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CHROMATIC WEAK DOMATIC PARTITION IN GRAPHS

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Abstract. In a simple graph G, a subset D of $V(G)$ is called a chromatic weak dominating set if D is a weak dominating set and $\chi(<D>) = \chi(G)$. Similar to domatic partition, chromatic weak domatic partition can be defined. The maximum cardinality of a chromatic weak domatic partition is called the chromatic weak domatic number of G. Bounds for this number are obtained and new results are derived involving chromatic weak domatic number and chromatic weak domination number.

Keywords: Domatic number, Weak domatic number, Chromatic weak domatic number.

AMS Subject Classification: 05C69

1. Introduction

Let $G = (V, E)$ be a simple graph. A subset D of V is said to be a *dominating set* if every vertex $u \in V - D$ is adjacent to some vertex $v \in D$ [5]. Further, D is a *strong dominating set* (*sd*-set) if every vertex $u \in V - D$ is strongly dominated by some v in D [12]. Similarly, we define a *weak dominating set* (*wd*-set) [12]. The *domination number* $\gamma(G)$ of G is the minimum cardinality of a dominating set of G [5]. Analogously, we define the *strong domination number* $\gamma_s(G)$ and the *weak domination number* $\gamma_w(G)$ of G [12].

A subset D of V is said to be a *dom-chromatic set* if D is a dominating set and $\chi(<D>) = \chi(G)$ [10]. Further, D is said to be a *chromatic strong dominating set* (*csd*-set) if D is a strong dominating set and $\chi(<D>) = \chi(G)$ [1]. Analogously, we define a *chromatic weak dominating set* (*cwd*-set) [13]. The *dom-chromatic number* $\gamma_{ch}(G)$ of G is the minimum cardinality of a dom-chromatic set [10]. Similarly, we define the *chromatic strong domination number* $\gamma_s^c(G)$ of G and the *chromatic weak domination number* $\gamma_w^c(G)$ of G [[1], [13]].

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A *domatic partition* (d-partition) of G is a partition of V into dominating sets [2]. A *chromatic strong domatic partition* (csd-partition) of a graph G is a partition of V into csd-sets [1]. The *domatic number* $d(G)$ of G is the maximum cardinality of a d-partition of G [2]. Similarly, we define the *chromatic strong domatic number* $d_s^c(G)$ of G [1]. For notations and terminologies we refer to Harary [3].

Partition of the vertex set into different types of sets has been studied by many authors. For example, proper coloring of vertices leads to partition of the vertex set into independent sets. Partition of the vertex set into irredundant sets has also been studied. Motivated by several types of partition, the strong domatic and weak domatic partitions are studied. In the case of partition into independent sets, the number of maximum independent sets occuring in the partition is also considered. Since domination (strong / weak domination) are super hereditary properties, maximum number of elements in a domatic (strong / weak domatic) partition is aimed. Further properties on dominating (strong / weak dominating) sets can be imposed. This leads to the study of chromatic weak domatic partition of the vertex set.

In this paper, chromatic weak domatic partition for standard graphs are studied, bounds for chromatic weak domatic partition number are obtained and the sum of chromatic weak domatic number of G and $\overline{G}$ is considered.

2. Partition into Chromatic Weak Domatic Sets

In this section, partition of the vertex set into maximum number of weak dominating sets, the chromatic number of whose induced subgraphs coincides with the chromatic number of the graph is studied.

Definition 2.1. [10] *The dom-chromatic number of a graph G, denoted by $d_{ch}(G)$, is defined as the maximum cardinality of the partition of $V(G)$ into dominating sets the chromatic number of whose induced subgraphs coincides with the chromatic number of the graph.*

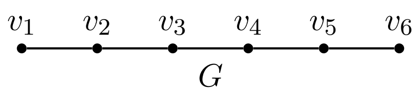
Definition 2.2. *A* **chromatic weak domatic partition** *(cwd-partition) of graph G is a partition of V into chromatic weak dominating sets. The existence of a chromatic weak domatic partition is guaranteed since V is a chromatic weak dominating set. The maximum cardinality of a partition of V into chromatic weak dominating sets is the chromatic weak domatic partition number (cwd-partition number) of G and is denoted by $d_w^c(G)$.*

