of the attention, particularly outside of theory and composition disciplines. The problem of this style of analysis is that it does not prepare the students for more sophisticated readings—they are not prepared to see music on the outside of the perceptual space. Thus, it is crucial to provide analyses showing not only directly noticeable aspects but also analyses of the musical gestures, special groupings, unusual structural functions like tonality, texture, melody, rhythm and meter. Advancing these analytical examples could prepare students for more complex and developed analytical models. This paper demonstrated one such alternative in how to read a piece of music other than stating the obvious. From this analysis, students can learn to interpret the structural elements and the gestures.

REFERENCES

- Beethoven, L. v. (n.d.). *Piano Sonatas, Vol. 1*. Leipzig, London, New York: Edition Peters.
- Bent, I. (2001). *Oxford Music Online*. Retrieved 05 04, 2020, from <https://www-oxfordmusiconline-com.login.ezproxy.library.ualberta.ca/grovemusic/view/10.1093/gmo/9781561592630.001.0001/omo-9781561592630-e0000041862?rskey=goBABB>
- Bribitzer Stull, M. (2006). Ab-C-E Complex: The Origin and Function of Chromatic Major Third Collections in Nineteenth-Century Music. *Music Theory Spectrum*, 28, 167-90.
- Cohn, R. (1997). Neo-Riemannian Operations, Parsimonious Trichords, and their Tonnetz Representations. *Journal of Music Theory*, 41 (1), 1–66.
- Cohn, R. (1998). Introduction to Neo-Riemannian Theory: A Survey and a Historical Perspective. *Journal of Music Theory*, Neo-Riemannian Theory , 42 (2), 167-180.
- Cohn, R. (2012). *Audacious Euphony: Chromaticism and the Triad's Second Nature* . New York: Oxford University Press.
- Drabkin, W. (1980). Glossary. In I. Bent, *Analysis* New York: W.W. Norton & Company. 109-143.
- Klumpenhouwer, H. (2002). Dualist Tonal Space and Transformation in Nineteenth-Century Musical Thought. In T. C. Theory, & T. Christensen (Ed.), *The Cambridge History of Music* (pp. 456–76). Cambridge: Cambridge University Press.
- Klumpenhouwer, H. (2011). Harmonic Dualism as Historical and Structural Imperative. In A. Rehding, & E. Gollin (Eds.), *The Oxford Handbook of Neo-Riemannian Music Theories* (pp. 194-217). New York: Oxford University Press.
- Rehding, A. (2011). Tonality between Rule and Repertory; Or, Riemann's Functions—Beethoven's Function. *Music Theory Spectrum*, 33 (2), 109-123.
- Riemann, H. (1918-19). *L.van Beethovens samtliche Klavier-Solosonaten Asthetische und formal-technische Analyse mit historischen Notizen*. Berlin: M. Hesse.