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THREE-DIMENSIONAL MODAL ANALYSIS OF BEAMS WITH TRIANGULAR AND HEXAGONAL CROSS-SECTIONS USING DIFFERENT CERAMIC MATERIALS

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ABSTRACT: This study deals with the investigation of the effects of cross-sections and ceramic materials on the free vibration behavior of beams. Fundamental frequency analyses were performed using finite element software ANSYS based on Taguchi's L8 orthogonal array with two control factors. Ceramic material types were used as the first control factor consisting of four levels such as Zirconia (ZrO_2), Silicon nitride (Si_3N_4), Alumina (Al_2O_3), and Silicon carbide (SiC) while cross-sections of beams were considered as the second control factor including two levels such as triangle and hexagon. To optimize the ceramic materials and the cross-sections, analysis of signal-to-noise (S/N) ratio was used. Important control factors and their percent contributions on numerical fundamental free vibration response were performed using analysis of variance (ANOVA). According to ANOVA results, the percent contribution ratios of the control factors on fundamental frequency of the beams are found as 67.45 for ceramic material and 31.50 for the cross-section of beams.

Keywords: Modal analysis, Cross-section, Ceramic, Finite element approach.

1. INTRODUCTION

Beam structures with various cross-sections can be broadly utilized in different engineering applications, such as different types of bridges, columns, and frames, amongst many others. In general, beams have been made using metal and ceramic materials. The ceramic materials have superb characteristics for heat resistance whereas the metal materials have superb strength and toughness [1]. Especially, three-dimensional free vibration characteristic of beams has attracted several engineering designers and researchers because of their widespread applications. In literature, various studies including free vibration analyses can be seen. Balhaddad and Onipede [2] presented a study about modal analysis of pre-twisted beams using three-dimensional approach. Evran [3] presented the modal analysis of the functionally graded tapered beams which having three layers using finite element software ANSYS and Taguchi method. Giunta et al. [4] reported modal analysis of sandwich beams with three-dimensions. Fang et al. [5] presented modal analysis of rotating functionally graded beams under cantilever boundary conditions in three dimensions. Evran [6] investigated the free vibration characteristic of layered functionally graded beams in three dimensions using numerical and statistical methods. In modal analysis, ANSYS software was used for finite element approach. Evran and Yılmaz [7] evaluated the impacts of ceramic and metal materials on the modal analysis of layered beams using finite element and Taguchi methods. Alshorbagy et al. [8] presented a study including modal analysis of a beam designed from functionally graded materials using the finite element approach. Yilmaz and Evran [9] investigated the first mode frequency characteristic of beams made from functionally graded materials in axial direction using experimental and numerical methods. As can be seen from above literature, there are many studies regarding free vibration. In this study, fundamental frequency analysis of the beams with triangular and hexagonal cross-sections were performed using different ceramic materials.

2. MATERIALS AND METHODS

In numerical analysis, beams with different cross sections were used, and each beam was made of different ceramic materials such as Zirconia (ZrO_2), Silicon nitride (Si_3N_4), Alumina (Al_2O_3), and Silicon carbide (SiC). Poisson's ratio for each material was taken to be constant and it was used as 0.3. Young module and density data for ceramic materials were given in Table 1.

Table 1. Ceramic Materials Properties [10].

Material Type	Properties	
	E (GPa)	**ρ (kg/m^3)**
Zirconia (ZrO_2)	151.00	3000
Silicon nitride (Si_3N_4)	322.27	2370
Alumina (Al_2O_3)	380.00	3800
Silicon carbide (SiC)	427.00	3210

The statistical analysis was performed using Minitab software. Numerical frequency analyses for the first mode were conducted under L8 orthogonal array which has two control factors, based on Taguchi method. The first control factor includes four levels while the second control factor consists of two levels. The first control factor was accepted as ceramic materials while the second control factor was taken as cross-sections of the beams and they were determined as triangle and hexagonal. The control factors and their levels were listed in Table 2.

Table 2. Control factors and levels.

Control Factors	Symbol	Levels			
		Level 1	**Level 2**	**Level 3**	**Level 4**
Ceramic Material	A	ZrO_2	Si_3N_4	Al_2O_3	SiC
Cross-Section of Beam	B	Triangle	Hexagon		

In order to find the optimum levels of ceramic materials and the cross-sections of the beams for the maximum free vibration behavior, "the larger is better" quality characteristic was used based on Taguchi method. This quality characteristic was identified in Equation 1 [11].