Proposition 2.1. *For any graph G,*

$$d_w^c(G) \leq d_{ch}(G).$$

Proof. Since every cwd-partition of a graph G is a dom-chromatic partition of G, we have $d_w^c(G) \leq d_{ch}(G)$. Hence the inequality follows. □

Illustration 2.1. *Let G be the graph given below:*



Then d_w^c-set of G is $\{v_1, v_4, v_5, v_6\}$ and hence $d_w^c(G) = 1$. Also d_{ch}-sets of G are $\{v_1, v_4, v_5\}$, $\{v_2, v_3, v_6\}$ and hence $d_{ch}(G) = 2$. Therefore $d_w^c(G) < d_{ch}(G)$. □

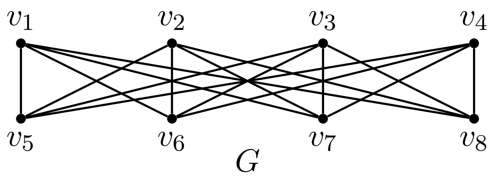
Remark 2.1. *For any graph G,*

$$d_w^c(G) \leq d_s^c(G) \leq d_{ch}(G) \leq d(G) \leq \delta(G) + 1.$$

Proposition 2.2. *For any graph G, $d_w^c(G).\gamma_w^c(G) \le n$.*

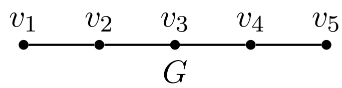
Proof. Let $\{V_1, V_2, \dots, V_k\}$ be a maximum chromatic weak domatic partition of G. Then $d_w^c(G) = k$. Since each V_i is a chromatic weak dominating set, $|V_i| \ge \gamma_w^c(G)$ for each i. Since $V = \cup_{i=1}^k V_i$, $n = \sum_{i=1}^k |V_i| \ge \sum_{i=1}^k \gamma_w^c(G)$. Therefore, $n \ge k\gamma_w^c(G)$. Hence $\gamma_w^c(G).d_w^c(G) \le n$. □

Illustration 2.2. *Let G be the graph given below:*



Then d_w^c-partition is $\{\{v_1, v_5\}, \{v_2, v_6\}, \{v_3, v_7\}, \{v_4, v_8\}\}$ and hence $d_w^c(G) = 4$. Also $D = \{v_1, v_5\}$ is a γ_w^c-set of G. Therefore $\gamma_w^c(G).d_w^c(G) = 8 = |V(G)|$. □

Illustration 2.3. *Let G be the graph given below:*



Then d_w^c-partition is $\{\{v_1, v_4, v_5\}\}$ and hence $d_w^c(G) = 1$. Also $D = \{v_1, v_4, v_5\}$ is a γ_w^c-set of G. Therefore $\gamma_w^c(G).d_w^c(G) = 3 < 5$. □

Remark 2.2. *If a graph G has $\gamma_w^c(G) > \frac{n}{2}$, then $d_w^c(G) = 1$.* □

Observation 2.1. *There exists a graph G for which $d_w^c(G) = \delta(G) + 1$. For instance, $d_w^c(\overline{K_2}) = 1$ and $\delta(\overline{K_2}) = 0$.* □

Definition 2.3. *A graph G is said to be* **chromatic weak domatically full** *if $d_w^c(G) = \delta(G) + 1$.*

Definition 2.4. [5] *A graph G is said to be χ-critical if $\chi(G - v) < \chi(G)$ for any vertex $v \in V(G)$.*

Proposition 2.3. *Let G be χ-critical. Then $\gamma_w^c(G) = |V(G)|$.*

Proof. Obvious. □

Remark 2.3. *The converse of the above Proposition 2.3 need not be true. For instance, let $G = K_{1,n-1}$ where $n \ge 3$. Then $\gamma_w^c(G) = |V(G)|$ but G is not χ-critical.* □

