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ANNIHILATOR DOMINATION NUMBER OF TENSOR PRODUCT OF PATH GRAPHS

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ABSTRACT. An annihilator dominating set (ADS) is a representative technique for finding the induced subgraph of a graph which can help to isolate the vertices. A dominating set of graph G is called ADS if its induced subgraph is containing only isolated vertices. The annihilator domination number of G, denoted by $\gamma_a(G)$ is the minimum cardinality of ADS. The tensor product of graphs G and H signified by $G \times H$ is a graph with vertex set $V = V(G) \times V(H)$ and edge $\{(u, v), (u', v')\} \in E$ whenever $(u, u') \in E(G)$ and $(v, v') \in E(H)$. In this paper, we deduce exact values of annihilator domination number of tensor product of P_m and P_n, $m, n \geq 2$. Further, we investigated some lower and upper bounds for annihilator domination number of tensor product of path graphs.

Keywords: Domination Number, Annihilator Dominating Set, Annihilator Domination Number, Paths, Tensor Product.

AMS Subject Classification: 05C69, 05C50, 05C76

1. Introduction and terminology

All graphs are considered to be only simple, undirected and finite graphs. Let $G = (V, G)$ be a graph. As usual, we denote $V(G)$ as set of vertices of G and $E(G)$ as set of edges of G. In 1977, Cockayne and Hedetniemi [2] introduced an outstanding survey of the results about dominating sets in graphs and used the notation $\gamma(G)$ for the domination number. The beginning of extensive study of dominating sets took place in nearly 1960. De jaenisch discovered and studied in depth the difficulties related to $(n \times n)$ chessboard problem that how many number of queens are required to lead. Domination applies in facility location problem like: hospitals, fire stations when a person needs to travel to get to the nearest facility, the number of amenities are permanent and its required to minimize the distance in one attempt. It also works on that problem where maximum space to a facility is permanent and needed one attempt to minimize the number of amenities compulsory. In case that everyone is serviced concepts from domination is also occur in problems including discovering sets of representative in the administering communication

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or electrical network and in land reconnoitering i.e. an evaluator must remain in order to acquire the estimations of height for a whole domain meanwhile the number of places are minimized.

A set $S \subseteq V(G)$ is a dominating set of graph G if every vertex of $V(G) - S$ is adjacent to atleast one vertex of set S. Domination number is the cardinality of the smallest dominating set of G which is denoted by $\gamma(G)$. A dominating set of cardinality $\gamma(G)$ is called a $\gamma-$set.

In 2016, motivated by the study of domination and split domination, Kavaturi and Vangipuram introduced a new parameter on domination which is said to be annihilator dominating set (ADS) and investigated some results of the annihilator domination number for some standard graphs like : $K_{m,n}$, P_n, C_n, W_n, S_n etc. A dominating set of graph G is called ADS if its induced subgraph is containing only isolated vertices. The annihilator domination number of G, denoted by $\gamma_a(G)$ is the minimum cardinality of ADS. Annihilator domination concept is widely applicable in real life situations like field of agriculture (control of pests) and the department of defense (armaments, weapons and fundamental commodities)[3].

In graph theory, many operations will be used like as: normal product, Cartesian product, tensor product, Corona product, etc. but here bring to light on the "Tensor Product".The tensor product of graphs G and H signified by $G \times H$ is a graph with vertex set $V = V(G) \times V(H)$ (where $\times$ denotes the tensor product of sets) and edge $\{(u, v), (u', v')\} \in E$ whenever $(u, u') \in E(G)$ and $(v, v') \in E(H)$.

In 1997, Kulli and Janakiram introduced the split domination in graphs [5]. Some important Vizing - like conjecture for direct product graphs were analyzed by Klavžar and Zmazek [4]. In 2007, Brešar et al. constructed a new approach to obtain dominating set of direct product of graphs [1]. Sampathkumar [7] gave the important results over Kronecker product of graphs. In 1981, Laskar and Walikar [6] proposed the various interesting results on domination related concept on graph theory and took different approaches to the problem. In this paper, we provide an approach to finding ADS of tensor product of path graphs and other graphs. It is based on the cardinality of the smallest annihilator dominating set of G. This paper is organized as follows: In Section 2 contains the main results of the paper and its graphs shown by example. In the Section 3, discussion and conclusion is given.

