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## TOPSIS for Multi Criteria Decision Making in Octagonal Intuitionistic Fuzzy Environment by Using Accuracy Function

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**Abstract** — Multi Criteria Decision Making (MCDM) enables a strong valid platform in domains where choosing the best of the best among various attributes is quite complicated. This paper provides a suitable methodology for solving MCDM problems in Intuitionistic Fuzzy region. In this paper we shall be dealing with the environment of octagonal intuitionistic fuzzy numbers. These numbers are more suitable to deal with uncertainties than other generalized form of fuzzy numbers. There are ways to solve MCDM in IF environment. Many have used  $\alpha$ -cuts of numbers which are complicated calculations usually ending up with deviation from the results. Despite of solving the problem using  $\alpha$ -cuts, we propose a new ranking technique in the procedure. This ranking technique is called an accuracy function for octagonal intuitionistic fuzzy numbers. Octagonal Intuitionistic fuzzy numbers are introduced along with its membership and non-membership values. For application, a numerical example is solved at the end of this paper.

**Keywords** — Fuzzy Numbers (FN's), Intuitionistic Fuzzy Numbers (IFN's), Octagonal Intuitionistic Fuzzy Number (OIFN's), Accuracy Function (AF), TOPSIS

### 1. Introduction

Multi Criteria Decision Making is based upon formation and designing decision and outlining problems composed of complex multi pattern. The whole purpose is to give decision makers a feasible solution to such problems. Predictably, there does not exist an exclusive optimal answer for such matter and it is mandatory to utilize the choice maker's performance to evaluate and characterize between solutions. MCDM is a dynamic region of research since the 1960's. Different approach has been proposed by distinct scholars to solve the MCDM problems.

The TOPSIS (Technique for order of preference by Similarity to Ideal Solution) is a Multi- Criteria Decision analysis method proposed by [1] which was further extended by [2]. TOPSIS is set upon the concept that the selected alternative should have the minimum distance from Positive Ideal Solution (PIS) and maximum distance from Negative Ideal Solution (NIS).

Fuzzy set theory was proposed by [3] to represent non exact information into a better form. Later, [4-5] gave the idea of Intuitionistic fuzzy set (IFS) as more compact and precise form of fuzzy set. Different types of fuzzy numbers and various actions on them were researched by many researchers. They investigated on various properties and fluctuations of intuitionistic fuzzy numbers and the first property of correlation between these numbers. Intuitionistic fuzzy sets are already proven to be commodious deal with vagueness

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and perplexity. Both the degree of membership and non-membership functions in IFS combined by the sum is less than one. Many researchers used fuzzy numbers in decision making by considering new parameters and present their précised application in MCGDM, consisting of medical and smart phone selections [6-7]. Prediction of games is very curious topic and fuzzy numbers can be used to predict sports is proposed by [8-9]. Soft sets are considered more precise in vague and hesitate environment. Many researchers discussed the applications, considering MCDM problems but in recent, using accuracy function in uncertain and vague environment a generalized TOPSIS is proposed [10]. But still there are some problems which are solved by fuzzy numbers due to their graphical representations. Ranking of optimal solution using octagonal numbers, is also proposed by [11-12]. Fuzzy numbers are used in the problems having fluctuations. Triangular, Trapezoidal, pentagonal numbers are used in uncertain environment to deal with the fluctuations [13-18]. Development of fuzzy to intuitionistic and into neutrosophic and then further divisions of numbers are done by [19-23]. Ye. Worked in intuitionistic environment and developed a new theory to tackle the problems having uncertain environment [24-26]. Nowadays researchers are also focused on the development of new theories to solve MCDM problems. Recently many researches are done in the field of fuzzy numbers but still there was a gap of octagonal numbers [27-31].

Here in this paper, we shall be working on octagonal intuitionistic fuzzy numbers and its Accuracy Function to solve the TOPSIS. Initially the rating of choice is represented as octagonal intuitionistic fuzzy numbers. The Accuracy Function is developed for the decision making applied to TOPSIS method with octagonal intuitionistic fuzzy numbers.

