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Fuzzy Orbit Irresolute Mappings

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Abstract − Fuzzy orbit topological space is a new structure very recently given by [1]. This new space is based on the notion of open fuzzy orbit sets. The aim of this paper is to provide applications of open fuzzy orbit sets. We introduce the notions of fuzzy orbit irresolute mappings and fuzzy orbit open (resp. irresolute open) mappings and studied some of their properties.

Keywords − *Fuzzy orbit, fuzzy orbit topology, fuzzy orbit closure, fuzzy orbit neighbourhood, fuzzy orbit irresolute mapping*

1. Introduction

The fuzzy set theory introduced by Zadeh [2] provides natural bases for building new branches of fuzzy mathematics. As a generalization of topological space in fuzzy setting, the concept of fuzzy topological space introduced by Chang [3] and studied further by many topologists (cf. [4, 5, 6, 7, 8, 9]). Malathi and Uma [10] in 2017 introduced the notions of the orbit of a fuzzy set under a mapping $\mathcal{J}: \mathcal{P} \longrightarrow \mathcal{P}$ and an open fuzzy orbit set in a fuzzy topological space$(\mathcal{P}, \sigma)$. Very recently, Majeed and El-Sheikh [1] studied the behavior of the collection of open fuzzy orbit sets and discussed the conditions on the mapping $\mathcal{J}: \mathcal{P} \longrightarrow \mathcal{P}$ to obtain a fixed orbit of these fuzzy sets. Majeed and El-Sheikh proved that the collection of all open fuzzy orbit sets under the mapping $\mathcal{J}: \mathcal{P} \longrightarrow \mathcal{P}$ construct a fuzzy topology, denoted by σ_{FO}, which is coarser than σ.Our purpose, in this work, is to define a new class of mappings between fuzzy topological spaces by using open fuzzy orbit sets. That is, we define the class of fuzzy orbit irresolute mappings on fuzzy topological spaces. This notion is independent from the notion of fuzzy continuous in the sense of Chang (see Examples 4.1 and 4.2). Also, we define and study fuzzy orbit open (resp. irresolute open) mappings.

2. Preliminaries

Throughout this paper, $\mathcal{P}$ will refer to the initial universe, $I = [0,1]$, $I_0 = (0,1]$, and $I^{\mathcal{P}}$ is the family of all fuzzy sets of $\mathcal{P}$. For $x \in \mathcal{P}$ and $t \in I_0$, a fuzzy point ($\mathcal{F}$-point, for short) x_t is defined as t if $x = y$ and 0 otherwise, $\forall\, y \in \mathcal{P}$. A $\mathcal{F}$ -point x_t is said to be belongs to a fuzzy set ω, denoted $x_t \in \omega$, if and only if $\omega(x) \geq t$. For $\delta, \omega \in I^{\mathcal{P}}$, δ is called quasi-coincident with ω, denoted by $\delta\, q \omega$ if $\delta(x) + \omega(x) > 1$ for some $x \in \mathcal{P}$, otherwise we write $\delta\, \bar{q}\, \omega$. And $\delta q \omega$ if and only if $\exists x_t;\ x_t \in \delta,\ x_t\, q\, \omega$.

Next, we list some definitions and basic properties about the notions of the orbit of fuzzy set and fuzzy orbit topological spaces and other related concepts.

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Definition 2.1. [10] Let $\mathcal{J}: \mathcal{P} \longrightarrow \mathcal{P}$ be a mapping and $\omega \in I^{\mathcal{P}}$. Then,

i. The fuzzy orbit ($fo.$, for short) of ω under $\mathcal{J}$, denoted by $O_{\mathcal{J}}(\omega)$ is defined as $O_{\mathcal{J}}(\omega) = \{\omega, \mathcal{J}(\omega), \mathcal{J}^2(\omega), \dots\}$.

ii. The Fuzzy orbit set (fo.s, for short) of ω under $\mathcal{J}$ is defined as $FO_{\mathcal{J}}(\omega) = \omega \wedge \mathcal{J}(\omega) \wedge \mathcal{J}^2(\omega) \wedge \dots$ the intersection of all members of $O_{\mathcal{J}}(\omega)$.

iii. If $(\mathcal{P}, \sigma)$ is a fuzzy topological space (*fts*, for short) and $\mathcal{J}: \mathcal{P} \longrightarrow \mathcal{P}$, then the *fo.s* under $\mathcal{J}$ which belongs to σ is called an open fuzzy orbit set under $\mathcal{J}$ (open-fo.s, for short). The complement of an open-fo.s is called a closed fuzzy orbit set under $\mathcal{J}$ (closed-fo.s, for short).

Definition 2.2. [10] Let $(\mathcal{P}, \sigma)$and $(\mathcal{Q}, \sigma^*)$ be two fts's. Let $\mathcal{J}: \mathcal{P} \longrightarrow \mathcal{P}$. A mapping $\psi: (\mathcal{P}, \sigma) \longrightarrow (\mathcal{Q}, \sigma^*)$ is called fo.continuous, if the inverse image of every open fuzzy set (open-fs, for short) in $\mathcal{Q}$ is an open-fo.s in $\mathcal{P}$.