Proposition 2.4. *If G is χ-critical, then $d_w^c(G) = 1$.*

Proof. Since G is χ-critical, V is the only chromatic weak dominating set of G. Therefore $d_w^c(G) = 1$. □

Observation 2.2. *Let G be a graph with $\Delta < n - 1$. Then there exist a graph G such that $\gamma_w^c(G) < \frac{n}{2}$ and $d_w^c(G) = 1$.*

Example 2.1. *Consider the Path P_{14}.*
Then $\gamma_w^c(P_{14}) = 6 < \frac{14}{2}$ but $d_w^c(P_{14}) = 1$. □

3. Cwd-partition number of Some Well Known Graphs

Some of the well-known graphs are the complete graph, star, complete bipartite graph, path, cycle, wheel and fan. The *cwd*-partition number for these graphs are derived in this section.

Theorem 3.1. *If a graph G has pendent vertices, then $d^c_w(G) = 1$.*

Proof. Let $V(G) = \{u_1, u_2, \ldots, u_k, \ldots, u_n\}$ be the vertex set of G. Let $T = \{u_1, u_2, \ldots, u_k\}$ be the set of all pendent vertices of G, where $k \leq n$. Let D be the *cwd*-set of G so that D contains all the pendent vertices and some vertices in $V(G) - T$. Then $|D| \geq |T|$. Thus $V - D$ has no pendent vertices and hence it is not a *wd*-set itself. Therefore $d^c_w(G) = 1$. □

Corollary 3.1. *For a Path P_n, $d^c_w(P_n) = 1$.* □

Corollary 3.2. *For a double star $D_{r,s}$, $d^c_w(D_{r,s}) = 1$.* □

Corollary 3.3. *Let T be a tree. Then $d^c_w(T) = 1$.* □

Proposition 3.1. *For a cycle C_n,*

$$d^c_w(C_n) = \begin{cases} 1 & \text{if } n \text{ is odd} \\ 2 & \text{if } n \text{ is even.} \end{cases}$$

Proof. Let G be a cycle C_n where $V(G) = \{v_1, v_2, \ldots, v_n\}$.
Case 1: n is odd.
In this case, $\gamma^c_w(G) = n > \frac{n}{2}$. By Remark 2.2, $d^c_w(G) = 1$.
Case 2: n is even.
Subcase 2(a): $n \equiv 1(\bmod 3)$
Then $n = 3k+1$. In this case, $\gamma^c_w(G) = \lceil \frac{n}{3} \rceil = k+1$. By Proposition 2.2, $d^c_w(G) \leq 2$. Obviously the sets $D = \{v_1, v_4, \ldots, v_n\}$ and $V - D$ are the *cwd*-sets of C_n. Hence $d^c_w(G) = 2$.
Subcase 2(b): $n \equiv 2(\bmod 3)$
Then $n = 3k+2$. In this case, $\gamma^c_w(G) = \lceil \frac{n}{3} \rceil + 1 = k+2$. By Proposition 2.2, $d^c_w(G) \leq 2$. Obviously the sets $D = \{v_1, v_4, \ldots, v_{n-2}, v_{n-1}\}$ and $V - D$ are the *cwd*-sets of C_n. Hence $d^c_w(G) = 2$.
Subcase 2(c): $n \equiv 0(\bmod 3)$
Then $n = 3k$. In this case, $\gamma^c_w(G) = \lceil \frac{n}{3} \rceil + 1 = k+1$. By Proposition 2.2, $d^c_w(G) \leq 2$. Obviously the sets $D = \{v_1, v_4, \ldots, v_{n-2}, v_{n-1}\}$ and $V - D$ are the *cwd*-sets of C_n. Hence $d^c_w(G) = 2$. □

Proposition 3.2. *For a complete graph K_n, $d^c_w(K_n) = 1$.*

Proof. Since $\gamma^c_w(K_n) = n$, the result follows. □

Proposition 3.3. *For a complete bipartite graph $K_{m,n}$,*

$$d^c_w(K_{m,n}) = \begin{cases} m & \text{if } m = n \\ 1 & \text{otherwise.} \end{cases}$$