## 2. Preliminaries

### Definition 2.1: Fuzzy Number [FN]

A fuzzy number is generalized form of a real number. It doesn't represent a single value, instead a group of values, where each entity has its membership value between [0,1]. Fuzzy number  $\bar{S}$  is a fuzzy set in  $R$  if it satisfies the given conditions.

- $\exists$  relatively one  $y \in R$  with  $\mu_{\bar{S}}(y) = 1$ .
- $\mu_{\bar{S}}(y)$  is piecewise continuous.
- $\bar{S}$  should be convex and normal.

### Definition 2.2: Triangular Fuzzy Number [TFN]

A Triangle fuzzy number  $\bar{S}$  is denoted by tuples,  $\bar{S}(x) = (\omega_1, \omega_2, \omega_3)$ , where  $\omega_1$ ,  $\omega_2$ , and  $\omega_3$  are real numbers and  $\omega_1 \leq \omega_2 \leq \omega_3$  with membership function defined as,

$$\mu_{\bar{S}}(x) = \begin{cases} \frac{x - \omega_1}{\omega_2 - \omega_1}, & \omega_1 \leq x \leq \omega_2 \\ \frac{\omega_3 - x}{\omega_3 - \omega_2}, & \omega_2 \leq x \leq \omega_3 \\ 0, & \text{otherwise} \end{cases}$$

### Definition 2.3: Octagonal Fuzzy Number [OFN]

A fuzzy number  $S$  is an octagonal fuzzy number denoted by  $\bar{S} = (\omega_1, \omega_2, \omega_3, \omega_4, \omega_5, \omega_6, \omega_7, \omega_8)$  where  $\omega_1, \omega_2, \omega_3, \omega_4, \omega_5, \omega_6, \omega_7$ , and  $\omega_8$  are real numbers and  $\omega_1 \leq \omega_2 \leq \omega_3 \leq \omega_4 \leq \omega_5 \leq \omega_6 \leq \omega_7 \leq \omega_8$  with membership function defined as,

$$\mu_{\bar{S}}(x) = \begin{cases} 0, & x < \omega_1 \\ k \left( \frac{x - \omega_1}{\omega_2 - \omega_1} \right), & \omega_1 \leq x \leq \omega_2 \\ k, & \omega_2 \leq x \leq \omega_3 \\ k + (1 - k) \left( \frac{x - \omega_3}{\omega_4 - \omega_3} \right), & \omega_3 \leq x \leq \omega_4 \\ 1, & \omega_4 \leq x \leq \omega_5 \\ k + (1 - k) \left( \frac{\omega_6 - x}{\omega_6 - \omega_5} \right), & \omega_5 \leq x \leq \omega_6 \\ k, & \omega_6 \leq x \leq \omega_7 \\ k \left( \frac{\omega_8 - x}{\omega_8 - \omega_7} \right), & \omega_7 \leq x \leq \omega_8 \\ 0, & x > \omega_8 \end{cases}$$

with  $0 < k < 1$ .

### 3. Material and Method

Fuzzy numbers are very helpful in problem solving like MCDM and MCGDM. These numbers are proposed here along with accuracy function (AF). An Intuitionistic fuzzy set  $\bar{S}^I(x)$  of  $S$  is defined as set of ordered triples as,

$$\bar{S}^I(x) = \{ \langle x, \mu_{\bar{S}^I}(x), \vartheta_{\bar{S}^I}(x) \rangle \mid x \in S \}$$

Where,  $\mu_{\bar{S}^I}(x), \vartheta_{\bar{S}^I}(x)$  are considered as MF's non- MF's such that  $\mu_{\bar{S}^I}(x), \vartheta_{\bar{S}^I}(x) : S \rightarrow [0,1]$ , and  $0 \leq \mu_{\bar{S}^I}(x) + \vartheta_{\bar{S}^I}(x) \leq 1, \forall x \in S$ .