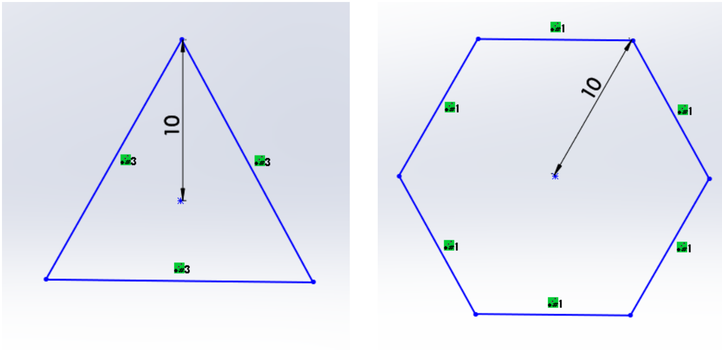
$$\left(\frac{S}{N}\right)_{HB} = -10.\log\left(n^{-1}\sum_{i=1}^{n}\left(y_i^2\right)^{-1}\right) \quad (1)$$

in here, n shows the number of numerical free vibration analysis in a trial and y_i symbolizes i^{th} data evaluated, for this study.

3. FINITE ELEMENT SOLUTION

3-D finite element solutions for free vibration analysis of the beams with triangular and hexagonal cross-sections were performed using finite element software ANSYS. Numerical analyses were carried out as modal analysis for the first mode. In the analysis, 3-D model element type called as SOLID186 was used. The element type is 3-D 20-Node Structural Solid and it demonstrates quadratic displacement performance and is defined by twenty nodes which have three degrees of freedom every nodes: translations for the nodal x, y, and z directions [12].

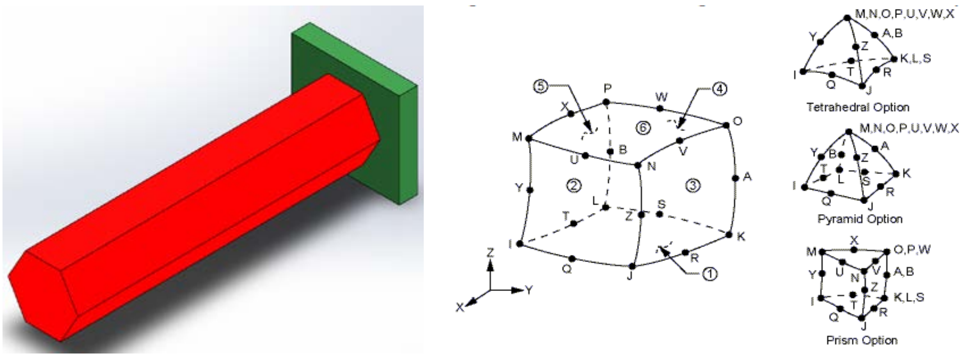
UX, UY, and UZ were used as degrees of freedom. Therefore, UX = UY = UZ was used as 0 for clamped boundary conditions. Length and diameter of cantilever beams were used as 100 in mm and 10 in mm. The cross-sections of beams were demonstrated in Figure 1.



a) b)

Figure 1. The cross-sections a) triangle and b) hexagon.

The beams were modelled under free and clamped boundary conditions. Sweep mesh type was employed for mesh operation. Cantilever beam and homogenous structural solid geometry for SOLID186 element were shown in Figure 2.



a) b)

Figure 2. a) Cantilever beam and b) SOLID186 homogenous structural solid geometry [12].