Proof. Let (U, V) be the bipartition of the complete bipartite graph $K_{m,n}$. Assume that $1 \leq m \leq n$ where $U = \{u_1, u_2, \ldots, u_m\}$ and $V = \{v_1, v_2, \ldots, v_n\}$. If $m = n$, then $\{\{u_1, v_1\}, \{u_2, v_2\}, \ldots, \{u_m, v_n\}\}$ is a *cwd*-partition of $K_{m,n}$ of maximum order. Hence $d^c_w(K_{m,n}) = 1$ for $m = n$. Otherwise, if $m < n$, then $\{\{u_1, v_1, v_2, \ldots, v_n\}\}$ is the only *cwd*-partition of $K_{m,n}$ of maximum order. Hence $d^c_w(K_{m,n}) = 1$ for $m < n$. □

Proposition 3.4. *For a wheel W_n $(n \geq 5)$, $d^c_w(W_n) = 1$.*

Proof. Let $V(W_n) = \{u, v_1, v_2, \ldots, v_{n-1}\}$ where u is the central vertex of W_n.
Case 1: n is even.
Then W_n is χ-critical for n even. Thus $d^c_w(W_n) = 1$ by Proposition 2.4.
Case 2: n is odd.
In this case, every *cwd*-set contains the central vertex of W_n. Since n is odd, it follows that $d^c_w(W_n) = 1$. □

Proposition 3.5. *Let F_n denote a fan graph. Then $d^c_w(F_n) = 1$.*

Proof. Let $V(F_n) = \{u, v_1, v_2, \ldots, v_{n-1}\}$ where u is the central vertex of F_n. Then in F_n, every *cwd*-set contains the central vertex. It follows that $d^c_w(F_n) = 1$. □

4. Results on cwd-partition number

Proposition 4.1. *G is non-trivial iff $\gamma^c_w(G) \geq 2$.* □

Proposition 4.2. *For any non-trivial graph G, $d^c_w(G) \leq \frac{n}{2}$.*

Proof. Proof follows from Proposition 2.2 and 4.1. □

Proposition 4.3. *For any graph G, $\gamma^c_w(G)+d^c_w(G) \leq n+1$. Further equality holds if and only if $d^c_w(G) = 1$ and $\gamma^c_w(G) = n$. Further χ-critical graphs, $\overline{K_n}$ and $K_{1,n-1}$ are some of the graphs for which $d^c_w(G) = 1$ and $\gamma^c_w(G) = n$.*

Proof. Suppose $n = 1$. Then $G = K_1$, $\gamma^c_w(G) = 1$, $d^c_w(G) = 1$. Therefore $\gamma^c_w(G) + d^c_w(G) = 2 = n + 1$. Let $n > 1$. Suppose $\gamma^c_w(G) = n$. Then $d^c_w(G) = 1$. Therefore $\gamma^c_w(G) + d^c_w(G) = n + 1$. Suppose $\gamma^c_w(G) < n$, that is $\gamma^c_w(G) \leq n - 1$.
Case 1: $\gamma^c_w(G) \leq \frac{n}{2}$.
Since $n > 1$, $d^c_w(G) \leq \frac{n}{2}$. Therefore $\gamma^c_w(G) + d^c_w(G) \leq n < n + 1$.
Case 2: $\gamma^c_w(G) > \frac{n}{2}$.
Since $\gamma^c_w(G).d^c_w(G) \leq n$, $d^c_w(G) \leq \frac{n}{n/2} = 2$.
Therefore $\gamma^c_w(G) + d^c_w(G) \leq n - 1 + 2 = n + 1$.
Suppose $\gamma^c_w(G) + d^c_w(G) = n + 1$. Suppose $\gamma^c_w(G) \leq \frac{n}{2}$. Then

$$\begin{aligned}\gamma^c_w(G) + d^c_w(G) &\leq \frac{n}{2} + \frac{n}{2} \quad (\text{since if } n > 1,\ d^c_w(G) \leq \frac{n}{2})\\ &= n < n + 1, \quad \text{a contradiction.}\end{aligned}$$

Therefore $\gamma^c_w(G) > \frac{n}{2}$. Then $d^c_w(G) = 1$. Therefore $\gamma^c_w(G) = n$.
The converse is obvious. □