For every IF set  $\hat{A}$  in  $S$ , if  $\pi_{\hat{A}}(S) = 1 - \mu_{\hat{A}}(S) - \nu_{\hat{A}}(S)$ , then  $\pi_{\hat{A}}(x)$  is called the indeterminacy degree  $[0,1]$ , or hesitancy degree of  $S$  to  $\hat{A}$ .

### 4. Calculations

#### Definition 4.1: Octagonal Intuitionistic Fuzzy Number [OIFN]

A Fuzzy Number denoted by:

$\bar{S}^I = \{(\omega_1, \omega_2, \omega_3, \omega_4, \omega_5, \omega_6, \omega_7, \omega_8), (\omega'_1, \omega'_2, \omega'_3, \omega_4, \omega_5, \omega'_6, \omega'_7, \omega'_8)\}$  where  $\omega_1, \omega_2, \omega_3, \omega_4, \omega_5, \omega_6, \omega_7, \omega_8, \omega'_1, \omega'_2, \omega'_3, \omega'_4, \omega'_5, \omega'_6, \omega'_7,$  and  $\omega'_8$  are real numbers with  $\omega_1 \leq \omega_2 \leq \omega_3 \leq \omega_4 \leq \omega_5 \leq \omega_6 \leq \omega_7 \leq \omega_8$  and  $\omega'_1 \leq \omega'_2 \leq \omega'_3 \leq \omega'_4 \leq \omega'_5 \leq \omega'_6 \leq \omega'_7 \leq \omega'_8$ . Its membership and non-membership functions are given by;

$$\mu_{\bar{S}}(x) = \begin{cases} 0, & x < \omega_1 \\ k \left( \frac{x - \omega_1}{\omega_2 - \omega_1} \right), & \omega_1 \leq x \leq \omega_2 \\ k, & \omega_2 \leq x \leq \omega_3 \\ k + (1 - k) \left( \frac{x - \omega_3}{\omega_4 - \omega_3} \right), & \omega_3 \leq x \leq \omega_4 \\ 1, & \omega_4 \leq x \leq \omega_5 \\ k + (1 - k) \left( \frac{\omega_6 - x}{\omega_6 - \omega_5} \right), & \omega_5 \leq x \leq \omega_6 \\ k, & \omega_6 \leq x \leq \omega_7 \\ k \left( \frac{\omega_8 - x}{\omega_8 - \omega_7} \right), & \omega_7 \leq x \leq \omega_8 \\ 0, & x > \omega_8 \end{cases}$$

with  $0 < k < 1$ .

$$\vartheta_{\bar{s}I}(x) = \begin{cases} 1, & x < \omega'_1 \\ 1 + (1 - k) \left( \frac{\omega'_1 - x}{\omega'_2 - \omega'_1} \right), & \omega'_1 \leq x \leq \omega'_2 \\ k, & \omega'_2 \leq x \leq \omega'_3 \\ k + k \left( \frac{\omega'_3 - x}{\omega'_4 - \omega'_3} \right), & \omega'_3 \leq x \leq \omega'_4 \\ 0, & \omega'_4 \leq x \leq \omega'_5 \\ k \left( \frac{x - \omega'_5}{\omega'_6 - \omega'_5} \right), & \omega'_5 \leq x \leq \omega'_6 \\ k, & \omega'_6 \leq x \leq \omega'_7 \\ k + (1 - k) \left( \frac{x - \omega'_7}{\omega'_8 - \omega'_7} \right), & \omega'_7 \leq x \leq \omega'_8 \\ 1, & x > \omega'_8 \end{cases}$$

where  $0 < k < 1$ .