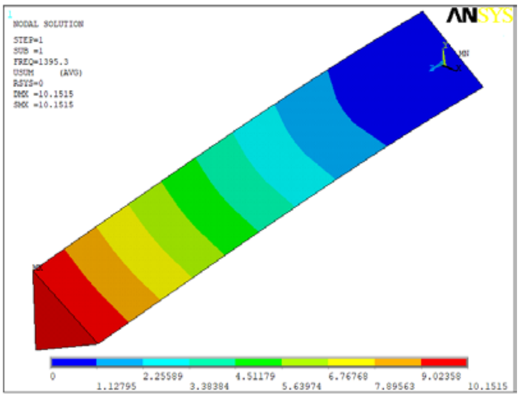
4. RESULTS AND DISCUSSIONS

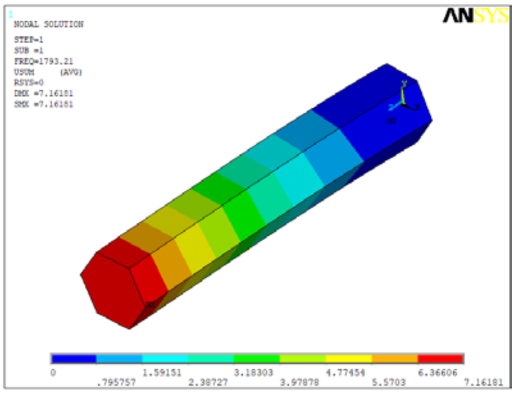
In order to optimize the levels of ceramic materials and the cross-sections of beams on free vibration, modal analyses were conducted using L8 orthogonal array based on Taguchi method. S/N ratio data were calculated for the finite element results using the statistical method. Results obtained for finite element and S/N ratio were tabulated in Table 3.

Table 3. Frequency and S/N ratio results for L8 orthogonal array.

Run	Designation	Control Factors		Results	
		Ceramic Type	**Cross-Section**	**Frequency (Hz)**	**S/N ratio (dB)**
1	A_1B_1	ZrO_2	Triangle	1395.30	62.8934
2	A_1B_2	ZrO_2	Hexagon	1793.21	65.0726
3	A_2B_1	Si_3N_4	Triangle	2293.38	67.2095
4	A_2B_2	Si_3N_4	Hexagon	2947.40	69.3888
5	A_3B_1	Al_2O_3	Triangle	1966.71	65.8748
6	A_3B_2	Al_2O_3	Hexagon	2527.57	68.0541
7	A_4B_1	SiC	Triangle	2268.31	67.1140
8	A_4B_2	SiC	Hexagon	2915.18	69.2933
Overall Mean ($\overline{T}_{\alpha}$)				2263.38	

Finite element approach was carried out for the fundamental frequency of each beam using ANSYS software. Results obtained for eight analyses were given in Figure 3 as visually.


1. modal analysis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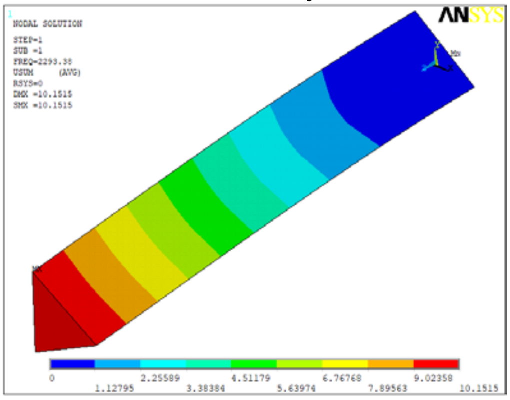
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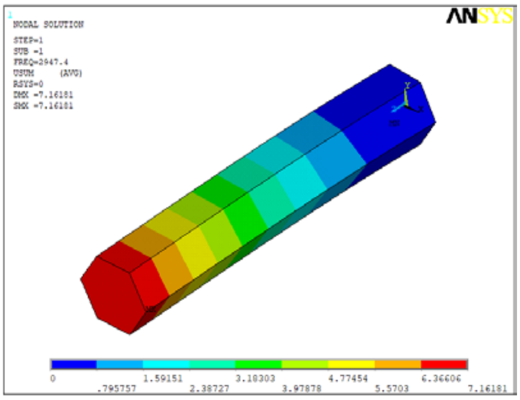
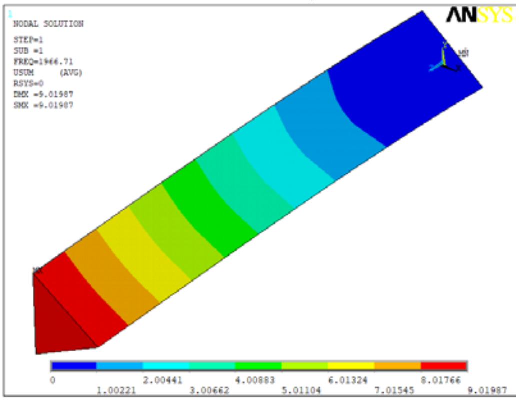
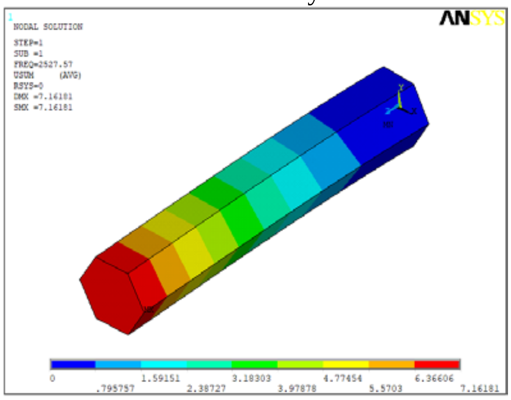
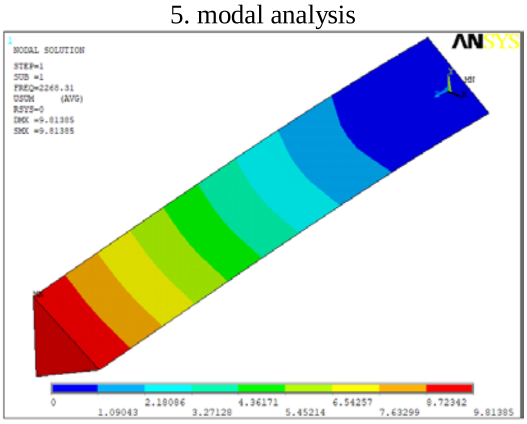