Definition 2.3. [3] Let $(\mathcal{P}, \sigma)$ and $(\mathcal{Q}, \sigma^*)$ be two fts's. A mapping $\psi: (\mathcal{P}, \sigma) \longrightarrow (\mathcal{Q}, \sigma^*)$ is called a fuzzy continuous (f.continuous, for short) if and only if the inverse image of each open-fs in $\mathcal{Q}$ is an open-fs in $\mathcal{P}$.

Majeed and El-Sheikh studied the collection of open-fo.s's and introduced some properties of these sets. They determined the cases on the mapping $\mathcal{J}: \mathcal{P} \longrightarrow \mathcal{P}$ becomes fixed open-fo.s (i.e., $I(\delta) = \delta$) for each open-fo.s δ, where $\mathcal{P}$ is a nonempty countable set. The following theorem explains that.

Theorem 2.1. [1] Let $(\mathcal{P}, \sigma)$be a fts and δ be an open-fo.s under the mapping $\mathcal{J}: \mathcal{P} \longrightarrow \mathcal{P}$. Then, $\mathcal{J}(\delta) = \delta$ whenever $\mathcal{J}$ is either bijective or constant mapping.

Remark 2.1. From now on, any mapping $\mathcal{J}: \mathcal{P} \longrightarrow \mathcal{P}$ will be considered as a bijective or constant mapping on a nonempty countable set $\mathcal{P}$.

Theorem 2.2. [1] Let $(\mathcal{P}, \sigma)$ be a fts and let σ_{FO} denotes the set of all open-fo.s's under the mapping $\mathcal{J}: \mathcal{P} \longrightarrow \mathcal{P}$. Then,$\sigma_{FO}$ constructs a fuzzy topology on $\mathcal{P}$ coarser than σ. The pair $(\mathcal{P}, \sigma_{FO})$ is called fuzzy orbit topological space (fo.ts, for short) associated with $(\mathcal{P}, \sigma)$.

Definition 2.4. [1] Let $(\mathcal{P}, \sigma_{FO})$ be a fo.ts and $\omega \in I^{\mathcal{P}}$. Then,

i. The fo.closure of ω, denoted by $cl_{FO}(\omega)$, is defined as,

$$cl_{FO}(\omega) = \wedge\{\delta \in I^{\mathcal{P}}: \delta \textit{ is a closed} - fo.s \textit{ and } \omega \leq \delta\}$$

ii. The fo.interior of ω, denoted by $Int_{FO}(\omega)$, is defined as,

$$Int_{FO}(\omega) = \vee\{\delta \in I^{\mathcal{P}}: \delta \textit{ is an open} - fo.s \textit{ and } \delta \leq \omega\}$$

Proposition 2.1. [1] Let $(\mathcal{P}, \sigma_{FO})$ be a fo.ts and $\omega \in I^{\mathcal{P}}$. Then,

$$Int_{FO}(\omega) \leq Int(\omega) \leq \omega \leq cl(\omega) \leq cl_{FO}(\omega).$$

Proposition 2.2. [1] Let $(\mathcal{P}, \sigma_{FO})$ be a fo.ts and $\omega, \delta \in I^{\mathcal{P}}$. Then,

i. $cl_{FO}(\overline{0}) = \overline{0}$ *(resp.* $Int_{FO}(\overline{0}) = \overline{0}$*)* and $cl_{FO}(\overline{1}) = \overline{1}$ *(resp.* $Int_{FO}(\overline{1}) = \overline{1}$*)*.
ii. $\omega \leq cl_{FO}(\omega)$ $(resp. Int_{FO}(\omega) \leq \omega)$.
iii. $cl_{FO}(\omega \vee \delta) = cl_{FO}(\omega) \vee cl_{FO}(\delta)$ $(resp. Int_{FO}(\omega \wedge \delta) = Int_{FO}(\omega) \wedge Int_{FO}(\delta))$.
iv. $If\ \omega \leq \delta, then\ cl_{FO}(\omega) \leq cl_{FO}(\delta)$ $(resp. Int_{FO}(\omega) \leq Int_{FO}(\delta))$.
v. $cl_{FO}(cl_{FO}(\omega)) = cl_{FO}(\omega)$ $(resp. Int_{FO}(Int_{FO}(\omega)) = Int_{FO}(\omega))$.
vi. $\omega\ is\ closed - fo. (resp. open) iff\ \omega = cl_{FO}(\omega) \big(resp. \omega = Int_{FO}(\omega)\big)$.

Theorem 2.3. [1] Let $(\mathcal{P}, \sigma_{FO})$ be a fo.ts and ω $I^{\mathcal{P}}$.Then,

i. $\overline{1} - Int_{FO}(\omega) = cl_{FO}(\overline{1} - \omega)$.
ii. $\overline{1} - cl_{FO}(\omega) = Int_{FO}(\overline{1} - \omega)$.