**Definition 4.2: Accuracy Function**

Let  $\bar{z} = (\mu_0, \vartheta_0)$  be an intuitionistic fuzzy number, then  $H(\bar{z})$  is the Accuracy Function of  $\bar{z}$  given by

$$H(\bar{z}) = \mu_0 + \vartheta_0$$

**4.2.1. Accuracy Function of an Octagonal Intuitionistic Fuzzy Number**

Let  $O_c = \{(\omega_1, \omega_2, \omega_3, \omega_4, \omega_5, \omega_6, \omega_7, \omega_8); (\omega'_1, \omega'_2, \omega'_3, \omega_4, \omega_5, \omega'_6, \omega'_7, \omega'_8)\}$  be an octagonal intuitionistic fuzzy number. Then its Accuracy Function  $H(O_c)$  is given by,

$$H(O_c) = \frac{\omega_1 + \omega_2 + \omega_3 + \omega_4 + \omega_5 + \omega_6 + \omega_7 + \omega_8}{8} + \frac{\omega'_1 + \omega'_2 + \omega'_3 + \omega_4 + \omega_5 + \omega'_6 + \omega'_7 + \omega'_8}{8}$$

**6. TOPSIS Algorithm**

**Step 1. Construction of Decision Matrix**

First of all, a decision matrix  $D_M = [X_{ij}]_{m \times n}$ , where  $i = 1, 2, 3, \dots, m$  and  $j = 1, 2, 3, \dots, n$  comprising of “ $m$ ” alternatives and “ $n$ ” criteria is designed as

$$D_M = \begin{bmatrix} X_{11} & X_{12} & \dots & X_{1n} \\ X_{21} & X_{22} & \dots & X_{2n} \\ \vdots & \vdots & \dots & \vdots \\ X_{m1} & X_{m2} & \dots & X_{mn} \end{bmatrix} \tag{1}$$

$[X_{ij}]_{m \times n}$  represents score of the  $i_{th}$  alternative regarding the  $j_{th}$  criteria. Our environment is intuitionistic fuzzy, so the initial decision matrix would be in IFN.

Whatever the intuitionistic fuzzy score, it can be reduced to crisp value using the accuracy formula for intuitionistic fuzzy number. The final decision matrix obtained in this step would now be in crisp environment after the application of Accuracy Function formula.

**Step 2. Normalization**

Decision Matrix is then normalized to form a normalized decision matrix  $R = [r_{ij}]_{m \times n}$  by

$$r_{ij} = \frac{X_{ij}}{\sqrt{\sum_{i=1}^m X_{ij}^2}} \tag{2}$$

where  $j = 1, 2, 3, \dots, n$ . "R" is the normalized score of decision matrix.

Normalized Decision Matrix thus obtained is of the form

$$R = \begin{bmatrix} r_{11} & \cdots & r_{1n} \\ \vdots & \ddots & \vdots \\ r_{m1} & \cdots & r_{mn} \end{bmatrix} \quad (3)$$

### Step 3. Computation of Weight Matrix

The weights accredited by the decision makers to the criteria are taken as a weight matrix, W. These weights are then used in step 4 for the calculation of WNDM.

$$W = [w_1 \quad w_2 \quad \cdots \quad w_j \quad \cdots \quad w_n]^T \quad (4)$$

where,  $\sum w_j = 1$ .

### Step 4. Computation of Weighted Normalized Decision Matrix

In this step, WNDM  $R' = [r'_{ij}]_{m \times n}$  is calculated by substituting the values of  $r_{ij}$  from matrix 3 and the weights from weight matrix 4 in below equation

$$R' = [r'_{ij}]_{m \times n} = W_j \times R_{ij} \quad (5)$$

Hence, WNDM is given by

$$R' = \begin{bmatrix} r_{11} & \cdots & r_{1n} \\ \vdots & \ddots & \vdots \\ r_{m1} & \cdots & r_{mn} \end{bmatrix} \begin{bmatrix} w_1 \\ \vdots \\ w_n \end{bmatrix} = \begin{bmatrix} w_1 \cdot r_{11} & \cdots & w_n \cdot r_{1n} \\ \vdots & \ddots & \vdots \\ w_1 \cdot r_{m1} & \cdots & w_n \cdot r_{mn} \end{bmatrix} \quad (6)$$

