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SPECIAL TYPES OF SINGLE VALUED NEUTROSOPHIC GRAPHS

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Abstract. Neutrosophic theory has many applications in graph theory, single valued neutrosophic graph (SVNG) is the generalization of fuzzy graph and intuitionistic fuzzy graph. In this paper, we introduced some types of SVNGs, which are subdivision SVNGs, middle SVNGs, total SVNGs and single valued neutrosophic line graphs (SVNLGs), also discussed the isomorphism, co weak isomorphism and weak isomorphism properties of subdivision SVNGs, middle SVNGs, total SVNGs and SVNLGs.

Keywords: Single valued neutrosophic line graph, Subdivision SVNG, middle SVNG, total SVNG.

AMS Subject Classification: 05C75.

1. Introduction

Neutrosopic sets were introduced by Smarandache [6], which are the generalization of fuzzy sets and intuitionistic fuzzy sets. Later on The Liu worked on neutrosophic sets ant its uses in decision making problems in [11, 13, 14, 8, 15]. The Neutrosophic sets have many applications in medical, management sciences, life sciences, engineering, graph theory, robotics, automata theory and computer science. The single valued neutrosophic graphs were introduced by Broumi, Talea, Bakali and Smarandache [9]. Recently in [12, 10] proposed some algorithms dealt with shortest path problem in a network (graph) where edge weights are characterized by a neutrosophic numbers including single valued neutrosophic numbers, bipolar neutrosophic numbers and interval valued neutrosophic numbers. Also the concept of single valued neutrosophic hyper-graphs generalized by Hassan et al in [1, 2, 4, 7]. Later on Malik and Hassan in [3] defined the concept of single valued neutrosophic trees and studied some of their properties. A graph is a convenient way of representing information involving relationship between objects. The objects are represented by vertices's and the relations by edges. When there is vagueness in the description of the objects or in its relations hips or in both, it is natural that we need to design a fuzzy graph Model. The special types and its truncations were paid the way by [5]. The SVNGs have many applications in path problems, networks and computer science. The strong

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SVNG and complete SVNG are the special types of SVNG. In this paper, we introduce the another types of SVNGs, which are subdivision SVNGs, middle SVNGs, total SVNGs and SVNLGs. These are all the strong SVNGs, also we discuss their relations based on isomorphism, co weak isomorphism and weak isomorphism.

2. Preliminaries

In this section we recall some basic concepts on SVNG and let G denotes SVNG and $G^* = (V, E)$ denotes its underlying crisp graph.

Definition 2.1. [6] *Let X be a crisp set, the single valued neutrosophic set (SVNS) A is characterized by three membership functions T_A, I_A and F_A. which are truth, indeterminacy and falsity membership functions, for every element in X its memberships contained in* $[0, 1]$.

Definition 2.2. [?] *The single valued neutrosophic graph (SVNG) is a pair $G = (C, D)$ of $G^* = (V, E)$, where C is SVNS on V and D is SVNS on E such that*

$$T_D(\alpha\beta) \leq \min(T_C(\alpha), T_C(\beta))$$

$$I_D(\alpha\beta) \geq \max(I_C(\alpha), I_C(\beta))$$

$$F_D(\alpha\beta) \geq \max(F_C(\alpha), F_C(\beta))$$

whenever

$$0 \leq T_D(\alpha\beta) + I_D(\alpha\beta) + F_D(\alpha\beta) \leq 3$$

$\forall\ \alpha, \beta \in V$. In this case D is SVN-Relation on C. The SVNG G is said to be complete (strong) SVNG, whenever

$$T_D(xy) = \min(T_C(x), T_C(y))$$

$$I_D(xy) = \max(I_C(x), I_C(y))$$

$$F_D(xy) = \max(F_C(x), F_C(y))$$

$\forall\ x, y \in V (\forall xy \in E)$. The order of G, which is denoted by $O(G)$, is defined by

$$O(G) = (O_T(G), O_I(G), O_F(G)),$$

where

$$O_T(G) = \sum_{\alpha\in V} T_C(\alpha),\ O_I(G) = \sum_{\alpha\in V} I_C(\alpha),\ O_F(G) = \sum_{\alpha\in V} F_C(\alpha).$$

The size of G, which is denoted $S(G)$, is defined by

$$S(G) = (S_T(G), S_I(G), S_F(G)),$$

where

$$S_T(G) = \sum_{\alpha\beta\in E} T_D(\alpha\beta),\ S_I(G) = \sum_{\alpha\beta\in E} I_D(\alpha\beta),\ S_F(G) = \sum_{\alpha\beta\in E} F_D(\alpha\beta).$$