Theorem 1.1. [3] *In a graph G, $\gamma(G) \leq \gamma_s(G) \leq \gamma_a(G)$ where $\gamma_s(G)$ be split domination number and $\gamma_a(G)$ be annihilator domination number.*

Theorem 1.2. [3] *If P_n be a path with n-vertices, then $\gamma_a(P_n) = \lfloor n/2 \rfloor$, where $\lfloor X \rfloor$ represents the greatest integer less than or equal to X.*

2. Annihilator domination of tensor product of path graphs

In this section, we determine the values of $\gamma_a(P_m \times P_n)$ when $m \geq 2$. Since $P_1 \times P_n \cong P_n$, we have:

Theorem 2.1. *For $n \geq 2$, let P_n be a path with n - vertices, then we have*
$\gamma_a(P_2 \times P_n) = 2\lfloor \frac{n}{2} \rfloor$.

Proof. Let $V(P_2) = \{u_i,\ i = 1, 2\}$, $V(P_n) = \{v_j,\ j = 1, 2, ..., n\}$,
$S_i = \{(u_i,\ v_j) \in V(P_2 \times P_n) \mid j = 1, 2, ...n,\ i = 1, 2\}$ and $(u_i,\ v_j) \rightarrow (u_i',\ v_j')$ if $u_i = u_i'$ or $v_j = v_j'$, since $\gamma_a(P_n) = \lfloor \frac{n}{2} \rfloor$.
Assume that the minimum annihilator dominating set of $(P_2 \times P_n)$ be

$$D = \{(u_1,\ v_{2k}) \cup (u_2,\ v_{2k})\} \text{ where } 1 \leq k \leq \lfloor \tfrac{n}{2} \rfloor.$$

It is clear that $|D| = 2\lfloor \frac{n}{2} \rfloor$.
We can check that D is annihilator dominating set for $(P_2 \times P_n)$, see Figure 1. If the

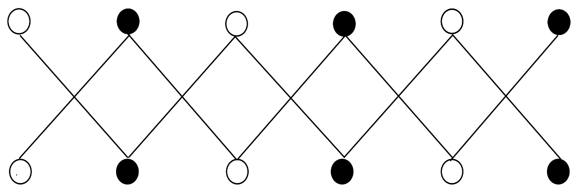

FIGURE 1. An annihilator dominating set for $(P_2 \times P_6)$

vertices of D will be deleted from $(P_2 \times P_n)$ then the resulting graph $< (V - D) >$ contains only isolated vertices. See Figure 2 for $(P_2 \times P_6)$.

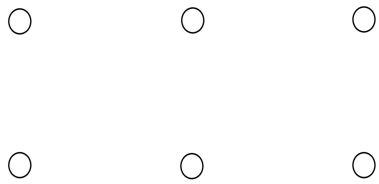

FIGURE 2. The induced subgraph $< (V - D) >$ for $(P_2 \times P_6)$

This completes the proof of this theorem. □

Theorem 2.2. *For $n \geq 2$, let P_n be a path with n - vertices, then we have* $\gamma_a(P_3 \times P_n) = n$.

Proof. Let $V(P_3) = \{u_i,\ i = 1, 2, 3\}$, $V(P_n) = \{v_j,\ j = 1, 2, ..., n\}$,
$S_i = \{(u_i,\ v_j) \in V(P_3 \times P_n) \mid j = 1, 2, ..., n,\ i = 1, 2, 3\}$ and $(u_i,\ v_j) \rightarrow (u_i',\ v_j')$ if $u_i = u_i'$ or $v_j = v_j'$, since $\gamma_a(P_n) = \lfloor \frac{n}{2} \rfloor$.
Assume that the minimum annihilator dominating set of $(P_3 \times P_n)$ be

$$D = \{(u_2,\ v_k)\} \text{ where } 1 \leq k \leq n.$$

It is clear that $|D| = n$.
We can check that D is annihilator dominating set for $(P_3 \times P_n)$, see Figure 3.

