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SOLUTION OF CONSTANT COEFFIENTS PARTIAL DERIVATIVE EQUATIONS WITH ELZAKI TRANSFORM METHOD

M. DUZ[1], T. M. ELZAKI[2], §

ABSTRACT. In this study, we apply Elzaki Transform method, which is the modified version of Laplace and Sumudu transforms, for general n.th order constant coefficients partial diferential equations. With this method we can solve all the equations that can be solved by the method of undeterminated coefficients.That is, this method is an alternative to the undetermined coefficients method.

Keywords: Elzaki transform.

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1. Introduction

Partial differential equations are used in many areas of engineering and basic sciences. For example, the heat equation, the wave equation and the Laplace equation are some of the well known partial differential equations used in these fields. There are some methods for solution of partial differential equations. Lagrange method, undetermined coefficients method, inverse operator method are some of the methods used to and special solutions of constant coefficients partial differential equations. Apart from these methods, partial differential equations can also be solved by means of Adomian decomposition, differential transformation, variational iteration methods, etc. [8, 9, 10, 11]. These equations also can be solved with aid of integral transforms such as Laplace transform, Fourier transform. In this paper, we have studied to obtain a formula for a special solution of in the most general case nth order partial differential equation which is two-variable, constant-coefficient, non-homogeneous. We have found the solution with Elzaki Transform Method.

The Elzaki transform, method which is used in several areas of mathematics is an integral transform. We can solve linear differential equations with Elzaki transform [1, 2, 3]. Moreover; integral equations and integro differential equations can be solved Elzaki Transform Method [6]. This method can not be suitable for solution of nonlinear differential because of nonlinear terms. However; nonlinear differential equations can be solved by using Elzaki transform aid with differential transform method and homotopy perturbation method [4, 5]. This paper is organized as follows: in Section 2, basic definitions and

[1] Department of Mathematics, Faculty of Science, Karabuk University, Karabuk, Turkey.
e-mail: mduz@karabuk.edu.tr; ORCID: https://orcid.org/0000-0003-2387-4045.
[2] Department of Mathematics, Faculty of Sciences and Arts-Alkamil, University of Jeddah, Kingdom of Saudi Arabia.
e-mail: tarig.alzaki@gmail.com; ORCID: https://orcid.org/0000-0002-6946-9267.
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theorems are given. In Section 3, we get a formalization to solve the constant coeffients partial differential equations nth order and some examples.

2. Basic Definitions and Theorems

Definition 2.1. *Let $F(t)$ be a function for $t>0$. Elzaki transform of $F(t)$ is defined as follows:*

$$E(F(t)) = v\int_0^\infty e^{-\frac{t}{v}}.f\,(t)\,dt$$

Theorem 2.1. [1, 5]*Elzaki transforms of some functions are given in following;*

$F(x,t)$	$E(f(t)) = T(x,v)$
$f(x)$	$f(x).v^2$
$f(x).t$	$f(x).v^3$
$f(x).t^n$	$f(x).n!v^{n+2}$
$f(x).e^{at}$	$f(x).\frac{v^2}{1-av}$
$f(x).\cos at$	$f(x).\frac{v^2}{1+a^2v^2}$
$f(x).\sin at$	$f(x).\frac{av^3}{1+a^2v^2}$

Theorem 2.2. [2] *Elzaki transforms of partial derivatives from first order of $f(x,t)$ are following:*

$$\begin{aligned} \textbf{\textit{i)}}\ E\left[\frac{\partial f}{\partial t}\right] &= \frac{1}{v}T(x,s) - vf(x,0), \\ \textbf{\textit{ii)}}\ E\left[\frac{\partial f}{\partial x}\right] &= \frac{\partial T(x,v)}{\partial x}, \end{aligned} \tag{1}$$

where $T(x,v) = E\left[f(x,t)\right]$.