The degree of a vertex α in G, which is denoted by $d_G(\alpha)$, is defined by

$$d_G(\alpha) = (d_T(\alpha), d_I(\alpha), d_F(\alpha)),$$

where

$$d_T(\alpha) = \sum_{\alpha\beta\in E} T_D(\alpha\beta),\ d_I(\alpha) = \sum_{\alpha\beta\in E} I_D(\alpha\beta),\ d_F(\alpha) = \sum_{\alpha\beta\in E} F_D(\alpha\beta).$$

3. Types of SVNGs

Definition 3.1. *Let $G_1 = (C_1, D_1)$ and $G_2 = (C_2, D_2)$ be two SVNGs of $G_1^* = (V_1, E_1)$ and $G_2^* = (V_2, E_2)$, respectively. Then the homomorphism $\chi : V_1 \to V_2$ is a mapping from V_1 into V_2 satisfying following conditions*

$$T_{C_1}(p) \leq T_{C_2}(\chi(p)),\ I_{C_1}(p) \geq I_{C_2}(\chi(p)),\ F_{C_1}(p) \geq F_{C_2}(\chi(p))$$

$\forall p \in V_1$.

$$T_{D_1}(pq) \leq T_{D_2}(\chi(p)\chi(q)),\ I_{D_1}(pq) \geq I_{D_2}(\chi(p)\chi(q)),\ F_{D_1}(pq) \geq F_{D_2}(\chi(p)\chi(q))$$

$\forall pq \in E_1$. *The weak isomorphism $\upsilon : V_1 \to V_2$ is a bijective homomorphism from V_1 into V_2 satisfying following conditions*

$$T_{C_1}(p) = T_{C_2}(\upsilon(p)),\ I_{C_1}(p) = I_{C_2}(\upsilon(p)),\ F_{C_1}(p) = F_{C_2}(\upsilon(p))$$

$\forall p \in V_1$. *The co-weak isomorphism $\kappa : V_1 \to V_2$ is a bijective homomorphism from V_1 into V_2 satisfying following conditions*

$$T_{D_1}(pq) = T_{D_2}(\kappa(p)\kappa(q)),\ I_{D_1}(pq) = I_{D_2}(\kappa(p)\kappa(q)),\ F_{D_1}(pq) = F_{D_2}(\kappa(p)\kappa(q))$$

$\forall pq \in E_1$. *An isomorphism $\psi : V_1 \to V_2$ is a bijective homomorphism from V_1 into V_2 satisfying following conditions*

$$T_{C_1}(p) = T_{C_2}(\psi(p)), I_{C_1}(p) = I_{C_2}(\upsilon(p)), F_{C_1}(p) = F_{C_2}(\psi(p))$$

$\forall p \in V_1$.

$$T_{D_1}(pq) = T_{D_2}(\psi(p)\psi(q)),\ I_{D_1}(pq) = I_{D_2}(\psi(p)\psi(q)),\ F_{D_1}(pq) = F_{D_2}(\psi(p)\psi(q))$$

$\forall pq \in E_1$.

Remark 3.1. *The weak isomorphism between two SVNGs preserves the orders.*

Remark 3.2. *The isomorphism between two SVNGs is an equivalence relation.*

Remark 3.3. *The isomorphism between two SVNGs is an equivalence relation.*

Remark 3.4. *The weak isomorphism between SVNGs is a partial order relation.*

Remark 3.5. *The co-weak isomorphism between two SVNGs preserves the sizes.*

Remark 3.6. *The co-weak isomorphism between SVNGs is a partial order relation.*

Remark 3.7. *The isomorphism between two SVNGs preserves the orders and sizes.*

Remark 3.8. *The isomorphism between two SVNGs preserves the degrees of their vertices's.*

Definition 3.2. *The subdivision SVNG $sd(G) = (C, D)$ of SVNG $G = (A, B)$, where C is a SVNS on $V \cup E$ and D is a SVNR on C, such that*
(1) *$C = A$ on V and $C = B$ on E.* **(2)** *If $v \in V$ lie on edge $e \in E$, then*

$$T_D(ve) = \min(T_A(v), T_B(e))$$
$$I_D(ve) = \max(I_A(v), I_B(e))$$
$$F_D(ve) = \max(F_A(v), F_B(e))$$

else

$$D(ve) = O = (0, 0, 0).$$

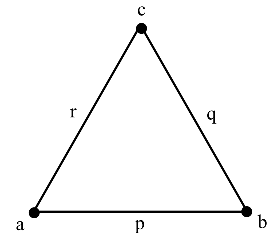

Figure 1. Crisp Graph of SVNG.