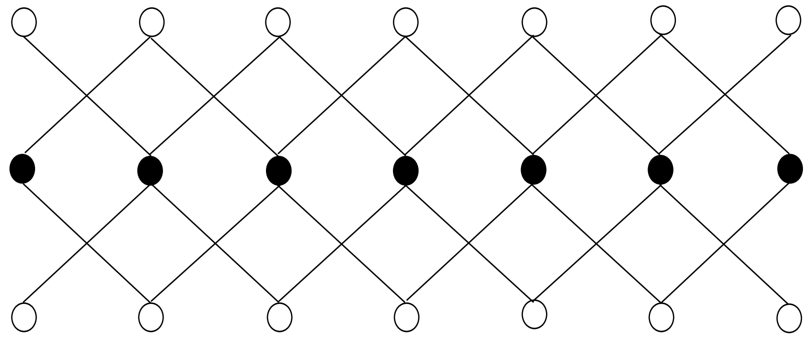

FIGURE 3. An annihilator dominating set for $(P_3 \times P_7)$

If the vertices of D will be deleted from $(P_3 \times P_n)$ then the resulting graph $< (V - D) >$ contains only isolated vertices. See Figure 4 for $(P_3 \times P_7)$.

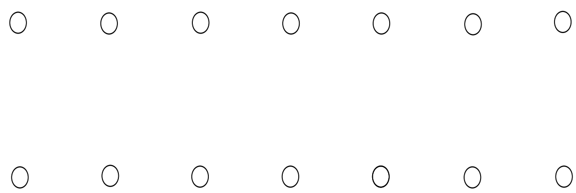

FIGURE 4. The induced subgraph $< (V - D) >$ for $(P_3 \times P_7)$

This completes the proof of this theorem. $\square$

Theorem 2.3. *For $n \geq 2$, let P_n be a path with n - vertices, then we have* $\gamma_a(P_4 \times P_n) = 4\lfloor \frac{n}{2} \rfloor$.

Proof. Let $V(P_4) = \{u_i,\ i = 1, 2, 3, 4\}$, $V(P_n) = \{v_j,\ j = 1, 2, ..., n\}$,
$S_i = \{(u_i,\ v_j) \in V(P_4 \times P_n) \mid j = 1, 2, ..., n,\ i = 1, 2, 3, 4\}$ and $(u_i,\ v_j) \to (u'_i,\ v'_j)$ if $u_i = u'_i$ or $v_j = v'_j$, since $\gamma_a(P_n) = \lfloor \frac{n}{2} \rfloor$.
Assume that the minimum annihilator dominating set of $(P_4 \times P_n)$ be

$$D = \{(u_1,\ v_{2k}) \cup (u_2,\ v_{2k}) \cup (u_3,\ v_{2k}) \cup (u_4,\ v_{2k})\} \text{ where } 1 \leq k \leq \lfloor \tfrac{n}{2} \rfloor.$$

It is clear that $|D| = 4\lfloor \frac{n}{2} \rfloor$.
We can check that D is annihilator dominating set for $(P_4 \times P_n)$, see Figure 5.

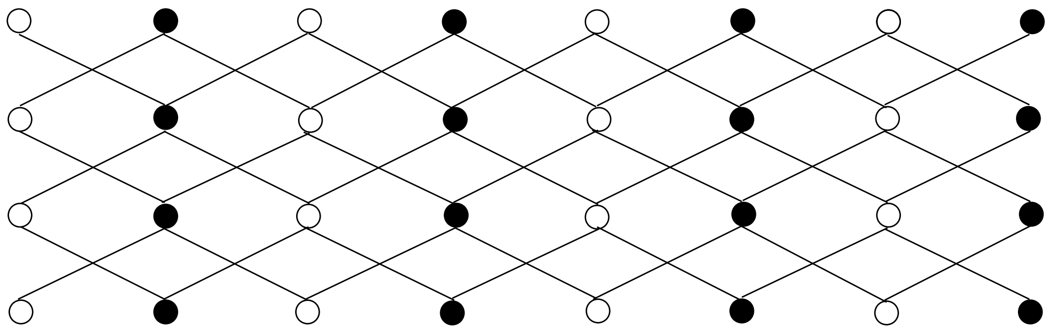

Figure 5. An annihilator dominating set for $(P_4 \times P_8)$

If the vertices of D will be deleted from $(P_4 \times P_n)$ then the resulting graph $< (V - D) >$ contains only isolated vertices. This completes the proof of this theorem.