Theorem 2.3. [2] *Elzaki transforms of partial derivatives from second order of $f(x,t)$ are following.*

$$\begin{aligned} \textbf{\textit{i)}}\ E\left[\frac{\partial^2 f}{\partial t^2}\right] &= \frac{1}{v^2}T(x,v) - f(x,0) - v\frac{\partial f}{\partial t}(x,0), \\ \textbf{\textit{ii)}}\ E\left[\frac{\partial^2 f}{\partial x^2}\right] &= \frac{\partial^2 T(x,v)}{\partial x^2}, \\ \textbf{\textit{iii)}}\ E\left[\frac{\partial^2 f}{\partial x \partial t}\right] &= \frac{\partial}{\partial x}\left(\frac{1}{v}T(x,v) - vf(x,0)\right), \end{aligned} \tag{2}$$

where $T(x,v) = E\left[f(x,t)\right]$.

Proof. There are proofs of (i) and (ii) in [2] .We get proof of (iii);

$$\begin{aligned} E\left[\frac{\partial^2 f}{\partial x \partial t}\right] &= v\int_0^\infty e^{-\frac{t}{v}}.\frac{\partial^2 f}{\partial x \partial t}dt \\ &= v.\frac{\partial}{\partial x}\int_0^\infty e^{-\frac{t}{v}}.\frac{\partial f}{\partial t}dt \\ &= \frac{\partial}{\partial x}E\left[\frac{\partial f}{\partial t}\right] \\ &= \frac{\partial}{\partial x}\left(\frac{1}{v}T(x,v) - vf(x,0)\right). \end{aligned}$$

□

Lemma 2.1. *Elzaki transforms of partial derivatives n.th order of $f(x,t)$ are following.*

$$
\begin{aligned}
i) E\left[\frac{\partial^n f}{\partial t^n}\right] &= \frac{1}{v^n}T(x,v) - \frac{1}{v^{n-2}}f(x,0) - \frac{1}{v^{n-3}}\frac{\partial f}{\partial t}(x,0) - ... - \frac{\partial^{n-2} f}{\partial t^{n-2}}(x,0) - v\frac{\partial^{n-1} f}{\partial t^{n-1}}(x,0) \\
ii) E\left[\frac{\partial^n f}{\partial x^n}\right] &= \frac{\partial^n T(x,v)}{\partial x^n} \qquad (3)
\end{aligned}
$$

Proof. i) Let's do proof with induction.

If $n = 1$ than

$$E\left[\frac{\partial f}{\partial t}\right] = \frac{1}{v}T(x,s) - vf(x,0).$$

Lemma is true for $n = 1$.

We assume that lemma is true for $n = k$. Let's

$$E\left[\frac{\partial^k f}{\partial t^k}\right] = \frac{1}{v^k}T(x,v) - \frac{1}{v^{k-2}}f(x,0) - \frac{1}{v^{k-3}}\frac{\partial f}{\partial t}(x,0) - ... - \frac{\partial^{k-2} f}{\partial t^{k-2}}(x,0) - v\frac{\partial^{k-1} f}{\partial t^{k-1}}(x,0)$$

We must show that lemma is true $n = k + 1$.

$$E\left[\frac{\partial^{k+1} f}{\partial t^{k+1}}\right] = v.\int_0^\infty e^{-\frac{t}{v}}.\frac{\partial^{k+1} f}{\partial t^{k+1}}dt$$

If we apply partial integration method than we get that

$$
\begin{aligned}
E\left[\frac{\partial^{k+1} f}{\partial t^{k+1}}\right] &= v\left[\left(\lim_{t\to\infty} e^{-\frac{t}{v}}\frac{\partial^k f}{\partial t^k}\right) - \frac{\partial^k f}{\partial t^k}(x,0) + \frac{1}{v}\int_0^\infty e^{-\frac{t}{v}}.\frac{\partial^k f}{\partial t^k}dt\right] \\
&= \frac{1}{v}E\left[\frac{\partial^k f}{\partial t^k}\right] - v\frac{\partial^k f}{\partial t^k}(x,0) \\
&= \frac{1}{v}\left[\frac{1}{v^k}T(x,v) - \frac{1}{v^{k-2}}f(x,0) - ... - \frac{\partial^{k-2} f}{\partial t^{k-2}}(x,0) - v\frac{\partial^{k-1} f}{\partial t^{k-1}}(x,0)\right] - v\frac{\partial^k f}{\partial t^k}(x,0) \\
&= \frac{1}{v^{k+1}}T(x,v) - \frac{1}{v^{k-1}}f(x,0) - ... - \frac{1}{v}\frac{\partial^{k-2} f}{\partial t^{k-2}}(x,0) - \frac{\partial^{k-1} f}{\partial t^{k-1}}(x,0) - v\frac{\partial^k f}{\partial t^k}(x,0)
\end{aligned}
$$