3. Fuzzy Orbit Neighbourhood

Definition 3.1. A fuzzy set ω in a fts $(\mathcal{P}, \sigma)$is said to be a fuzzy orbit neighbourhood (fo.nbhd, for short) of a $\mathcal{F}$-point x_t if and only if there exists an open-fo.s δ such that $x_t \in \delta \leq \omega$.

Theorem 3.1. Let $(\mathcal{P}, \sigma)$ be a fts and $\omega \in I^{\mathcal{P}}$. Then, ω is an open-fo.s if and only if ω is a fo.nbhd for any $\mathcal{F}$-point $x_t \in$ ω.

Proof. Suppose ω is an open-fo.s and let $x_t \in$ ω. Since ω ≤ ω and ω is an open-fo.s, then ω is a fo.nbhd of x_t.

Conversely, since for all $x_t \in$ ω, there exists an open-fo.s δ_k such that $x_t \in \delta_k \leq \omega$. Then, $\bigvee x_t \leq \bigvee_{k\in\omega} \delta_k \leq \omega$. Since every fuzzy set can be represented by the union of its $\mathcal{F}$-points, then $\bigvee x_t = \omega$. Also, by Theorem 2.2, $\bigvee_{k\in\omega} \delta_k$ is an open-fo.s. Thus, ω is an open-fo.s.

Definition 3.2. A fuzzy set ω in a fts $(\mathcal{P}, \sigma)$is said to be a fuzzy orbit Q-neighbourhood (fo.Q-nbhd, for short) of a $\mathcal{F}$-point x_t if ∃ an open-fo.s δ such that $x_t q \delta \leq \omega$.

Definition 3.3. Let $(\mathcal{P}, \sigma)$ and $(\mathcal{Q}, \sigma')$ be any two fts's. Let $\mathcal{I}_1: \mathcal{P} \longrightarrow \mathcal{P}$ and $\mathcal{I}_2: \mathcal{Q} \longrightarrow \mathcal{Q}$ be any two mappings. A mapping $\psi: (\mathcal{P}, \sigma_{FO}) \longrightarrow (\mathcal{Q}, \sigma'_{FO})$ is said to be f.continuous, if the inverse image of any open-fo.s under the mapping $\mathcal{I}_2$ in $(\mathcal{Q}, \sigma')$ is an open-fo.s under the mapping $\mathcal{I}_1$ *in* $(\mathcal{P}, \sigma)$.

Theorem 3.2. Let $\psi: (\mathcal{P}, \sigma_{FO}) \longrightarrow (\mathcal{Q}, \sigma'_{FO})$ and $g: (\mathcal{Q}, \sigma'_{FO}) \longrightarrow (Z, \sigma''_{FO})$ be two mappings. Then, $go\psi$ is f. continuous mapping if ψ and g are f.continuous.

Proof. The proof is straightforward.

4. Fuzzy Orbit Irresolute (Irresolute Open) Mappings

Our goal here is to introduce and study the concept of irresolute (resp. irresolute open) mappings in fst's by using the concepts of open-fo.s's.

Definition 4.1. Let $(\mathcal{P}, \sigma)$ and $(\mathcal{Q}, \sigma^*)$ be any two fts's. Let $\mathcal{I}_1: \mathcal{P} \longrightarrow \mathcal{P}$ and $\mathcal{I}_2: \mathcal{Q} \longrightarrow \mathcal{Q}$ be any two mappings. A mapping $\psi: (\mathcal{P}, \sigma) \longrightarrow (\mathcal{Q}, \sigma^*)$ is said to be fuzzy orbit irresolute (fo.irresolute, for short), if the inverse image of every open-fo.s under the mapping $\mathcal{I}_2$ in $(\mathcal{Q}, \sigma^*)$ is an open-fo.s under the mapping $\mathcal{I}_1$ in $(\mathcal{P}, \sigma)$.

The concept of f.continuous in the sense of Chang and fo.irresolute are independent. The next two examples explain that.

Example 4.1. Let $\mathcal{P} = \{k_1, k_2, k_3\}$ and $\mathcal{Q} = \{s_1, s_2, s_3\}$. Define $\sigma = \{\overline{0}, \overline{1}, \omega\}$ and $\sigma^* = \{\overline{0}, \overline{1}, \delta_1, \delta_2\}$ where $\omega \in I^{\mathcal{P}}$and $\delta_1, \delta_2 \in I^{\mathcal{Q}}$ such that $\omega = \{(k_1, 0.2), (k_2, 0.3), (k_3, 0.3)\}$, $\delta_1 = \{(s_1, 0.2), (s_2, 0.3), (s_3, 0.3)\}$ and $\delta_2 = \{(s_1, 0.6), (s_2, 0.5), (s_3, 0.7)\}$. Clearly, $(\mathcal{P}, \sigma)$ and $(\mathcal{Q}, \sigma^*)$ are fts's.