4. modal analysis


5. modal analysis


6. modal analysis


7. modal analysis

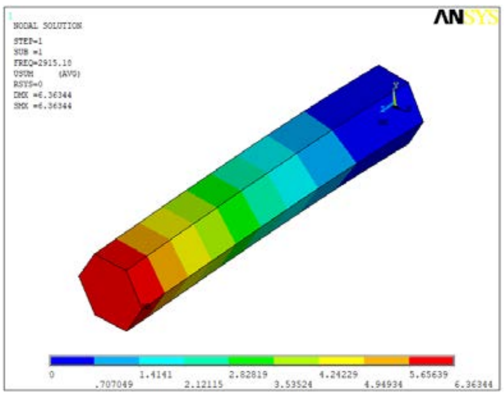

8. modal analysis

Figure 3. Fundamental frequency analysis for finite element approach.

4.1. Examination of Optimal Levels

In order to examine the optimal levels of control factors such as ceramic material types and cross-sections of the beams, average finite element data and their S/N ratio data for each level of all control factors based on numerical fundamental frequency and S/N ratio data were calculated using Minitab statistical software. Results found were presented in Table 4.

Table 4. Response table for S/N ratio and frequency.

Level	S/N ratio in dB		Mean in Hz	
	A	**B**	**A**	**B**
1	63.98	65.77	1594	1981
2	68.30	67.95	2620	2546
3	66.96		2247	
4	68.20		2592	
Delta	4.32	2.18	1026	565
Rank	1	2	1	2

As can be seen from Table 4, optimum levels of ceramic material types and cross-section of the beams are found as second levels. Therefore, the maximum numerical fundamental frequency result was obtained hexagonal beam made of Silicon nitride. In order to see the impacts of every levels of control factors on the numerical fundamental free vibration analysis, average S/N ratio data for each level of control factors were plotted and were presented in Figure 4.