Thus proof of (i) is completed.

Now let's prove proof of (ii).

If $n = 1$ then from Theorem 2.2

$$E\left[\frac{\partial f}{\partial x}\right] = \frac{\partial T}{\partial x}.$$

Lemma is true for $n = 1$.

We assume that lemma is true for $n = k$. Let's

$$E\left[\frac{\partial^k f}{\partial x^k}\right] = \frac{\partial^k T(x,v)}{\partial x^k}$$

We must show that lemma is true $n = k + 1$.

$$
\begin{aligned}
E\left[\frac{\partial^{k+1} f}{\partial x^{k+1}}\right] &= v.\int_0^\infty e^{-\frac{t}{v}}.\frac{\partial^{k+1} f}{\partial x^{k+1}}dt \\
&= \frac{\partial}{\partial x}\left(v\int_0^\infty e^{-\frac{t}{v}}.\frac{\partial^k f}{\partial x^k}dt\right) \\
&= \frac{\partial}{\partial x}\frac{\partial^k T(x,v)}{\partial x^k} = \frac{\partial^{k+1} T(x,v)}{\partial x^{k+1}}
\end{aligned}
$$

□

Theorem 2.4. *ELzaki transforms of partial derivatives* $(n+m)$*.th order of* $f(x,t)$ *are following.*

$$E\left[\frac{\partial^{n+m}f}{\partial x^n\partial t^m}\right]=\frac{\partial^n}{\partial x^n}\left(\frac{1}{v^m}T(x,v)-\frac{1}{v^{m-2}}f(x,0)-...-\frac{\partial^{m-2}f}{\partial t^{m-2}}(x,0)-v\frac{\partial^{m-1}f}{\partial t^{m-1}}(x,0)\right) \tag{4}$$

Proof.

$$\begin{aligned}E\left[\frac{\partial^{n+m}f}{\partial x^n\partial t^m}\right] &= v.\int_0^\infty e^{-\frac{t}{v}}.\frac{\partial^{n+m}f}{\partial x^n\partial t^m}dt\\ &= v.\frac{\partial^n}{\partial x^n}\int_0^\infty e^{-\frac{t}{v}}.\frac{\partial^m f}{\partial t^m}dt\\ &= \frac{\partial^n}{\partial x^n}E\left[\frac{\partial^m f}{\partial t^m}\right]\\ &= \frac{\partial^n}{\partial x^n}\left(\frac{1}{v^m}T(x,v)-\frac{1}{v^{m-2}}f(x,0)-...-\frac{\partial^{m-2}f}{\partial t^{m-2}}(x,0)-v\frac{\partial^{m-1}f}{\partial t^{m-1}}(x,0)\right)\end{aligned}$$

□

3. Solution of constant coeffients partial derivative equations from nth order

Theorem 3.1. *Let* $A_{i,j}$ *are real constants* $(1\le i\le n, 1\le j\le n, i+j\le n)$*,* $u=u(x,y)$ *is a polynomial of* x,y*. Then a solution of*