### Step 5. Calculation of PIS and NIS

Positive Ideal Solution:

$$O_j^+ = S_1^+, S_2^+, S_3^+, \dots, S_n^+$$

where  $S_i^+ = \{\max(S_{ij}); j \in J^+, \min(S_{ij}); j \in J^-\}$

Negative Ideal Solution:

$$O_j^- = S_1^-, S_2^-, \dots, S_n^-$$

where  $S_i^- = \{\min(S_{ij}); j \in J^-, \max(S_{ij}); j \in J^-\}$

Here,

$J^+ = \{j = 1, 2, 3, \dots, n; j \text{ linked with benefit criteria}\}$

$J^- = \{j = 1, 2, 3, \dots, n; j \text{ linked with cost criteria}\}$

### Step 6. Determination of separation measure for each Alternative

Separation Measure of each alternative is to be measured from PIS and NIS respectively.

$$T_i^+ = \sqrt{\sum_{j=1}^n (S_j^+ - S_{ij})^2}, \quad i = 1, 2, 3 \dots m \quad (7)$$

$$T_i^- = \sqrt{\sum_{j=1}^n (S_i^- - S_{ij})^2}, \quad i = 1, 2, 3 \dots m \tag{8}$$

**Step 7. Computation of Relative Closeness to Ideal Solution  $C_i$**

For each alternative, Closeness coefficient is calculated by

$$C_i = \frac{T_i^-}{T_i^+ + T_i^-} \tag{9}$$

Where  $0 \leq C_i \leq 1$ ;  $i = 1, 2, 3, m$

**Step 8. Result**

Alternatives get ranked depending upon the closeness coefficient from most beneficial to least value. The alternative possessing highest value of closeness coefficient is then taken into account.

**7. Numerical Analysis**

**Statemen 7.1:** The problem is to exalt safety by mitigating liabilities and focusing on enhancing the safety of system. Here we have to inspect the use and application of MCDM in Safety Assessment. We have three companies ( $A_i = 1,2,3$ ). Four Criteria are  $C_1 =$  Detailed information about crew members and their behavior;  $C_2 =$  planning, preview and scenarios of risk management;  $C_3 =$  Comparison with industry and  $C_4 =$  Cost Control. The following data form 7.1 is constructed.

**Table1.** Initial decision matrix in octagonal intuitionistic fuzzy environment

	$C_1$	$C_2$	$C_3$	$C_4$
$A_1$	{(1,2,3,4,6,8,9,10); (0,2,3,4,6,7,11,13)}	{(4,6,8,10,11,12,14,15); (2,3,4,10,11,13,14,16)}	{(7,8,10,11,13,14,16,18); (5,6,8,11,13,15,17,19)}	{(6,8,10,14,16,17,19,20); (5,7,9,10,14,16,18,19)}
$A_2$	{(3,4,6,8,10,12,14,16); (2,5,7,8,10,11,13,15)}	{(5,6,9,12,15,17,18,20); (3,7,10,12,15,16,19,20)}	{(8,10,12,14,16,18,19,20); (6,7,9,14,16,17,18,19)}	{(9,10,12,13,15,16,18,19); (7,9,11,13,15,16,17,18)}
$A_3$	{(2,6,7,8,9,10,11,12); (1,2,6,8,9,13,14,15)}	{(6,7,9,10,12,13,15,17); (4,5,6,10,12,14,16,18)}	{(1,2,3,4,5,6,7,8); (0,1,2,4,5,6,7,8)}	{(4,7,10,13,16,18,19,20); (3,6,9,13,16,17,18,20)}