□

Theorem 2.4. *Let P_n be a path with n - vertices, for every $n \geq 2$ and P_m be also a path with m - vertices, for every $m \equiv 0 \ (mod\ 2)$, then we have*

$$\gamma_a(P_m \times P_n) = \begin{cases} \frac{mn}{2} & \text{if } n \equiv 0 \ (mod\ 2) \\ \frac{m(n-1)}{2} & \text{if } n \equiv 1 \ (mod\ 2). \end{cases}$$

Proof. Let $V(P_m) = \{u_i,\ i = 1, 2, 3, ..., m\}$, $V(P_n) = \{v_j,\ j = 1, 2, ..., n\}$,
$S_i = \{(u_i,\ v_j) \in V(P_m \times P_n) \mid j = 1, 2, ..., n,\ i = 1, 2, 3, ..., m\}$ and $(u_i,\ v_j) \to (u'_i,\ v'_j)$ if $u_i = u'_i$ or $v_j = v'_j$, since $\gamma_a(P_n) = \lfloor \frac{n}{2} \rfloor$.
According to the values of n, we consider following two cases:

Case I : If $m \equiv 0 \ (mod\ 2)$ (say $2r$) and $n \equiv 0 \ (mod\ 2)$ (say $2s$) where $m, n \geq 2$.
Assume that the minimum annihilator dominating set of $(P_m \times P_n)$ be

$$D = \{(u_{2k_1-1},\ v_{2k_2-1})\ , (u_{2k_1},\ v_{2k_2-1})\} \text{ where } 1 \leq k_1 \leq m/2\ ;\ 1 \leq k_2 \leq n/2.$$

It is clear that $|D| = \frac{mn}{2}$.
We can check that D is annihilator dominating set for $(P_m \times P_n)$, see Figure 6.
If the vertices of D will be deleted from $(P_m \times P_n)$ then the resulting graph $< (V - D) >$ contains only isolated vertices.

Case II : If $m \equiv 0 \ (mod\ 2)$ (say $2r$) and $n \equiv 1 \ (mod\ 2)$ (say $2s + 1$) where $m, n \geq 2$.

Assume that the minimum annihilator dominating set of $(P_m \times P_n)$ be

$$D = \{(u_{2k_1-1},\ v_{2k_2})\ , (u_{2k_1},\ v_{2k_2})\} \text{ where } 1 \leq k_1 \leq m/2\ ;\ 1 \leq k_2 \leq n/2.$$

It is clear that $|D| = \frac{m(n-1)}{2}$.
We can check that D is annihilator dominating set for $(P_m \times P_n)$, see Figure 7.
If the vertices of D will be deleted from $(P_m \times P_n)$ then the resulting graph $< (V - D) >$ contains only isolated vertices.

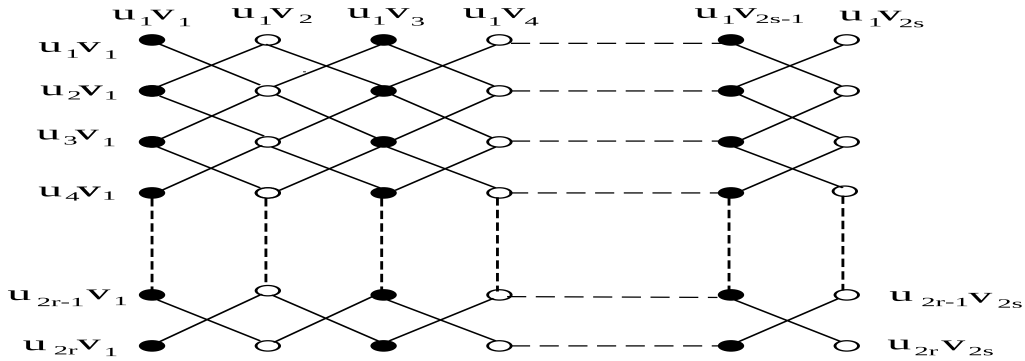

FIGURE 6. An annihilator dominating set for $(P_m \times P_n)$ (Case I)