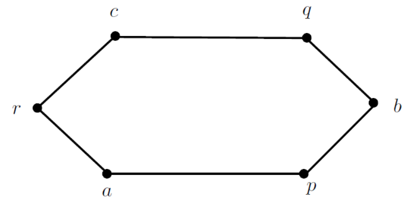

Figure 2. Crisp Graph of SDSVNG.

Example 3.1. *The crisp graph* $G^* = (V, E)$ *of SVNG* $G = (A, B)$*, which is shown in Figure 1. The SVNSs* A *and* B *over* $V = \{a, b, c\}$ *and* $E = \{p = ab, q = bc, r = ac\}$ *are defined in Table 1.*

A	T_A	I_A	F_A	B	T_B	I_B	F_B
a	0.2	0.1	0.4	p	0.2	0.4	0.5
b	0.3	0.2	0.5	q	0.3	0.8	0.6
c	0.4	0.7	0.6	r	0.1	0.7	0.9

Table 1. SVNSs of SVNG.

The crisp graph of SDSVNG $sd(G) = (C, D)$ *of a SVNG* G*,is shown in Figure 2. By calculations the SVNSs* C *and* D *are given in Table 2.*

C	T_C	I_C	F_C	D	T_D	I_D	F_D
a	0.2	0.1	0.4	ap	0.2	0.4	0.5
p	0.2	0.4	0.5	pb	0.2	0.4	0.5
b	0.3	0.2	0.5	bq	0.3	0.8	0.6
q	0.3	0.8	0.6	qc	0.3	0.8	0.6
c	0.4	0.7	0.6	cr	0.1	0.7	0.9
r	0.1	0.7	0.9	ra	0.1	0.7	0.9

Table 2. SVNSs of SDSVNG.

Proposition 3.1. *Let* G *be a SVNG and* $sd(G)$ *be the subdivision SVNG of a SVNG* G*, then*
(**1**) $O(sd(G)) = O(G) + S(G)$.
(**2**) $S(sd(G)) = 2S(G)$.

Proposition 3.2. *If* G *is complete SVNG, then* $sd(G)$ *need not to be complete SVNG.*

Definition 3.3. *The total single valued neutrosophic graph (TSVNG)* $T(G) = (C, D)$ *of* $G = (A, B)$*, where* C *is a SVNS on* $V \cup E$ *and* D *is a SVNR on* C *defined by*

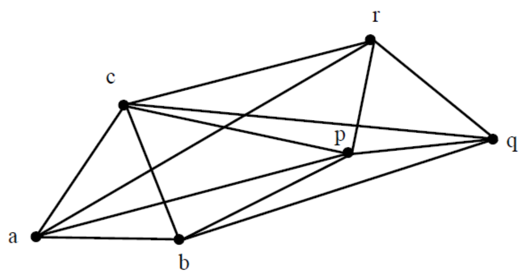

FIGURE 3. Crisp Graph of TSVNG.

(**1**) *$C = A$ on V and $C = B$ on E.*
(**2**) *If $v \in V$ lie on edge $e \in E$, then*

$$T_D(ve) = \min(T_A(v), T_B(e))$$
$$I_D(ve) = \max(I_A(v), I_B(e))$$
$$F_D(ve) = \max(F_A(v), F_B(e))$$

else

$$D(ve) = O = (0, 0, 0).$$

(**3**) *If $\alpha, \beta \in E$, then*

$$T_D(\alpha\beta) = T_B(\alpha\beta),\ I_D(\alpha\beta) = I_B(\alpha\beta),\ F_D(\alpha\beta) = F_B(\alpha\beta).$$

(**4**) *If $e, f \in E$ have a common vertex, then*

$$T_D(ef) = \min(T_B(e), T_B(f))$$
$$I_D(ef) = \max(I_B(e), I_B(f))$$
$$F_D(ef) = \max(F_B(e), F_B(f))$$

else

$$D(ef) = O = (0, 0, 0).$$

D	T_D	I_D	F_D	D	T_D	I_D	F_D
ab	0.2	0.4	0.5	ap	0.2	0.4	0.5
bc	0.3	0.8	0.6	pb	0.2	0.4	0.5
ca	0.1	0.7	0.9	bq	0.3	0.8	0.6
pq	0.2	0.8	0.6	qc	0.3	0.8	0.6
qr	0.1	0.8	0.9	cr	0.1	0.7	0.9
rp	0.1	0.7	0.9	ra	0.1	0.7	0.9

TABLE 3. SVNS of TSVNG.