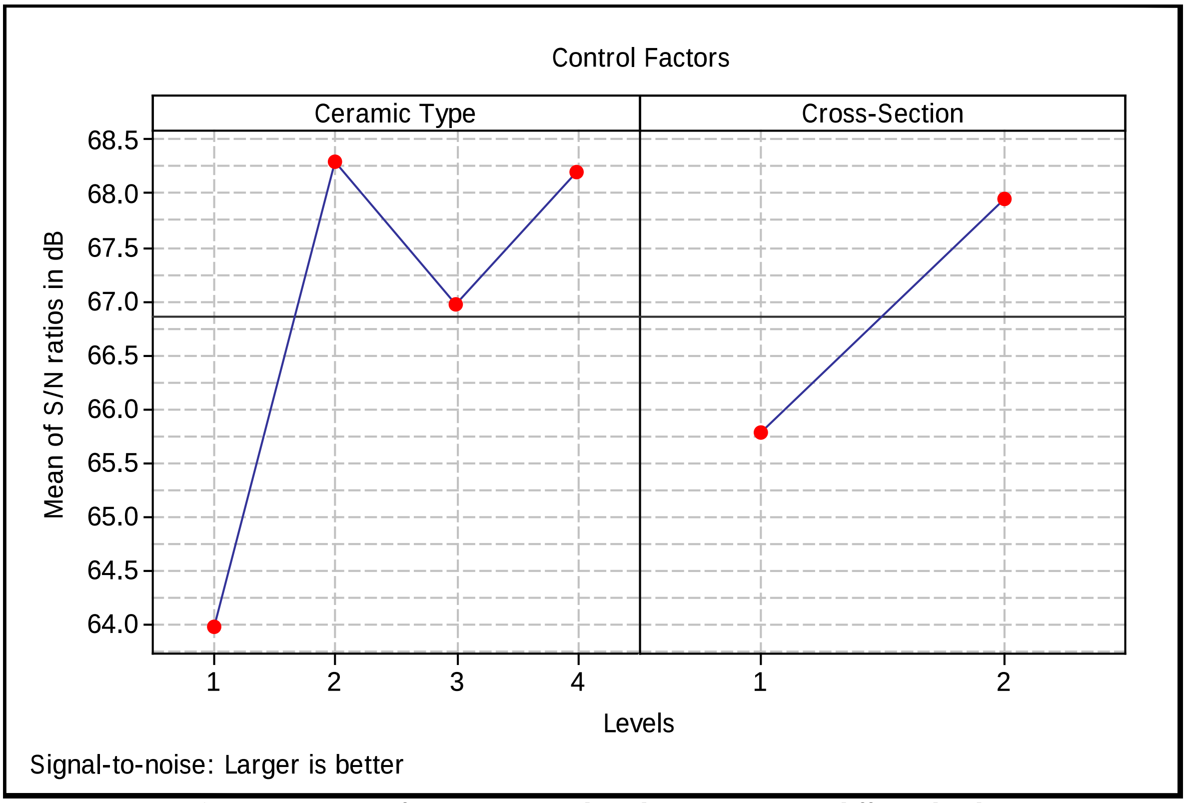


Figure 4. Impacts of ceramic materials and cross sections at different levels.

It can be seen from Figure 4 that free vibration of beams increases from the first level to the second level whereas decrease from the second level to third level. However, free vibration of beams increases from the third level to the fourth level. For the cross-sections of beams, free vibration increases from level 1 to level 2.

4.2. Analysis of Variance

Analysis of variance (ANOVA) was employed to see impact ratios of control factors on fundamental frequency results and to obtain significant control factors at 95 % confidence level using Minitab statistical software. Result solved for R-Sq = 98.95% and R-Sq (adj) = 97.55% were illustrated in Table 5.

Table 5. ANOVA for fundamental free vibration.

Source	DF	Seq SS	Adj MS	F	P	% Effect
A	3	1366543	455514	64.210	0.003	67.45
B	1	638258	638258	89.970	0.002	31.50
Error	3	21282	7094			1.05
Total	7	2026083				

Table 5 indicates that the control factors have powerful impacts on free vibration of the beams owing to $P < 0.05$ data. Also, the percent contribution ratios of control factors on fundamental frequency of the beams are found as 67.45 for ceramic material and 31.50 for the cross-section of beams.