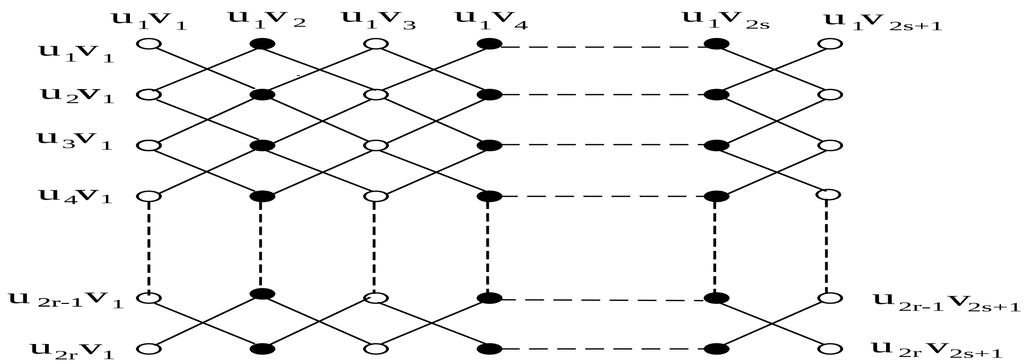

FIGURE 7. An annihilator dominating set for $(P_m \times P_n)$ (Case II)

This completes the proof of this theorem. $\square$

Theorem 2.5. *Let P_n be a path with n - vertices, for every $n \geq 2$ and P_m be also a path with m - vertices, for every $m \equiv 1$ (mod 2), then*

$$\gamma_a(P_m \times P_n) = \begin{cases} \frac{(m-1)n}{2} & \text{if } n \equiv 0 \ (mod\ 2) \\ m\lfloor\frac{n}{2}\rfloor & \text{if } n = 3,\ 5,\ m \neq 3 \\ n\lfloor\frac{m}{2}\rfloor & \text{if } n = odd,\ n \geq 7. \end{cases}$$

Proof. Let $V(P_m) = \{u_i,\ i = 1, 2, 3, ..., m\}$, $V(P_n) = \{v_j,\ j = 1, 2, ..., n\}$, $S_i = \{(u_i,\ v_j) \in V(P_m \times P_n) \mid j = 1, 2, ..., n,\ i = 1, 2, 3, ..., m\}$ and $(u_i,\ v_j) \to (u_i',\ v_j')$ if $u_i = u_i'$ or $v_j = v_j'$, since $\gamma_a(P_n) = \lfloor\frac{n}{2}\rfloor$, see Figure 8.

Following three cases may arise according to the values of n.

Case I : If $m \equiv 1$ $(mod\ 2)$ and $n \equiv 0$ $(mod\ 2)$ where $m, n \geq 2$.
Assume that the minimum annihilator dominating set of $(P_m \times P_n)$ be

$$D = \{(u_{2k_1},\ v_{2k_2-1}), (u_{2k_1},\ v_{2k_2})\} \text{ where } 1 \leq k_1 \leq \lfloor m/2\rfloor\ ;\ 1 \leq k_2 \leq \lfloor n/2\rfloor.$$

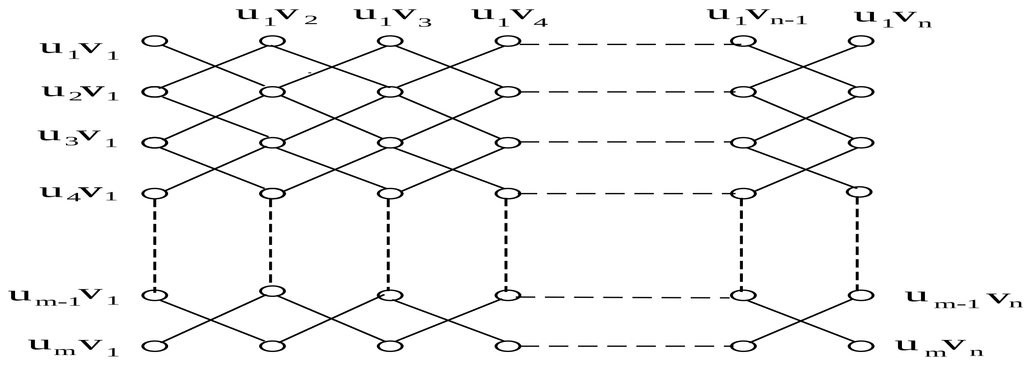

FIGURE 8. $(P_m \times P_n)$

It is clear that $|D| = \frac{(m-1)n}{2}$.
We can check that D is annihilator dominating set for $(P_m \times P_n)$,
If the vertices of D will be deleted from $(P_m \times P_n)$ then the resulting graph $< (V - D) >$ contains only isolated vertices.
Case II : If $m \equiv 1 \ (mod\ 2)$, $m \neq 3$ and $n = 3,\ 5$.
Assume that the minimum annihilator dominating set of $(P_m \times P_n)$ be

$$D = \{(u_{k_1},\ v_{2k_2})\} \text{ where } 1 \leq k_1 \leq m\ ;\ 1 \leq k_2 \leq \lfloor n/2 \rfloor.$$