Example 3.2. *Consider the Example 3.1 the crisp graph of TSVNG $T(G) = (C, D)$, is shown in Figure 3. Here C is same as given in Example 3.1. By calculations the SVNS D is defined in Table 3.*

Proposition 3.3. *Let G be a $SVNG$ and $T(G)$ be the TSVNG of G, then*
(**1**) $O(T(G)) = O(G) + S(G) = O(sd(G))$.
(**2**) $S(sd(G)) = 2S(G)$.

Proposition 3.4. *If G is a SVNG, then $sd(G)$ is weak isomorphic to $T(G)$.*

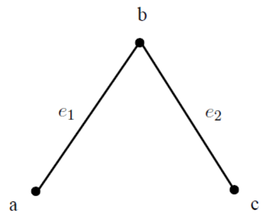

FIGURE 4. Crisp Graph of SVNG.

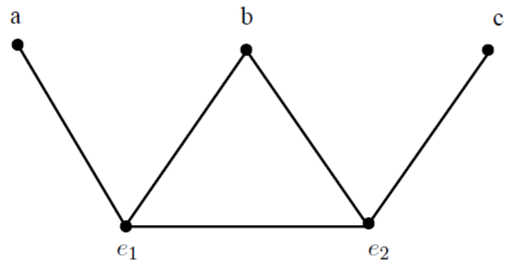

FIGURE 5. Crisp Graph of MSVNG.

Definition 3.4. *The middle single valued neutrosophic graph (MSVNG) $M(G) = (C, D)$ of $G = (A, B)$, where C is a SVNS on $V \cup E$ and D is a SVNR on C defined by*
(**1**) $C = A$ *on* V *and* $C = B$ *on* E, *else* $C = O = (0, 0, 0)$.
(**2**) *If* $v \in V$ *lie on edge* $e \in E$, *then*

$$T_D(ve) = T_B(e),\ I_D(ve) = I_B(e),\ F_D(ve) = F_B(e)$$

else

$$D(ve) = O = (0, 0, 0).$$

(**3**) *If* $u, v \in V$, *then*

$$D(uv) = O = (0, 0, 0).$$

(**4**) *If* $e, f \in E$ *and* e *and* f *are adjacent in* G, *then*

$$T_D(ef) = T_B(uv),\ I_D(ef) = I_B(uv),\ F_D(ef) = F_B(uv).$$

Example 3.3. *Consider the SVNG $G = (A, B)$ of a G^*, which is shown in Figure 4. The SVNSs A and B are defined in Table 4. The crisp graph of MSVNG $M(G) = (C, D)$, is shown in Figure 5. By calculations, the SVNSs C and D are defined in Table 5.*

A	T_A	I_A	F_A
a	0.3	0.4	0.4
b	0.7	0.6	0.3
c	0.9	0.7	0.2
B	T_B	I_B	F_B
e_1	0.2	0.6	0.6
e_2	0.4	0.8	0.7

TABLE 4. SVNSs of SVNG.

C	T_C	I_C	F_C	D	T_D	I_D	F_D
a	0.3	0.4	0.5	e_1e_2	0.2	0.8	0.7
b	0.7	0.6	0.3	ae_1	0.2	0.6	0.6
c	0.9	0.7	0.2	be_1	0.2	0.6	0.6
e_1	0.2	0.6	0.6	be_2	0.2	0.6	0.6
e_2	0.4	0.8	0.7	ce_2	0.4	0.8	0.7

Table 5. SVNSs of MSVNG.

Remark 3.9. *Let G be a SVNG and $M(G)$ be the MSVNG of G, then* $O(M(G)) = O(G) + S(G)$.