Define $\psi: (\mathcal{P}, \sigma) \longrightarrow (\mathcal{Q}, \sigma^*)$, $\mathcal{I}_1: \mathcal{P} \longrightarrow \mathcal{P}$ and $\mathcal{I}_2: \mathcal{Q} \longrightarrow \mathcal{Q}$ as $\psi(k_1) = s_1, \psi(k_2) = s_3, \psi(k_3) = s_2, \mathcal{I}_1(k_1) = k_1, \mathcal{I}_1(k_2) = k_3$, $\mathcal{I}_1(k_3) = k_2$and $\mathcal{I}_2(s_1) = s_1, \mathcal{I}_2(s_2) = s_3$, $\mathcal{I}_2(s_3) = s_2$. Then, ψ is fo.irresolute but not f.continuous mapping, since δ_2 is an open-fs in $\mathcal{Q}$, however $\psi^{-1}(\delta_2) \notin \sigma$.

Example 4.2. Let $\mathcal{P} = \{k_1, k_2, k_3\}$ and $\mathcal{Q} = \{s_1, s_2, s_3\}$. Define $\sigma = \{\bar{0}, \bar{1}, \omega\}$ and $\sigma^* = \{\bar{0}, \bar{1}, \delta\}$ where $\omega \in I^{\mathcal{P}}$ and $\delta \in I^{\mathcal{Q}}$ such that $\omega = \{(k_1, 0.4), (k_2, 0.4), (k_3, 0.7)\}$, $\delta = \{(s_1, 0.7), (s_2, 0.4), (s_3, 0.4)\}$. Clearly, $(\mathcal{P}, \sigma)$ and $(\mathcal{Q}, \sigma^*)$ are fts's.

Define $\psi: (\mathcal{P}, \sigma) \longrightarrow (\mathcal{Q}, \sigma^*)$, $\mathcal{I}_1: \mathcal{P} \longrightarrow \mathcal{P}$ and $\mathcal{I}_2: \mathcal{Q} \longrightarrow \mathcal{Q}$ as $\psi(k_1) = s_2, \psi(k_2) = s_3, \psi(k_3) = s_1$, $\mathcal{I}_1(k_1) = k_1, \mathcal{I}_1(k_2) = k_3$, $\mathcal{I}_1(k_3) = k_2$ and $\mathcal{I}_2(s_1) = s_1, \mathcal{I}_2(s_2) = s_3$, $\mathcal{I}_2(s_3) = s_2$. Then, ψ is f.continuous but not fo.irresolute mapping, since δ is an open-fo.s under $\mathcal{I}_2$ in $\mathcal{Q}$, however $\psi^{-1}(\delta) = \omega$ is not an open-fo.s under $\mathcal{I}_1$ in $\mathcal{P}$.

Theorem 4.1. Let $(\mathcal{P}, \sigma)$ and $(\mathcal{Q}, \sigma^*)$ be any two fts's, let $(\mathcal{P}, \sigma_{FO})$ and $(\mathcal{Q}, \sigma^*_{FO})$ be its associative fo.ts's with $(\mathcal{P}, \sigma)$ and $(\mathcal{Q}, \sigma^*)$ respectively. Let $\mathcal{I}_1: \mathcal{P} \longrightarrow \mathcal{P}$ and $\mathcal{I}_2: \mathcal{Q} \longrightarrow \mathcal{Q}$ be any two mappings. Then $\psi: (\mathcal{P}, \sigma) \longrightarrow (\mathcal{Q}, \sigma^*)$ is fo.irresolute mapping iff $\psi: (\mathcal{P}, \sigma_{FO}) \longrightarrow (\mathcal{Q}, \sigma^*_{FO})$ is f.continuous mapping.

Proof. Straightforward.

Theorem 4.2. Let $(\mathcal{P}, \sigma)$ and $(\mathcal{Q}, \sigma^*)$ be any two fts's. Let $\psi: (\mathcal{P}, \sigma) \longrightarrow (\mathcal{Q}, \sigma^*)$ be any mapping. If ψ is fo.continuous mapping, then ψ is fo.irresolute mapping.

Proof. The proof is straightforward by Definition 4.1 and Definition 2.2.

The inverse direction of Theorem 4.2 may not be held, In Example 4.1, ψ *is* fo.irresolute mapping, however, it is not fo.continuous since δ_2 is an open-fs in $\mathcal{Q}$, but its inverse image is not open-fo.s in $\mathcal{P}$.

Some characterizations of fo.irresolute mapping are given next.