Proposition 4.4. *Let G be any graph with even order n. Then $d^c_w(G) = \frac{n}{2}$ if and only if $G = K_{\frac{n}{2},\frac{n}{2}}$ or $\overline{K_2}$.*

Proof. If $G = K_1$, then $d^c_w(G) = 1 = n \neq \frac{n}{2}$. Therefore $G \neq K_1$. Let $G \neq \overline{K_2}$ and $d^c_w(G) = \frac{n}{2}$. Let $V_1, V_2, \ldots, V_{\frac{n}{2}}$ be a cwd-partition of G. Then $|V_i| \leq 2$ for all i.

Since $n \geq 2$, $|V_i| \geq 2$ for all i. (Therefore $|V_i| = 1 \Rightarrow G = K_1$). Therefore $|V_i| = 2$ for all i. If V_i is independent for some i, then $\chi(G) = \chi(< V_i >) = 1$. Hence, $G = \overline{K_n}$ and $d^c_w(\overline{K_n}) = 1 = \frac{n}{2}$. Thus, $G = \overline{K_2}$, which is a contradiction to $G \neq \overline{K_2}$. Therefore V_i is not independent for every i.

Therefore, $\chi(G) = \chi(< V_i >) = 2$. Therefore G is nontrivial bipartite.

Let X, Y be the bipartition of G. Let $X \cap V_i = \{x_i\}$ and $Y \cap V_i = \{y_i\}$. Since $V_1, V_2, \ldots, V_{\frac{n}{2}}$ is a partition of V, $|X| = |Y| = \frac{n}{2}$. Since $V_i = \{x_i, y_i\}$ is a dominating set and X, Y are independent sets, each y_j is adjacent to x_i and each x_j is adjacent to y_i. Since i is arbitrary, G is a complete bipartite graph. Thus, $G = K_{\frac{n}{2},\frac{n}{2}}$. $\square$

Proposition 4.5. *Let G be a graph such that G and $\overline{G}$ are not chromatic weak domatically full. Then $d_w^c(G) + d_w^c(\overline{G}) \leq n - 1$.*

Proof. Since G and $\overline{G}$ are not chromatic weak domatically full, $d_w^c(G) \leq \delta(G)$ and $d_w^c(\overline{G}) \leq \delta(\overline{G})$. Therefore $d_w^c(G) + d_w^c(\overline{G}) \leq \delta(G) + \delta(\overline{G}) = n - 1$. $\square$

Proposition 4.6. *If a graph G has $d_w^c(G) \geq 2$, then $\gamma_w^c(G) + d_w^c(G) \leq \left\lfloor \frac{n}{2} \right\rfloor + 2$.*

Proof. Let G be a graph with $d_w^c(G) \geq 2$. Then $\gamma_w^c(G) \leq \left\lfloor \frac{n}{2} \right\rfloor$. Since $G \neq K_1$, $\gamma_w^c(G) \geq 2$ and so $d_w^c(G) \leq \left\lfloor \frac{n}{2} \right\rfloor$. If either $\gamma_w^c(G) = 2$ or $d_w^c(G) = 2$, then the bound is sharp. If $\gamma_w^c(G) \geq 4$ and $d_w^c(G) \geq 4$, then since $\gamma_w^c(G).d_w^c(G) \leq n$, $\gamma_w^c(G) \leq \left\lfloor \frac{n}{d_w^c(G)} \right\rfloor$ and $d_w^c(G) \leq \left\lfloor \frac{n}{\gamma_w^c(G)} \right\rfloor$. Thus $\gamma_w^c(G) \leq \left\lfloor \frac{n}{4} \right\rfloor$.