Remark 3.10. *Let G be a SVNG, then $M(G)$ is a strong SVNG.*

Remark 3.11. *If G is complete SVNG, then $M(G)$ need not to be complete SVNG.*

Proposition 3.5. *Let G be a SVNG, then $sd(G)$ is weak isomorphic with $M(G)$.*

Proposition 3.6. *Let G be a SVNG, then $M(G)$ is weak isomorphic with $T(G)$.*

Proposition 3.7. *Let G be a SVNG, then $T(G)$ is isomorphic with $G \cup M(G)$.*

Definition 3.5. *Let the intersection graph be $P(X) = (X, Y)$ of a G^*, let C_1 and D_1 be SVNSs over V and E. Also let C_2 and D_2 be SVNSs over X and Y. Then the single valued neutrosophic intersection graph (SVNIG) of a SVNG $G = (C_1, D_1)$ is a SVNG $P(G) = (C_2, D_2)$, such that*

$$T_{C_2}(X_i) = T_{C_1}(v_i), I_{C_2}(X_i) = I_{C_1}(v_i), F_{C_2}(X_i) = F_{C_1}(v_i)$$

$$T_{D_2}(X_iX_j) = T_{D_1}(v_iv_j), I_{D_2}(X_iX_j) = I_{D_1}(v_iv_j), F_{D_2}(X_iX_j) = F_{D_1}(v_iv_j)$$

$\forall\ X_i, X_j \in X$ *and* $X_iX_j \in Y$.

Proposition 3.8. *Let $G = (A_1, B_1)$ be a SVNG of $G^* = (V, E)$, and let $P(G) = (A_2, B_2)$ be a SVNIG, then*
(**1**) *The SVNIG is a SVNG.*
(**2**) *The SVNG is isomorphic to SVNIG.*

Proof. (**1**) By the definition of SVNIG, we have

$$\begin{aligned} T_{B_2}(S_iS_j) &= T_{B_1}(v_iv_j) \le \min(T_{A_1}(v_i), T_{A_1}(v_j)) = \min(T_{A_2}(S_i), T_{A_2}(S_j)) \\ I_{B_2}(S_iS_j) &= I_{B_1}(v_iv_j) \ge \max(I_{A_1}(v_i), I_{A_1}(v_j)) = \max(I_{A_2}(S_i), I_{A_2}(S_j)) \\ F_{B_2}(S_iS_j) &= F_{B_1}(v_iv_j) \ge \max(F_{A_1}(v_i), F_{A_1}(v_j)) = \max(F_{A_2}(S_i), F_{A_2}(S_j)) \end{aligned}$$

this shows that SVNIG is a SVNG.
(**2**) Define $f : V \to X$ by $f(v_i) = S_i$ for $i = 1, 2, 3, \ldots, n$ clearly f is bijective. Now $v_iv_j \in E$ if and only if $S_iS_j \in T$, and $T = \{f(v_i)f(v_j) : v_iv_j \in E\}$, also

$$\begin{aligned} T_{A_2}(f(v_i)) &= T_{A_2}(S_i) = T_{A_1}(v_i) \\ I_{A_2}(f(v_i)) &= I_{A_2}(S_i) = I_{A_1}(v_i) \\ F_{A_2}(f(v_i)) &= F_{A_2}(S_i) = F_{A_1}(v_i) \end{aligned}$$

$\forall\ v_i \in V$.

$$T_{B_2}(f(v_i)f(v_j)) = T_{B_2}(S_iS_j) = T_{B_1}(v_iv_j)$$
$$I_{B_2}(f(v_i)f(v_j)) = I_{B_2}(S_iS_j) = I_{B_1}(v_iv_j)$$
$$F_{B_2}(f(v_i)f(v_j)) = F_{B_2}(S_iS_j) = F_{B_1}(v_iv_j)$$

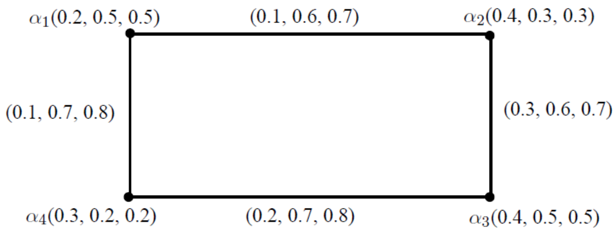

FIGURE 6. The SVNG.

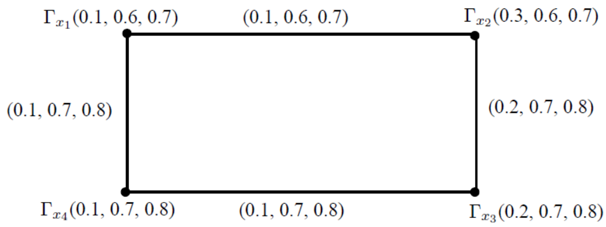

FIGURE 7. The SVNLG.