Theorem 4.3. Let $(\mathcal{P}, \sigma)$ and $(\mathcal{Q}, \sigma^*)$ be any two fts's. Let $\psi: (\mathcal{P}, \sigma) \longrightarrow (\mathcal{Q}, \sigma^*)$ be any mapping. Then, the following statements are equivalent:

(a) ψ is fo.irresolute mapping,

(b) For every $\mathcal{F}$-point x_t of $\mathcal{P}$ and every open-fo.s δ in $\mathcal{Q}$ such that $\psi(x_t) \in \delta$, there is an open-fo.s ω in $\mathcal{P}$ such that $x_t \in \omega$ and $\psi(\omega) \leq \delta$,

(c) For every closed-fo. ν in $\mathcal{Q}$, $\psi^{-1}(\nu)$ is closed-fo.s in $\mathcal{P}$,

(d) For every $\mathcal{F}$-point x_t of $\mathcal{P}$ and every fo.nbhd δ in $\mathcal{Q}$ of $\psi(x_t)$, $\psi^{-1}(\delta)$ is a fo.nbhd of x_t in $\mathcal{P}$,

(e) For every $\mathcal{F}$*-point* x_t of $\mathcal{P}$ and every *fo.nbhd* δ *in* $\mathcal{Q}$ *of* $\psi(x_t)$, *there is a fo.nbhd* ω in $\mathcal{P}$ of x_t such that $\psi(\omega) \leq \delta$,

(f) For every $\mathcal{F}$*-point* x_t of $\mathcal{P}$ and every open-fo.s δ in $\mathcal{Q}$ such that $\psi(x_t) q\, \delta$, there is an open-fo.s ω in $\mathcal{P}$ such that $x_t q \omega$ and $\psi(\omega) \leq \delta$,

(g) For every $\mathcal{F}$-point x_t of $\mathcal{P}$ and every fo.Q-nbhd δ in $\mathcal{Q}$ of $\psi(x_t)$, $\psi^{-1}(\delta)$ is fo.Q-nbhd of x_t in $\mathcal{P}$,

(h) For every $\mathcal{F}$-point x_t of $\mathcal{P}$ and every fo.Q-nbhd δ in $\mathcal{Q}$ of $\psi(x_t)$, there is a fo.Q-nbhd ω of x_t such that $\psi(\omega) \leq \delta$,

(i) $\psi(cl_{FO}(\omega)) \leq cl_{FO}(\psi(\omega))$, for every fuzzy set ω of $\mathcal{P}$,

(j) $cl_{FO}(\psi^{-1}(\delta)) \leq \psi^{-1}(cl_{FO}(\delta))$, for every fuzzy set δ of $\mathcal{Q}$,

(k) $\psi^{-1}(Int_{FO}(\delta)) \leq Int_{FO}(\psi^{-1}(\delta))$, for every fuzzy set δ of $\mathcal{Q}$.

PROOF.

(a)⟹(b) Let x_t *be a* $\mathcal{F}$*-point* of $\mathcal{P}$ and δ be an *open-fo.s* in $\mathcal{Q}$ under the mapping $\mathcal{I}_2$ such that $\psi(x_t) \in \delta$. Put $\omega = \psi^{-1}(\delta)$. Then, by (a) ω is an *open-fo.s* in $\mathcal{P}$ under the mapping $\mathcal{I}_1$ such that $x_t \in \omega$ and $\psi(\omega) = \psi(\psi^{-1}(\delta)) \leq \delta$. Hence, $\psi(\omega) \leq \delta$.

(b)⟹(a) Let δ be an open-fo.s in $\mathcal{Q}$. Let $x_t \in \psi^{-1}(\delta)$. Then, $\psi(x_t) \in$ δ. Now by *(b)* there is an open-fo.s ω in $\mathcal{P}$ such that $x_t \in \omega$ and $\psi(\omega) \leq$ δ. Then, $x_t \in$ ω $\leq \psi^{-1}(\delta)$. Hence by Theorem 3.1 $\psi^{-1}(\delta)$ is an open-fo.s in $\mathcal{P}$. Thus, ψ is fo.irresolute mapping.

(a)⇔(c) Obvious.

(a)⟹(d) Let x_t be a $\mathcal{F}$*-point* of $\mathcal{P}$ and let δ be a *fo.nbhd* of $\psi(x_t)$. Then, there is an *open-fo.s* ν in $\mathcal{Q}$ such that $\psi(x_t) \in \nu \leq \delta$. *Now* $\psi^{-1}(\nu)$ *is an open-fo.s in* $\mathcal{P}$, because ψ *is a fo.*irresolute mapping and $x_t \in \psi^{-1}(\nu) \leq \psi^{-1}(\delta)$. *Thus,* $\psi^{-1}(\delta)$ is a *fo.nbhd* of x_t in $\mathcal{P}$.

(d)⟹(e) Let x_t be a $\mathcal{F}$*-point* of $\mathcal{P}$ and let δ be a *fo.nbhd* of $\psi(x_t)$. Then, by hypothesis $\omega = \psi^{-1}(\delta)$ is a *fo.nbhd* of x_t and $\psi(\omega) = \psi(\psi^{-1}(\delta)) \leq \delta$. Hence, $\psi(\omega) \leq \delta$.