$$\begin{aligned}& A_{n,0}\frac{\partial^n u}{\partial x^n}+A_{n-1,1}\frac{\partial^n u}{\partial x^{n-1}\partial y}+A_{n-2,2}\frac{\partial^n u}{\partial x^{n-2}\partial y^2}+...+A_{0,n}\frac{\partial^n u}{\partial y^n}\\ & +A_{n-1,0}\frac{\partial^{n-1}u}{\partial x^{n-1}}+A_{n-2,1}\frac{\partial^{n-1}u}{\partial x^{n-2}\partial y}+A_{n-3,2}\frac{\partial^{n-1}u}{\partial x^{n-3}\partial y^2}+...+A_{0,n-1}\frac{\partial^{n-1}u}{\partial y^{n-1}}\\ & +...+A_{1,0}\frac{\partial u}{\partial x}+A_{0,1}\frac{\partial u}{\partial y}+A_{0,0}u\\ =\ & F(x,y)\end{aligned}$$

$$\frac{\partial^k u}{\partial y^k}(x,0)=f_k(x), 0\le k\le n-1$$

is

$$u=\frac{H(x,v)}{L(D)}.$$

Here

$$\begin{aligned}H(x,v) &= E(F(x,y))\\ &+\sum A_{n-k,k}\frac{\partial^{n-k}}{\partial x^{n-k}}\left[\frac{u(x,0)}{v^{k-2}}+\frac{1}{v^{k-3}}\frac{\partial u}{\partial y}(x,0)+...+\frac{\partial^{k-2}u}{\partial y^{k-2}}(x,0)+v.\frac{\partial^{k-1}u}{\partial y^{k-1}}(x,0)\right]\\ &+\sum A_{n-1-k,k}\frac{\partial^{n-1-k}}{\partial x^{n-1-k}}\left[\frac{u(x,0)}{v^{k-2}}+\frac{1}{v^{k-3}}\frac{\partial u}{\partial y}(x,0)+...+\frac{\partial^{k-2}u}{\partial y^{k-2}}(x,0)+v.\frac{\partial^{k-1}u}{\partial y^{k-1}}(x,0)\right]\\ &+...+\sum A_{1-k,k}\frac{\partial^{1-k}}{\partial x^{1-k}}\left[\frac{u(x,0)}{v^{k-2}}+\frac{1}{v^{k-3}}\frac{\partial u}{\partial y}(x,0)+...+\frac{\partial^{k-2}u}{\partial y^{k-2}}(x,0)+v.\frac{\partial^{k-1}u}{\partial y^{k-1}}(x,0)\right]\end{aligned}$$

$$\begin{aligned} L(D) &= A_{n,0}D^n + \left(\frac{A_{n-1,1}}{v} + A_{n-1,0}\right)D^{n-1} + \left(\frac{A_{n-2,2}}{v^2} + \frac{A_{n-2,1}}{v} + A_{n-2,0}\right)D^{n-2} + ... \\ &+ \left(\frac{A_{1,n-1}}{v^{n-1}} + \frac{A_{1,n-2}}{v^{n-2}} + ... + \frac{A_{1,1}}{v} + A_{1,0}\right)D + \left(\frac{A_{0,n}}{v^n} + \frac{A_{0,n-1}}{v^{n-1}} + ... + \frac{A_{0,1}}{v} + A_{0,0}\right) \end{aligned}$$

Proof.

$$\sum_{k=0}^{n} A_{n-k,k}\frac{\partial^n u}{\partial x^{n-k}\partial y^k} + \sum_{k=0}^{n-1} A_{n-1-k,k}\frac{\partial^{n-1} u}{\partial x^{n-1-k}\partial y^k} + ... + \sum_{k=0}^{1} A_{1-k,k}\frac{\partial u}{\partial x^{1-k}\partial y^k} + A_{0,0}u = F(x,y)$$

We let's use Elzaki transform for above equality.

$$\sum_{k=0}^{n} A_{n-k,k}E\left(\frac{\partial^n u}{\partial x^{n-k}\partial y^k}\right) + \sum_{k=0}^{n-1} A_{n-1-k,k}E\left(\frac{\partial^{n-1} u}{\partial x^{n-1-k}\partial y^k}\right) + ... + A_{0,0}E(u) = E(F(x,y))$$

From Lemma 2.1.