$\forall\ v_iv_j \in E.$ □

Definition 3.6. *Let $G^* = (V, E)$ and $L(G^*) = (X, Y)$ be its line graph, where A_1 and B_1 be SVNSs over V and E. Let A_2 and B_2 be SVNSs over X and Y. The single valued neutrosophic line graph (SVNLG) of SVNG $G = (A_1, B_1)$ is SVNG $L(G) = (A_2, B_2)$, such that*

$$T_{A_2}(S_x) = T_{B_1}(x) = T_{B_1}(u_xv_x)$$
$$I_{A_2}(S_x) = I_{B_1}(x) = I_{B_1}(u_xv_x)$$
$$F_{A_2}(S_x) = F_{B_1}(x) = F_{B_1}(u_xv_x)$$

$\forall\ S_x, S_y \in X$ *and*

$$T_{B_2}(S_xS_y) = \min(T_{B_1}(x), T_{B_1}(y))$$
$$I_{B_2}(S_xS_y) = \max(I_{B_1}(x), I_{B_1}(y))$$
$$F_{B_2}(S_xS_y) = \max(F_{B_1}(x), F_{B_1}(y))$$

$\forall\ S_xS_y \in Y.$

Example 3.4. *Consider the $G^* = (V, E)$ where $V = \{\alpha_1, \alpha_2, \alpha_3, \alpha_4\}$ and $E = \{x_1 = \alpha_1\alpha_2, x_2 = \alpha_2\alpha_3, x_3 = \alpha_3\alpha_4, x_4 = \alpha_4\alpha_1\}$ and $G = (A_1, B_1)$ is SVNG of $G^* = (V, E)$, which is shown in Figure 6. Consider the $L(G^*) = (X, Y)$, such that $X = \{\Gamma_{x_1}, \Gamma_{x_2}, \Gamma_{x_3}, \Gamma_{x_4}\}$ and $Y = \{\Gamma_{x_1}\Gamma_{x_2}, \Gamma_{x_2}\Gamma_{x_3}, \Gamma_{x_3}\Gamma_{x_4}, \Gamma_{x_4}\Gamma_{x_1}\}$. Let A_2 and B_2 be SVNSs over X and Y. Then by calculations, SVNLG $L(G)$ is shown in Figure 7.*

Proposition 3.9. *Every SVNLG is a strong SVNG.*

Proposition 3.10. *The $L(G) = (A_2, B_2)$ is a SVNLG corresponding to SVNG $G = (A_1, B_1)$.*

Proposition 3.11. *The $L(G) = (A_2, B_2)$ is a SVNLG of some SVNG $G = (A_1, B_1)$ if and only if*

$$T_{B_2}(S_xS_y) = \min(T_{A_2}(S_x), T_{A_2}(S_y))$$
$$I_{B_2}(S_xS_y) = \max(I_{A_2}(S_x), I_{A_2}(S_y))$$

$$F_{B_2}(S_xS_y) = \max(F_{A_2}(S_x), F_{A_2}(S_y))$$

$\forall\ S_xS_y \in Y$.

Proof. Assume that

$$T_{B_2}(S_xS_y) = \min(T_{A_2}(S_x), T_{A_2}(S_y))$$
$$I_{B_2}(S_xS_y) = \max(I_{A_2}(S_x), I_{A_2}(S_y))$$
$$F_{B_2}(S_xS_y) = \max(F_{A_2}(S_x), F_{A_2}(S_y))$$

$\forall\ S_xS_y \in Y$. We define

$$T_{A_1}(x) = T_{A_2}(S_x),\ I_{A_1}(x) = I_{A_2}(S_x),\ F_{A_1}(x) = F_{A_2}(S_x)$$

$\forall\ x \in E$, then

$$T_{B_2}(S_xS_y) = \min(T_{A_2}(S_x), T_{A_2}(S_y)) = \min(T_{A_2}(x), T_{A_2}(y))$$
$$I_{B_2}(S_xS_y) = \max(I_{A_2}(S_x), I_{A_2}(S_y)) = \max(I_{A_2}(x), I_{A_2}(y))$$
$$F_{B_2}(S_xS_y) = \max(F_{A_2}(S_x), F_{A_2}(S_y)) = \max(F_{A_2}(x), F_{A_2}(y))$$