(e) ⟹(b) Let x_t be a $\mathcal{F}$*-point* of $\mathcal{P}$ and let δ be an open-fo.s in $\mathcal{Q}$ containing $\psi(x_t)$. Then, δ is a *fo.nbhd* of $\psi(x_t)$, so there is a *fo.nbhd* ω of x_t of $\mathcal{P}$ such that $x_t \in \omega$ and $\psi(\omega) \leq \delta$. Therefore, there exists an open-fo.s ω' in $\mathcal{P}$ such that $x_t \in \omega' \leq \omega$. Clearly, $\psi(\omega') \leq \psi(\omega) \leq \delta$.

(a)⟹(f) Let x_t be a $\mathcal{F}$*-point* of $\mathcal{P}$ and δ be an *open-fo.s* in $\mathcal{Q}$ such that $\psi(x_t)q\delta$. *Let* $\omega = \psi^{-1}(\delta)$, *then* ω *is an open-fo.s in P and* $x_t q\ \omega$ *and* $\psi(\omega) = \psi(\psi^{-1}(\delta)) \leq \delta$.

(f)⟹(g) Let x_t be a $\mathcal{F}$*-point* of $\mathcal{P}$ and δ be a *fo.Q-nbhd* of $\psi(x_t)$ in $\mathcal{Q}$. Then, there exists an *open-fo.s* ν in $\mathcal{Q}$ such that $\psi(x_t)q\nu \leq \delta$. By hypothesis there is an *open-fo.* ω in $\mathcal{P}$ such that $x_t q\omega$ and $\psi(\omega) \leq \nu$. Thus $x_t q\omega \leq \psi^{-1}(\nu) \leq \psi^{-1}(\delta)$. Hence, $\psi^{-1}(\delta)$ is a *fo.Q-nbhd* of x_t.

(g)⟹(h) Let x_t be a $\mathcal{F}$*-point* of $\mathcal{P}$ and δ be a *fo.Q-nbhd* of $\psi(x_t)$ *in* $\mathcal{Q}$. Then, $\omega = \psi^{-1}(\delta)$ is a *fo.Q-nbhd* of x_t and $\psi(\omega) \leq \psi(\psi^{-1}(\delta)) \leq \delta$.

(h)⟹(f) Let x_t be a $\mathcal{F}$*-point* of $\mathcal{P}$ and δ be an open-fo.s in $\mathcal{Q}$ such that $\psi(x_t)q\delta$. *Then,* δ is a *fo.Q-nbhd* of $\psi(x_t)$. So, there is a *fo.Q-nbhd* ω of x_t such that $\psi(\omega) \leq \delta$. Therefore, there exists an *open-fo.s* ν in $\mathcal{P}$ such that $x_t q\nu \leq \omega$. Hence, $x_t q\nu$ and $\psi(\nu) \leq \psi(\omega) \leq \delta$.

(f)⟹(a) Let η be an *open-fo.s* in $\mathcal{Q}$ and $x_t \in \psi^{-1}(\eta)$. Clearly, $\psi(x_t) \in \eta$. Choose the $\mathcal{F}$*-point* $\overline{1} - x_t$. Then, $\psi(\overline{1} - x_t)q\eta$. And so by *(f)* there exists an *open-fo.s* ω such that $\overline{1} - x_t q\omega$ and $\psi(\omega) \leq \eta$. Now, $\overline{1} - x_t q\omega$ this implies $x_t \in \omega$. Thus, $x_t \in \omega \leq \psi^{-1}(\eta)$. Hence, by Theorem 3.1, $\psi^{-1}(\eta)$ is an *open-fo.s* in $\mathcal{P}$.

(i)⟹(c) Let δ be any closed-fo.s in $\mathcal{Q}$. Then, from *(i)*, $\psi(cl_{FO}(\psi^{-1}(\delta))) \leq cl_{FO}(\psi(\psi^{-1}(\delta)) \leq cl_{FO}(\delta) = \delta$. By taking the inverse of the equality we get $cl_{FO}(\psi^{-1}(\delta)) \leq \psi^{-1}(\delta)$. Since $\psi^{-1}(\delta) \leq cl_{FO}(\psi^{-1}(\delta))$. Then, we have $\psi^{-1}(\delta) = cl_{FO}(\psi^{-1}(\delta))$. Hence, $\psi^{-1}(\delta)$ is a closed- fo.s in $\mathcal{P}$.

(c)⟹(i) Suppose that *(c)* holds. Let ω be a fuzzy set of $\mathcal{P}$. Since $\omega \leq \psi^{-1}(\psi(\omega))$, then $\omega \leq \psi^{-1}(\psi(cl_{FO}(\omega)))$. Now, $cl_{FO}(\psi(\omega))$ is a closed-fo.s contains ω. Consequently, $cl_{FO}(\omega) \leq cl_{FO}(\psi^{-1}(cl_{FO}(\psi(\omega)))) = \psi^{-1}(cl_{FO}(\psi(\omega)))$ and so $\psi(cl_{FO}(\omega)) \leq cl_{FO}(\psi(\omega))$.