Hence $\gamma_w^c(G) + d_w^c(G) \leq 2\left\lfloor \frac{n}{4} \right\rfloor < \left\lfloor \frac{n}{2} \right\rfloor + 2$. Let $d_w^c(G) = 3$ or $\gamma_w^c(G) = 3$. Then $\gamma_w^c(G) + d_w^c(G) \leq 3 + \left\lfloor \frac{n}{3} \right\rfloor$. Since $3 = d_w^c(G)$ or $\gamma_w^c(G) \leq \left\lfloor \frac{n}{2} \right\rfloor$, $n \geq 6$. For $n \geq 6$, $3 + \left\lfloor \frac{n}{3} \right\rfloor \leq \left\lfloor \frac{n}{2} \right\rfloor + 2$. Therefore $\gamma_w^c(G) + d_w^c(G) \leq \left\lfloor \frac{n}{2} \right\rfloor + 2$. $\square$

Proposition 4.7. *For any graph G, $d_w^c(G) + d_w^c(\overline{G}) \leq n$, with equality holds if and only if $G = K_2$ or $\overline{K_2}$.*

Proof. Let $n \geq 2$. Then $d_w^c(G) + d_w^c(\overline{G}) \leq \frac{n}{2} + \frac{n}{2} = n$. $d_w^c(G) + d_w^c(\overline{G}) = n$ iff $d_w^c(G) = d_w^c(\overline{G}) = \frac{n}{2}$. That is, iff $G = K_{\frac{n}{2},\frac{n}{2}}$ or $\overline{K_2}$ and $\overline{G} = K_{\frac{n}{2},\frac{n}{2}}$ or $\overline{K_2}$. Let $G = K_{\frac{n}{2},\frac{n}{2}}$. Then $\overline{G} = K_{\frac{n}{2}} \cup K_{\frac{n}{2}}$. In this case, $d_w^c(G) = \frac{n}{2}$ and $d_w^c(\overline{G}) = 1$. Therefore $d_w^c(G) + d_w^c(\overline{G}) = \frac{n}{2} + 1 = n$ iff $n = 2$. Therefore $G = K_2$ and $\overline{G} = \overline{K_2}$. If $G = \overline{K_2}$, then $\overline{G} = K_2 = K_{\frac{n}{2},\frac{n}{2}}$ and $d_w^c(G) + d_w^c(\overline{G}) = 2 = n$. $\square$

Proposition 4.8. *For any graph G, $d_w^c(G).d_w^c(\overline{G}) \leq \frac{n^2}{4}$ with equality holding if and only if $G = K_2$ or $\overline{K_2}$.*

Proof. Since $n > 1$, both G and $\overline{G}$ having chromatic weak domatic number at least 1. Thus, $1 \leq d_w^c(G).d_w^c(\overline{G})$. Then the lower bound is sharp may be seen by taking $G = K_2$ or $\overline{K_2}$. Since $d_w^c(G) \leq \frac{n}{2}$ and $d_w^c(\overline{G}) \leq \frac{n}{2}$, then the upper bound is attained if $n > 1$. $d_w^c(G).d_w^c(\overline{G}) = \frac{n^2}{4}$ if and only if $d_w^c(G) = \frac{n}{2}$ and $d_w^c(\overline{G}) = \frac{n}{2}$. That is if and only if $G = K_{\frac{n}{2},\frac{n}{2}}$ or $\overline{K_2}$ and $d_w^c(\overline{G}) = 1$. Therefore $d_w^c(G).d_w^c(\overline{G}) = \frac{n}{2} = \frac{n^2}{4}$ if and only if $n = 2$. That is $G = K_2$. Let $G = \overline{K_2}$. Then $\overline{G} = K_2 = K_{\frac{n}{2},\frac{n}{2}}$. Therefore $d_w^c(G).d_w^c(\overline{G}) \leq \frac{n^2}{4}$ if and only if $G = K_2$ or $\overline{K_2}$. $\square$

5. Conclusion

In this paper, a study of a new parameter $d_w^c(G)$ is initiated. Characterization of graphs for which $d_w^c(G) = 1$ is a result to be derived. If $G_1 \oplus G_2 \oplus G_3 = K_n$, then is it true that $d_w^c(G_1) + d_w^c(G_2) + d_w^c(G_3) \leq 2n + 1$? Finding an upper bound for the product $d_w^c(G)$ and $\chi(G)$ is yet another problem. The relationship between $d_w^c(G)$ and other graph theoretic parameters can also be investigated.

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