A SVNS A_1 that yields the property

$$T_{B_1}(xy) \leq \min(T_{A_1}(x), T_{A_1}(y))$$
$$I_{B_1}(xy) \geq \max(I_{A_1}(x), I_{A_1}(y))$$
$$F_{B_1}(xy) \geq \max(F_{A_1}(x), F_{A_1}(y))$$

will suffice. Converse is straight forward. □

Proposition 3.12. *If $L(G) = (A_2, B_2)$ is SVNLG of SVNG $G = (A_1, B_1)$, then $L(G^*)$ is the crisp line graph of G^*.*

Proof. Since $L(G)$ be a SVNLG,

$$T_{A_2}(S_x) = T_{B_1}(x),\ I_{A_2}(S_x) = I_{B_1}(x),\ F_{A_2}(S_x) = F_{B_1}(x)$$

$\forall\ x \in E$, and so $S_x \in X$ if and only if $x \in E$, also

$$T_{B_2}(S_xS_y) = \min(T_{B_1}(x), T_{B_1}(y))$$
$$I_{B_2}(S_xS_y) = \max(I_{B_1}(x), I_{B_1}(y))$$
$$F_{B_2}(S_xS_y) = \max(F_{B_1}(x), F_{B_1}(y))$$

$\forall\ S_xS_y \in Y$ and so, $Y = \{S_xS_y : S_x \cap S_y \neq \phi, x, y \in E, x \neq y\}$. □

Proposition 3.13. *If $L(G) = (A_2, B_2)$ is SVNLG of SVNG $G = (A_1, B_1)$ if and only if $L(G^*) = (X, Y)$ is the line graph and*

$$T_{B_2}(xy) = \min(T_{A_2}(x), T_{A_2}(y))$$
$$I_{B_2}(xy) = \max(I_{A_2}(x), I_{A_2}(y))$$
$$F_{B_2}(xy) = \max(F_{A_2}(x), F_{A_2}(y))$$

$\forall\ xy \in Y$.

Proof. It follows from Propositions 3.11 and 3.12. □

Proposition 3.14. *Let G be a SVNG, then $M(G)$ is isomorphic with $sd(G) \cup L(G)$.*

Theorem 3.1. *Let $L(G) = (A_2, B_2)$ be SVNLG corresponding to SVNG $G = (A_1, B_1)$.*
(a) *If G is weak isomorphic onto $L(G)$ if and only if $\forall\ v \in V$, $x \in E$ and G^* to be a cycle, such that*

$$T_{A_1}(v) = T_{B_1}(x),\ I_{A_1}(v) = T_{B_1}(x),\ F_{A_1}(v) = T_{B_1}(x).$$

(b) *If G is weak isomorphic onto $L(G)$, then G and $L(G)$ are isomorphic.*

Proof. By hypothesis G^* is a cycle. Let $V = \{v_1, v_2, v_3, \ldots, v_n\}$ and $E = \{x_1 = v_1v_2, x_2 = v_2v_3, \ldots, x_n = v_nv_1\}$ where $P : v_1v_2v_3 \ldots v_n$ is a cycle, characterize a SVNS A_1 by $A_1(v_i) = (p_i, q_i, r_i)$ and B_1 by $B_1(x_i) = (a_i, b_i, c_i)$ for $i = 1, 2, 3, \ldots, n$ and $v_{n+1} = v_1$, then for $p_{n+1} = p_1$, $q_{n+1} = q_1$, $r_{n+1} = r_1$, we have

$$a_i \leq \min(p_i, p_{i+1}),\ b_i \geq \max(q_i, q_{i+1}),\ c_i \geq \max(r_i, r_{i+1})$$

for $i = 1, 2, 3, \ldots, n$. Now $X = \{\Gamma_{x_1}, \Gamma_{x_2}, \ldots, \Gamma_{x_n}\}$ and $Y = \{\Gamma_{x_1}\Gamma_{x_2}, \Gamma_{x_2}\Gamma_{x_3}, \ldots, \Gamma_{x_n}\Gamma_{x_1}\}$, thus for $a_{n+1} = a_1$, we obtain

$$A_2(\Gamma_{x_i}) = B_1(x_i) = (a_i, b_i, c_i)$$

and

$$B_2(\Gamma_{x_i}\Gamma_{x_{i+1}}) = (\min(a_i, a_{i+1}), \max(b_i, b_{i+1}), \max(c_i, c_{i+1}))$$

for $i = 1, 2, 3, \ldots, n$ and $v_{n+1} = v_1$. Since f preserves adjacency, hence it induce permutation π of $\{1, 2, 3, \ldots, n\}$,