$$\begin{aligned} &\sum_{k=0}^{n} A_{n-k,k}\frac{\partial^{n-k}}{\partial x^{n-k}}\left(\frac{T}{v^k} - \frac{u(x,0)}{v^{k-2}} - \frac{1}{v^{k-3}}u_y(x,0) - ... - \frac{\partial^{k-2}u}{\partial y^{k-2}}(x,0) - v.\frac{\partial^{k-1}u}{\partial y^{k-1}}(x,0)\right) \\ &+ \sum_{k=0}^{n-1} A_{n-1-k,k}\frac{\partial^{n-1-k}}{\partial x^{n-k}}\left(\frac{T}{v^k} - \frac{u(x,0)}{v^{k-2}} - \frac{1}{v^{k-3}}u_y(x,0) - ... - \frac{\partial^{k-2}u}{\partial y^{k-2}}(x,0) - v.\frac{\partial^{k-1}u}{\partial y^{k-1}}(x,0)\right) \\ &+ ... + A_{1,0}\frac{\partial T}{\partial x} + A_{0,1}\left(\frac{T}{v} - v.u(x,0)\right) + A_{0,0}T \\ = \; & E(F(x,y)) \end{aligned}$$

From above equality, we can write following equality

$$\begin{aligned} &\left[A_{n,0}D^n + \left(\frac{A_{n-1,1}}{v} + A_{n-1,0}\right)D^{n-1} + ... + \left(\frac{A_{0,n}}{v^n} + \frac{A_{0,n-1}}{v^{n-1}} + ... + \frac{A_{0,1}}{v} + A_{0,0}\right)\right]T \\ = \; & E(F(x,y)) + \sum_{k=0}^{n} A_{n-k,k}\frac{\partial^{n-k}}{\partial x^{n-k}}\left[\frac{u(x,0)}{v^{k-2}} + \frac{1}{v^{k-3}}\frac{\partial u}{\partial y}(x,0) + ... + \frac{\partial^{k-2}u}{\partial y^{k-2}}(x,0) + v.\frac{\partial^{k-1}u}{\partial y^{k-1}}(x,0)\right] \\ &+ \sum_{k=0}^{n-1} A_{n-1-k,k}\frac{\partial^{n-1-k}}{\partial x^{n-1-k}}\left[\frac{u(x,0)}{v^{k-2}} + \frac{1}{v^{k-3}}\frac{\partial u}{\partial y}(x,0) + ... + \frac{\partial^{k-2}u}{\partial y^{k-2}}(x,0) + v.\frac{\partial^{k-1}u}{\partial y^{k-1}}(x,0)\right] \\ &+ ... + \sum_{k=0}^{1} A_{1-k,k}\frac{\partial^{1-k}}{\partial x^{1-k}}\left[\frac{u(x,0)}{v^{k-2}} + \frac{1}{v^{k-3}}\frac{\partial u}{\partial y}(x,0) + ... + \frac{\partial^{k-2}u}{\partial y^{k-2}}(x,0) + v.\frac{\partial^{k-1}u}{\partial y^{k-1}}(x,0)\right] \end{aligned}$$

Therefore,

$$\begin{aligned} H(x,v) &= E(F(x,y)) + \sum_{k=0}^{n} A_{n-k,k}\frac{\partial^{n-k}}{\partial x^{n-k}}\left[\frac{u(x,0)}{v^{k-2}} + \frac{1}{v^{k-3}}\frac{\partial u}{\partial y}(x,0) + ... + v.\frac{\partial^{k-1}u}{\partial y^{k-1}}(x,0)\right] \\ &+ \sum_{k=0}^{n-1} A_{n-1-k,k}\frac{\partial^{n-1-k}}{\partial x^{n-1-k}}\left[\frac{u(x,0)}{v^{k-2}} + \frac{1}{v^{k-3}}\frac{\partial u}{\partial y}(x,0) + ... + \frac{\partial^{k-2}u}{\partial y^{k-2}}(x,0) + v.\frac{\partial^{k-1}u}{\partial y^{k-1}}(x,0)\right] \\ &+ ... + \sum_{k=0}^{1} A_{1-k,k}\frac{\partial^{1-k}}{\partial x^{1-k}}\left[\frac{u(x,0)}{v^{k-2}} + \frac{1}{v^{k-3}}\frac{\partial u}{\partial y}(x,0) + ... + \frac{\partial^{k-2}u}{\partial y^{k-2}}(x,0) + v.\frac{\partial^{k-1}u}{\partial y^{k-1}}(x,0)\right] \end{aligned}$$