(i)⟹(j) Let δ be a fuzzy set of *Q*. Then, $\psi^{-1}(\delta)$ is a fuzzy set of $\mathcal{P}$. Therefore by *(i)*, $\psi(cl_{FO}(\psi^{-1}(\delta))) \leq cl_{FO}(\psi(\psi^{-1}(\delta))) \leq cl_{FO}(\delta)$. Hence, $cl_{FO}(\psi^{-1}(\delta)) \leq \psi^{-1}(cl_{FO}(\delta))$.

(j)⟹(i) Let $\delta = \psi(\omega)$ where ω is a fuzzy set of $\mathcal{P}$, and we know that $\omega \leq \psi^{-1}(\delta)$ which implies $cl_{FO}(\omega) \leq cl_{FO}(\psi^{-1}(\delta))$. Thus, $cl_{FO}(\omega) \leq cl_{FO}(\psi^{-1}(\delta)) \leq \psi^{-1}(cl_{FO}(\delta)) \leq \psi^{-1}(cl_{FO}(\psi(\omega)))$. Therefore, $\psi(cl_{FO}(\omega)) \leq cl_{FO}(\psi(\omega))$.

(a)⟹(k) Let δ be an *open-fo.s* in $\mathcal{Q}$. Clearly $\psi^{-1}(Int_{FO}(\delta))$ is an *open-fo.s* in $\mathcal{P}$ and we have $\psi^{-1}(Int_{FO}(\delta)) \leq Int_{FO}(\psi^{-1}(Int_{FO}(\delta))) \leq Int_{FO}(\psi^{-1}(\delta))$.

(k)⟹(a) Let δ be an open-fo.s in $\mathcal{Q}$. Then, $Int_{FO}(\delta) = \delta$ and $\psi^{-1}(\delta) = \psi^{-1}(Int_{FO}(\delta)) \leq Int_{FO}(\psi^{-1}(\delta))$. Hence, we have $\psi^{-1}(\delta) = Int_{FO}(\psi^{-1}(\delta))$. This means that $\psi^{-1}(\delta)$ is an *open-fo.s* in $\mathcal{P}$. Hence, the proof is complete.

Theorem 4.4. Let $(\mathcal{P},\sigma)$, $(\mathcal{Q},\sigma^*)$ and (Z,σ^{**}) be fts's. Let $\psi: (\mathcal{P},\sigma) \longrightarrow (\mathcal{Q},\sigma^*)$ and $g: (\mathcal{Q},\sigma^*) \longrightarrow (Z,\sigma^{**})$ be two mappings. Then, $go\psi$ is

i. fo.irresolute mapping if ψ and g are fo.irresolute,
ii. fo.continuous if ψ is fo.irresolute and g is fo.continuous.

Proof. From Definition 4.1 and Definition 2.2 we can obtain the result.

Definition 4.2 Let $(\mathcal{P},\sigma)$ and $(\mathcal{Q},\sigma^*)$ be any two *fts's.* Let $\mathcal{I}_1: \mathcal{P} \longrightarrow \mathcal{P}$ and $\mathcal{I}_2: \mathcal{Q} \longrightarrow \mathcal{Q}$ be any two mappings. A mapping $\psi: (\mathcal{P},\sigma) \longrightarrow (\mathcal{Q},\sigma^*)$ is said to be

i. fuzzy orbit open (resp. closed) mapping (fo.open (resp. closed)) mapping, if the image of every open-(resp. closed-)fs in $\mathcal{P}$ is an open-(resp. closed-) fo.s in Q.

ii. fuzzy orbit irresolute open (resp. closed) mapping (fo.irresolute open (resp. irresolute closed), for short) mapping, if the image of every open-(resp. closed-) fo.s in $\mathcal{P}$ is an open-(resp. closed-) fo.s in Q.

The relationship between fo.open mappings and fo.irresolute open mappings is given in the following theorem.

Theorem 4.5. Let $(\mathcal{P},\sigma)$ and $(\mathcal{Q},\sigma^*)$ be any two *fts's.* If $\psi: (\mathcal{P},\sigma) \longrightarrow (\mathcal{Q},\sigma^*)$ *is* fo.open mapping, then ψ is fo.irresolute open mapping.

Proof. The proof uses only the fact every open-fo.s is an open-fs and the hypothesis.

The converse of Theorem 4.5 does not hold. We show that in the following example.