It is clear that $|D| = m\lfloor \frac{n}{2} \rfloor$.
We can check that D is annihilator dominating set for $(P_m \times P_n)$.
If the vertices of D will be deleted from $(P_m \times P_n)$ then the resulting graph $< (V - D) >$ contains only isolated vertices.
Case III : If $m \equiv 1 \ (mod\ 2)$ and $n = odd,\ n \geq 7$.
Assume that the minimum annihilator dominating set of $(P_m \times P_n)$ be

$$D = \{(u_{2k_1},\ v_{k_2})\} \text{ where } 1 \leq k_1 \leq \lfloor n/2 \rfloor\ ;\ 1 \leq k_2 \leq n.$$

It is clear that $|D| = n\lfloor \frac{m}{2} \rfloor$.
We can check that D is annihilator dominating set for $(P_m \times P_n)$.
If the vertices of D will be deleted from $(P_m \times P_n)$ then the resulting graph $< (V - D) >$ contains only isolated vertices. This completes the proof of this theorem. □

Corollary 2.1. *Let P_n be a path with n - vertices and P_m be a path with m - vertices. Then $\gamma_a(P_m \times P_n) = m\left\lfloor \frac{n}{2} \right\rfloor = m\gamma_a(P_n)$, where m is even and $n \geq 2$.*

Proof. Let the vertices of $V(P_m) = \{u_i,\ i = 1, 2, 3, ..., m\}$, $V(P_n) = \{v_j,\ j = 1, 2, ..., n\}$. Since, we have already proved in Theorem 2.4 that $\gamma_a(P_m \times P_n) = \frac{mn}{2}$, where m, n are both even and $\frac{m(n-1)}{2}$ where m is even and n is odd. By combining above results, we get

$$\begin{aligned} \gamma_a(P_m \times P_n) &= m\left\lfloor \frac{n}{2} \right\rfloor \\ &= m\gamma_a(P_n) \qquad \text{(by Theorem 1.2)} \end{aligned}$$

□

Theorem 2.6. *If $m, n \geq 2$, then $\gamma_a(P_m \times P_n) \leq 3\gamma_a(P_m)\gamma_a(P_n)$.*

Proof. Let the vertices of $V(P_m) = \{u_i,\ i = 1, 2, 3, ..., m\}$, $V(P_n) = \{v_j,\ j = 1, 2, ..., n\}$.
Case I : If m is even
By Corollary 2.1, $\gamma_a(P_m \times P_n) = m\left\lfloor \frac{n}{2} \right\rfloor$. We know that

$$\frac{m}{3} \leq \left\lfloor \frac{m}{2} \right\rfloor \Rightarrow m \leq 3\left\lfloor \frac{m}{2} \right\rfloor$$

By combining above results, we get

$$\gamma_a(P_m \times P_n) \leq 3\left\lfloor \frac{m}{2} \right\rfloor \left\lfloor \frac{n}{2} \right\rfloor$$
$$\gamma_a(P_m \times P_n) \leq 3\gamma_a(P_m)\gamma_a(P_n)$$

Case II : If m is odd
By Theorem 2.5, $\gamma_a(P_m \times P_n) = \frac{(m-1)n}{2}$ *or* $m\left\lfloor \frac{n}{2} \right\rfloor$ *or* $n\left\lfloor \frac{m}{2} \right\rfloor$
Subcase I : $\gamma_a(P_m \times P_n) = (m-1)\frac{n}{2}$. We know that

$$\frac{m-1}{3} \leq \left\lfloor \frac{m}{2} \right\rfloor$$
$$m - 1 \leq 3\left\lfloor \frac{m}{2} \right\rfloor$$

and
$$\frac{n}{2} = \left\lfloor \frac{n}{2} \right\rfloor \qquad (n \textit{ is even})$$

By combining above results, we get

$$\gamma_a(P_m \times P_n) \leq 3\gamma_a(P_m)\gamma_a(P_n)$$

.