$$L(D) = A_{n,0}D^n + \left(\frac{A_{n-1,1}}{v} + A_{n-1,0}\right)D^{n-1} + \left(\frac{A_{n-2,2}}{v^2} + \frac{A_{n-2,1}}{v} + A_{n-2,0}\right)D^{n-2}$$
$$+ ... + \left(\frac{A_{1,n-1}}{v^{n-1}} + \frac{A_{1,n-2}}{v^{n-2}} + ... + \frac{A_{1,1}}{v} + A_{1,0}\right)D + \left(\frac{A_{0,n}}{v^n} + \frac{A_{0,n-1}}{v^{n-1}} + ... + \frac{A_{0,1}}{v} + A_{0,0}\right)$$

$$T(x,v) = \frac{H(x,v)}{L(D)}$$

$u(x,y) = E^{-1}[T(x,v)]$. □

Conclusion 3.1. *Let* A, B, C *are real constants,* $u = u(x,y)$ *is a polynomial of* x, y *. Then a solution of*

$$A\frac{\partial u}{\partial x} + B\frac{\partial u}{\partial y} + Cu = F(x,y)$$
$$u(x,0) = f(x)$$

is

$$u = E^{-1}\left[\frac{1}{e^{\frac{x}{A}\left(\frac{B}{v}+c\right)}}\int \left(\frac{F^*}{A} + \frac{B}{A}v.f(x)\right)e^{\frac{x}{A}\left(\frac{B}{v}+c\right)}dx\right] \tag{5}$$

Conclusion 3.2. *Let* A, B, C, E, F, G *be real constants and let* $H(x,y)$ *be a polynomial of* x, y *. Then a solution of*

$$A\frac{\partial^2 u}{\partial x^2} + B\frac{\partial^2 u}{\partial x \partial y} + C\frac{\partial^2 u}{\partial y^2} + E\frac{\partial u}{\partial x} + F\frac{\partial u}{\partial y} + Gu = H(x,y)$$
$$u(x,0) = f(x), u_y(x,0) = g(x)$$

is

$$u(x,y) = E^{-1}(T(x,v)) = E^{-1}\left[\frac{H^*(x,v) + Bv.\frac{df(x)}{dx} + Cf(x) + Cv.g(x)}{A.D^2 + \left(\frac{B}{v} + E\right)D + \left(\frac{C}{v^2} + \frac{F}{v} + G\right)}\right]$$

Example 3.1. *Find the solution of initial value problem in following*

$$\begin{aligned} \frac{\partial u}{\partial x} &= 2\frac{\partial u}{\partial y} - u \\ u(x,0) &= 6.e^{-3x} \end{aligned} \tag{6}$$

Solution 3.1. *From Conclusion 3.1 coefficients of equation are* $A = 1, B = -2, C = -1$ *and* $F(x,y) = 0$*. By Conclusion 3.1 following solution is obtained*

$$\begin{aligned} u &= E^{-1}\left[\frac{1}{e^{\frac{x}{1}\left(\frac{-2}{v}+1\right)}}\int \frac{-2}{1}v.6.e^{-3x}e^{\frac{x}{1}\left(\frac{-2}{v}+1\right)}dx\right] \\ &= E^{-1}\left[-12v.e^{x\left(\frac{2}{v}-1\right)}\int e^{\left(-2-\frac{2}{v}\right)x}dx\right] \\ &= E^{-1}\left[\frac{-12v}{-2-\frac{2}{v}}.e^{-3x}\right] = 6e^{-3x}E^{-1}\left[\frac{v^2}{1+v}\right] \\ &= 6.e^{-3x}.e^{-y} \end{aligned}$$

Example 3.2. [2, 7] *Consider the following Laplace equation* $u_{xx} + u_{yy} = 0$ *with the conditions* $u(x,0) = 0, u_y(x,0) = \cos x$.