4.3. Estimation of Optimum Fundamental Frequency

Prediction of optimal fundamental frequency was carried out using the significant control factors including the optimal levels for the maximum response. According to analyses of signal-to-noise and variance, the optimal result for the free vibration characteristic at the maximum level was found using control factors called A and B at the second levels. The predicted mean of free vibration characteristic for the first mode can be computed using Equation 2 [11].

$$\mu_\alpha = \overline{A_2} + \overline{B_2} - \overline{T}_\alpha \tag{2}$$

For Equation 2, $\overline{A_2}$ and $\overline{B_2}$ were calculated to be 2620 Hz and 2546 Hz, respectively following analysis of signal-to-noise. These data for means were presented in Table 4. Also, $\overline{T}_\alpha$ is the overall mean for Taguchi L8 orthogonal array with two control factors and this data was given as 2263.38 Hz in Table 3. Substituting numerical data of different terms in Equation 2, μ_α is computed to be 2902.62 Hz. Confirmation analysis and the population at the 95 % confidence intervals were computed following Equation 3 and Equation 4 [11], respectively.

$$CI_{CA} = \left(F_{\alpha;1;n_2} V_{error}\left[\frac{1}{n_{eff}} + \frac{1}{R}\right]\right)^{0.5} \tag{3}$$

$$CI_{POP} = \left(\frac{F_{\alpha;1;n_2} V_{error}}{n_{eff}}\right)^{0.5} \tag{4}$$

$$n_{eff} = \frac{N}{(1 + T_{DOF})} \tag{5}$$

where, $\alpha = 0.05$ describes the risk and $n_2 = 3$ points to the error value for degree of freedom in ANOVA in Table 5. $F_{0.05;1;3}$ is taken to be 10.13 [11] and this data was given in F ratio table associated with 95 % confidence interval. $V_{error} = 7094$ implies the error value for variance according to ANOVA data. R was utilized the sample size of confirmation numerical analysis of free vibration for the first mode and this value is operated as 1. T_{DOF} symbolizes the total number of degrees of freedom for the powerful control factors in ANOVA and this value was employed to be 4. N implies the total number of finite element analysis and it was determined to be 8 for Taguchi's L8 orthogonal array as shown in Table 3. Thus, n_{eff} was found to be 1.6. CI_{CA} and CI_{POP} were analyzed as 341.72 ± and ± 211.93, respectively. The expected confidence interval for confirmation numerical analysis [11] is:

$$\text{Mean } \mu_\alpha - CI_{CA} < \mu_\alpha < CI_{CA} + \text{Mean } \mu_\alpha$$

The population connected to the 95 % confidence interval [11] is:

$$\text{Mean } \mu_\alpha - CI_{POP} < \mu_\alpha < CI_{POP} + \text{Mean } \mu_\alpha$$

The fundamental frequency for the first mode and predictive results for the optimum level using predicted confidence intervals were listed in Table 6.

Table 6. Optimum response.

Designation	Numerical Result	Predictive Result	Predicted Confidence Intervals for 95% Confidence Level
A_2B_2	2947.40 Hz	2902.62 Hz	$2560.9 < \mu_\alpha < 3244.34$ for CI_{CA} $2690.69 < \mu_\alpha < 3114.55$ for CI_{POP}

5. CONCLUSIONS

Impacts of the cross-sections and ceramic materials on the fundamental free vibration behavior of beams were analyzed using 3-D finite element model based on ANSYS software. The numerical analyses were performed using Taguchi's L8 orthogonal array which has two control factors. The levels of the first control factor were assumed as triangular and hexagonal cross-sections while the levels of the second control factor were considered as ceramic materials such as Zirconia (ZrO_2), Silicon nitride (Si_3N_4), Alumina (Al_2O_3), and Silicon carbide (SiC). The significant levels and percent contribution rate of the cross-sections and ceramic materials on fundamental frequency analysis were carried out using ANOVA while the effects of the control factors were performed using S/N ratio analysis. The following conclusions are noted from this study:

- The maximum fundamental frequency result was obtained using hexagonal cross-section of beam designed from silicon nitride.
- The percent contribution ratios of control factors on numerical fundamental free vibration of the beams were found as 67.45 for ceramic material and 31.50 for cross-section of beams.
- Ceramic materials and the cross-sections were carried out to be the significant control factors owing to $P < 0.05$.
- The maximum impact was occurred in free end of beams while the minimum impact was determined in clamped end.
- The free vibration behavior of the beams with triangular cross-sections are smaller compared to the beams which have hexagonal cross-sections.
- Estimated optimum fundamental frequency at 95 % confidence intervals was obtained to be $2560.9 < \mu_\alpha < 3244.34$ for CI_{CA} and $2690.69 < \mu_\alpha < 3114.55$ for CI_{POP}.

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