Subcase II : $\gamma_a(P_m \times P_n) = m\left\lfloor \frac{n}{2} \right\rfloor$. Also $\frac{m}{3} \leq \left\lfloor \frac{m}{2} \right\rfloor$. Using this inequality, we get $\gamma_a(P_m \times P_n) \leq 3\gamma_a(P_m)\gamma_a(P_n)$.

Subcase III : $\gamma_a(P_m \times P_n) = n\left\lfloor \frac{m}{2} \right\rfloor$. Also $\frac{n}{3} \leq \left\lfloor \frac{n}{2} \right\rfloor$. Using this inequality, we get $\gamma_a(P_m \times P_n) \leq 3\gamma_a(P_m)\gamma_a(P_n)$. Hence the result follows. □

Theorem 2.7. *If $m, n \geq 2$, then $\gamma_a(P_m \times P_n) \geq 2\gamma_a(P_m)\gamma_a(P_n)$.*

Proof. Let the vertices of $V(P_m) = \{u_i,\ i = 1, 2, 3, ..., m\}$, $V(P_n) = \{v_j,\ j = 1, 2, ..., n\}$.
Case I : If m is even
By Corollary 2.1, $\gamma_a(P_m \times P_n) = m\left\lfloor \frac{n}{2} \right\rfloor$. We know that

$$\frac{m}{2} = \left\lfloor \frac{m}{2} \right\rfloor \Rightarrow m = 2\left\lfloor \frac{m}{2} \right\rfloor \qquad \text{(m is even)}$$

By combining above results, we get

$$\gamma_a(P_m \times P_n) = 2\left\lfloor \frac{m}{2} \right\rfloor \left\lfloor \frac{n}{2} \right\rfloor$$
$$\gamma_a(P_m \times P_n) = 2\gamma_a(P_m)\gamma_a(P_n)$$

Case II : If m is odd
By Theorem 2.5, $\gamma_a(P_m \times P_n) = \frac{(m-1)n}{2}$ *or* $m\left\lfloor \frac{n}{2} \right\rfloor$ *or* $n\left\lfloor \frac{m}{2} \right\rfloor$
Subcase I : $\gamma_a(P_m \times P_n) = (m-1)\frac{n}{2}$. We know that

$$\frac{m-1}{2} = \left\lfloor \frac{m}{2} \right\rfloor$$
$$m - 1 = 2\left\lfloor \frac{m}{2} \right\rfloor$$

and $\frac{n}{2} = \left\lfloor \frac{n}{2} \right\rfloor$ $(n\ is\ even)$

By combining above results, we get

$$\gamma_a(P_m \times P_n) = (m-1)\frac{n}{2} = 2\left\lfloor \frac{m}{2} \right\rfloor \left\lfloor \frac{n}{2} \right\rfloor = 2\gamma_a(P_m)\gamma_a(P_n)$$

.

Subcase II : $\gamma_a(P_m \times P_n) = m\left\lfloor \frac{n}{2} \right\rfloor$. Also $\frac{m}{2} \geq \left\lfloor \frac{m}{2} \right\rfloor$. Using this inequality, we get $\gamma_a(P_m \times P_n) \geq 2\gamma_a(P_m)\gamma_a(P_n)$.

Subcase III : $\gamma_a(P_m \times P_n) = n\left\lfloor \frac{m}{2} \right\rfloor$. Also $\frac{n}{2} \geq \left\lfloor \frac{n}{2} \right\rfloor$. Using this inequality, we get $\gamma_a(P_m \times P_n) \geq 2\gamma_a(P_m)\gamma_a(P_n)$. Hence the result follows. □

Theorem 2.8. *If k is even and $n \geq 2$, then we get $\gamma_a(P_{2k} \times P_n) = 2\gamma_a(P_k \times P_n)$.*

Proof. By Corollary 2.1

$$\begin{aligned}\gamma_a(P_{2k} \times P_n) &= 2k\left\lfloor \frac{n}{2} \right\rfloor \\ &= 2\gamma_a(P_k \times P_n)\end{aligned}$$

Hence the result follows.

□

3. Conclusions

In this paper, an approach is presented to find the annihilator domination number of tensor product of P_m and P_n, $m, n \geq 2$. Our main emphasis is to obtain the annihilator domination number which are related to tensor product of path graphs. The main result gives the value of $\gamma_a(P_m \times P_n)$ for different cases. Further, we investigated some lower and upper bounds for annihilator domination number of tensor product of path graphs.

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