$$f(v_i) = \Gamma_{v_{\pi(i)}v_{\pi(i)+1}}$$

and

$$v_iv_{i+1} \to f(v_i)f(v_{i+1}) = \Gamma_{v_{\pi(i)}v_{\pi(i)+1}}\Gamma_{v_{\pi(i+1)}v_{\pi(i+1)+1}}$$

for $i = 1, 2, 3, \ldots, n-1$. Thus

$$p_i = T_{A_1}(v_i) \leq T_{A_2}(f(v_i)) = T_{A_2}(\Gamma_{v_{\pi(i)}v_{\pi(i)+1}}) = T_{B_1}(v_{\pi(i)}v_{\pi(i)+1}) = a_{\pi(i)}$$

similarly, $q_i \geq b_{\pi(i)}$ and $r_i \geq c_{\pi(i)}$ and

$$\begin{aligned} a_i &= T_{B_1}(v_iv_{i+1}) \leq T_{B_2}(f(v_i)f(v_{i+1})) \\ &= T_{B_2}(\Gamma_{v_{\pi(i)}v_{\pi(i)+1}}\Gamma_{v_{\pi(i+1)}v_{\pi(i+1)+1}}) \\ &= \min(T_{B_1}(v_{\pi(i)}v_{\pi(i)+1}), T_{B_1}(v_{\pi(i+1)}v_{\pi(i+1)+1})) \\ &= \min(a_{\pi(i)}, a_{\pi(i)+1}) \end{aligned}$$

similarly $b_i \geq \max(b_{\pi(i)}, b_{\pi(i)+1})$ and $c_i \geq \max(c_{\pi(i)}, c_{\pi(i)+1})$ for $i = 1, 2, 3, \ldots, n$. Therefore

$$p_i \leq a_{\pi(i)}, q_i \geq b_{\pi(i)}, r_i \geq c_{\pi(i)}$$

and

$$a_i \leq \min(a_{\pi(i)}, a_{\pi(i)+1}),\ b_i \geq \max(b_{\pi(i)}, b_{\pi(i)+1}),\ c_i \geq \max(c_{\pi(i)}, c_{\pi(i)+1})$$

thus

$$a_i \leq a_{\pi(i)}, b_i \geq b_{\pi(i)}, c_i \geq c_{\pi(i)}$$

and so

$$a_{\pi(i)} \leq a_{\pi(\pi(i))}, b_{\pi(i)} \geq b_{\pi(\pi(i))}, c_{\pi(i)} \geq c_{\pi(\pi(i))}$$

$\forall\ i = 1, 2, 3, \ldots, n$. Next to extend,

$$\begin{gathered} a_i \leq a_{\pi(i)} \leq \ldots \leq a_{\pi^j(i)} \leq a_i \\ b_i \geq b_{\pi(i)} \geq \ldots \geq b_{\pi^j(i)} \geq b_i \\ c_i \geq c_{\pi(i)} \geq \ldots \geq c_{\pi^j(i)} \geq c_i \end{gathered}$$

where π^{j+1} identity. Hence

$$a_i = a_{\pi(i)}, b_i = b_{\pi(i)}, c_i = c_{\pi(i)}$$

$\forall\ i = 1, 2, 3, \ldots, n$. Thus we conclude that

$$a_i \leq a_{\pi(i+1)} = a_{i+1}, b_i \geq b_{\pi(i+1)} = b_{i+1}, c_i \geq c_{\pi(i+1)} = c_{i+1}$$

which together with

$$a_{n+1} = a_1, b_{n+1} = b_1, c_{n+1} = c_1$$

which implies that $a_i = a_1, b_i = b_1, c_i = c_1 \ \forall \ i = 1, 2, 3, \ldots, n$. Thus we have

$$a_1 = a_2 = \ldots = a_n = p_1 = p_2 = \ldots = p_n$$

$$b_1 = b_2 = \ldots = b_n = q_1 = q_2 = \ldots = q_n$$

$$c_1 = c_2 = \ldots = c_n = r_1 = r_2 = \ldots = r_n$$

Therefore (a) and (b) holds, since converse of result (a) is straight forward. □

4. Conclusion

The neutrosophic graphs have many applications in path problems, networks and computer science. Strong SVNG and complete SVNG are the types of SVNG. In this paper, we discussed the special types of SVNGs, subdivision SVNGs, middle SVNGs, total SVNGs and SVNLGs of the given SVNGs. We investigated isomorphism properties of subdivision SVNGs, middle SVNGs, total SVNGs and SVNLGs. In our future research, we will focus on types of bipolar SVNGs and interval valued neutrosophic graphs.

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