Example 4.3. Let $\mathcal{P} = \{k_1, k_2, k_3\}$ and $\mathcal{Q} = \{s_1, s_2, s_3\}$. Define $\sigma = \{\bar{0}, \bar{1}, \omega_1, \omega_2\}$ and $\sigma^* = \{\bar{0}, \bar{1}, \delta_1, \delta_2\}$ where $\omega_1, \omega_2 \in I^{\mathcal{P}}$ and $\delta_1, \delta_2 \in I^{\mathcal{Q}}$ such that

$$\omega_1 = \{(k_1, 0.9), (k_2, 0.5), (k_3, 0.6)\}, \omega_2 = \{(k_1, 0.2), (k_2, 0.2), (k_3, 0.2)\},$$

$$\delta_1 = \{(s_1, 0.5), (s_2, 0.6), (s_3, 0.9)\}, \delta_2 = \{(s_1, 0.2), (s_2, 0.2), (s_3, 0.2)\}$$

Clearly, $(\mathcal{P},\sigma)$ and $(\mathcal{Q},\sigma^*)$ are fts's. Define $\psi: (\mathcal{P},\sigma) \longrightarrow (\mathcal{Q},\sigma^*)$, $\mathcal{I}_1: \mathcal{P} \longrightarrow \mathcal{P}$ and $\mathcal{I}_2: \mathcal{Q} \longrightarrow \mathcal{Q}$ as $\psi(k_1) = s_3$, $\psi(k_2) = s_1$, $\psi(k_3) = s_2$, $\mathcal{I}_1(k_1) = k_3$, $\mathcal{I}_1(k_2) = k_2$, $\mathcal{I}_1(k_3) = k_1$ and $\mathcal{I}_2(s_1) = s_1$, $\mathcal{I}_2(s_2) = s_2$, $\mathcal{I}_2(s_3) = s_3$. Then, ω_2 is an open-fo.s in $\mathcal{P}$ and $\psi(\omega_2) = \delta_2$ which is also open-fo.s in Q, so ψ is fo.irresolute open. But ψ does not fo.open mapping, since there is an open-fs ω_1 in $\mathcal{P}$ and $\psi(\omega_1) = \delta_1$ is not open-fo.s in Q.

Theorem 4.6. Let $(\mathcal{P},\sigma)$ and $(\mathcal{Q},\sigma^*)$ be any two *fts's* and let $\psi: (\mathcal{P},\sigma) \longrightarrow (\mathcal{Q},\sigma^*)$ is fo.irresolute open. If δ is a fuzzy set of Q and ω is a closed-fo.s in $\mathcal{P}$ containing $\psi^{-1}(\delta)$, then there exists a closed-fo.s η of Q containing δ such that $\psi^{-1}(\eta) \leq \omega$.

Proof. Let δ be a fuzzy set of Q and ω be a closed-fo.s in $\mathcal{P}$ such that $\psi^{-1}(\delta) \leq \omega$. Then, $\bar{1} - \omega$ is an open-fo.s in $\mathcal{P}$. By hypothesis $\psi(\bar{1} - \omega)$ is an open-fo.s in Q. Let $\eta = \bar{1} - \psi(\bar{1} - \omega)$ (i.e., η is a closed-fo.s in Q). Since $\psi^{-1}(\delta) \leq \omega$, we have $\bar{1} - \omega \leq \bar{1} - \psi^{-1}(\delta)$, implies $\psi(\bar{1} - \omega) \leq \psi(\bar{1} - \psi^{-1}(\delta)) = \bar{1} - \psi(\psi^{-1}(\delta)) \leq \bar{1} - \delta$. Hence, $\delta \leq \bar{1} - \psi(\bar{1} - \omega) = \eta$. Since ψ is fo.irresolute open, then η is a closed-fo.in Q and $\psi^{-1}(\eta) = \psi^{-1}(\bar{1} - \psi(\bar{1} - \omega)) = \bar{1} - \psi^{-1}(\psi(\bar{1} - \omega)) \leq \omega$. Consequently, $\psi^{-1}(\eta) \leq \omega$.

Theorem 4.7. Let $(\mathcal{P},\sigma)$ and (Q,σ^*) be any two *fts's.* A mapping $\psi:(\mathcal{P},\sigma)\longrightarrow(Q,\sigma^*)$ is fo.irresolute open iff $\psi(Int_{FO}(\omega))\leq Int_{FO}(\psi(\omega))$, for every fuzzy set ω of $\mathcal{P}$.

PROOF. Suppose ψ is fo.irresolute open. Then, $\psi(Int_{FO}(\omega))$ is an open-fo.s in Q. Hence, $\psi(Int_{FO}(\omega))=Int_{FO}(\psi(Int_{FO}(\omega)))\leq Int_{FO}(\psi(\omega))$.

Sufficiency, let ω be an open-fo.s in $\mathcal{P}$, then by hypothesis $\psi(Int_{FO}(\omega))\leq Int_{FO}(\psi(\omega))$. Hence, $\psi(\omega)$ is an open-fo.s in Q.

Theorem 4.8. Let $(\mathcal{P},\sigma)$, (Q,σ^*) and (Z,σ^{**}) be *fts's.* Let $\psi:(\mathcal{P},\sigma)\longrightarrow(Q,\sigma^*)$ and $g:(Q,\sigma^*)\longrightarrow(Z,\sigma^{**})$ be fo.irresolute open mappings. Then, $go\psi$ is fo.irresolute open.

PROOF. Straightforward from Definition 4.2.

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