Solution 3.2. *From conclusion 3.2 coefficients of equation are* $A = 1, C = 1, B = D = E = F = G = 0, H(x,y) = 0$. *Moreover boundary conditions of equation are* $u(x,0) = f(x) = 0, u_t(x,0) = g(x) = \cos x$. *If these variables and constants are written at solution then solution is obtained as that*

$$\begin{aligned} u(x,y) &= E^{-1}\left(\frac{v.\cos x}{D^2+\frac{1}{v^2}}\right) \\ &= E^{-1}\left[\frac{v^3}{1+v^2D^2}(\cos x)\right] \\ &= E^{-1}\left[v^3\left(1-v^2D^2+v^4D^4-v^6D^6+...\right)(\cos x)\right] \\ &= \cos x.E^{-1}\left(\frac{v^3}{1-v^2}\right) = \frac{\cos x}{2}E^{-1}\left(\frac{v^2}{1-v}-\frac{v^2}{1+v}\right) \\ &= \cos x\left(\frac{e^y-e^{-y}}{2}\right) = \cos x.\sinh y \end{aligned}$$

Example 3.3. [11] *Find the solution of initial value problem in following*

$$\begin{aligned} \frac{\partial^2 u}{\partial x^2} - 16\frac{\partial^2 u}{\partial y^2} &= x^2y^2 \\ u(x,0) &= \frac{4}{45}x^6 \\ u_y(x,0) &= 0 \end{aligned}$$

Solution 3.3. *From Conclusion 3.2 coefficients of equation are* $A = 1, B = 0, C = -16, D = E = F = 0, G = x^2y^2$. *Therefore we can solution of equation as follow:*

$$\begin{aligned} u(x,y) &= E^{-1}\left[\frac{2x^2v^4-\frac{64}{45}x^6}{D^2-\frac{16}{v^2}}\right] \\ &= E^{-1}\left[-\frac{v^2}{16}\left(1+\frac{v^2D^2}{16}+\frac{v^4D^4}{16^2}+\frac{v^6D^6}{16^3}+...\right)\left(2x^2v^4-\frac{64}{45}x^6\right)\right] \\ &= E^{-1}\left[-\frac{v^2}{16}\left(2x^2v^4-\frac{64}{45}x^6+\frac{v^2}{16}\left(4v^4-\frac{64.6.5.x^4}{45}\right)+\frac{v^4}{16^2}\left(-\frac{64.6.5.4.3.x^2}{45}\right)-\frac{v^6.64.6!}{16^3.45}\right)\right] \\ &= E^{-1}\left[\frac{4}{45}v^2x^6+\frac{1}{6}v^4.x^4\right] \\ &= \frac{4}{45}x^6+\frac{y^2x^4}{12} \end{aligned}$$

4. Conclusion

Elzaki transform method is applied for general nth order constant coefficients partial differential equations. It may be concluded that the proposed technique is very powerful and efficient in finding the analytic solutions for a large class of partial differential equations explicitly and efficiency of the proposed method.

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Assist.Prof. Murat DUZ graduated from Department of Mathematics, Kırıkkale University, Kırıkkale, Turkey in 1998. He received his MS degree in Mathematics from Kırıkkale University in 2001 and his PhD from Uludag University in 2006, respectively. During his MS, he worked as a research assistant. He has been a member of Department of Mathematics in Karabuk University, Karabuk, Turkey since 2009. His research interests focus mainly on differential equations, complex functions theory, and special transforms.

Tarig Mohyeldin Elzaki Farah (Ph.D) is a Professor in Applied Mathematics in the Department of Mathematics, Faculty of Sciences and Arts-Alkamil, University of Jeddah, Jeddah-Kingdom of Saudi Arabia. He is reviewer more than 10 Scientific Journals, and Editorial Board Member of the journal; International Research Journal of Electrical and Electronics Engineering (https://premierpublishers.org/irjeee/editorial-board). He has published more than 60 publications. His research interests are applied mathematics, differential equations, integral transforms and complex analysis.