**Step1.** By the use of Accuracy Function, we defuzzified the above values into crisp notation given by Table 1.

**Table 2.** Defuzzified decision matrix

■	$C_1$	$C_2$	$C_3$	$C_4$
$A_1$	11.125	19.125	23.875	26
$A_2$	18	25.5	27.875	27.25
$A_3$	16.625	19.625	8.625	26.125

**Step2.** Normalized Decision Matrix is given by Table 3.

**Table 3.** Normalized decision matrix

■	$C_1$	$C_2$	$C_3$	$C_4$
$A_1$	0.413	0.511	0.633	0.567
$A_2$	0.669	0.681	0.739	0.594
$A_3$	0.618	0.524	0.229	0.570
$W$	0.3	0.4	0.1	0.2

**Step 3.** Weights assigned by DM’s to the criteria are given by the matrix;

$$w = \begin{pmatrix} 0.25 \\ 0.15 \\ 0.25 \\ 0.35 \end{pmatrix}^T \tag{10}$$

**Step 4.** Weighted Normalized Decision Matrix is given by Table 4.

**Table 4.** Weighted normalized decision matrix

■	$C_1$	$C_2$	$C_3$	$C_4$
$A_1$	0.124	0.204	0.0631	0.113
$A_2$	0.201	0.272	0.074	0.119
$A_3$	0.185	0.210	0.023	0.114

**Step 5.** Calculation of PIS (Positive Ideal Solution) and NIS (Negative Ideal Solution):

$$A^+ = \{0.201, 0.272, 0.074, 0.113\} \text{ and } A^- = \{0.124, 0.204, 0.023, 0.119\}$$

**Step 6.** Determination of Separation Measure is given by Table 5 and Table 6.

**Table 5.** Separation measure  $T_i^+$

■	$T_i^+$
$A_1$	0.1033
$A_2$	0.006
$A_3$	0.08

**Table 6.** Separation Measure  $T_i^-$

■	$T_i^-$
$A_1$	0.04
$A_2$	0.114
$A_3$	0.061

**Step 7.** Determination of RCC to ideal solution  $C_i^*$

$$A_1 = 0.279$$

$$A_2 = 0.950$$

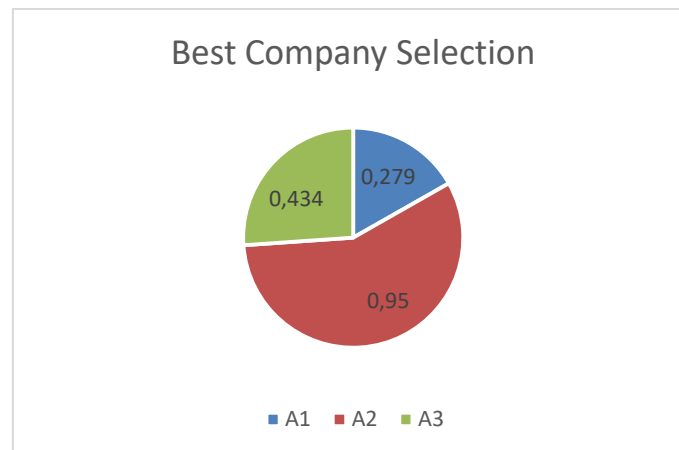
$$A_3 = 0.434$$

**Result:**  $A_2 > A_3 > A_1$

Hence it is concluded that the second company is the best choice.

## 8. Result Discussion

The proposed method and algorithm are applied on MCDM type problem. The problem is to exalt safety by mitigating liabilities and focusing on enhancing the safety of system. Here we have the inspected results as shown below,



Hence it is concluded that the second company is the best choice.

## 9. Conclusion

This research focuses on Multi Criteria Decision Making issues in intuitionistic fuzzy region in which the assessment of choices is represented as Octagonal intuitionistic fuzzy numbers. The Accuracy Function is made for Multi Criteria Decision Making as an alternate to alpha cuts of intuitionistic fuzzy numbers which sum up to complicated calculations. Accuracy Function is applied to TOPSIS technique with OIFNs which reduces the complexity of the environment from complex intuitionistic fuzzy to crisp. The derived results help us conclude that customer can have the safety measures by using their various choice factors.

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