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ON TRANS-SASAKIAN MANIFOLD EQUIPPED WITH m-PROJECTIVE CURVATURE TENSOR

J. P. JAISWAL [1], A. S. YADAV[2], §

ABSTRACT. The work towards of the attending paper is to interpret the trans-Sasakian manifold equipped with m-projective curvature tensor and its various geometric properties. First, we observe that m-projectively flat trans-Sasakian manifold is Einstein. In order, we discussed m-projectively conservative and ϕ-m-projectively flat trans-Sasakian manifold. Following, we found the sufficient condition for quasi m-projectively flat trans-Sasakian manifold to be m-projectively flat. In the end, the m-projectively and ϕ-m-projectively symmetric trans-Sasakian manifolds are analyzed.

Keywords: Trans-Sasakian manifold, m-projectively flat, Einstein manifold, m-projective conservative.

AMS Subject Classification: 53C15, 53B05.

1. INTRODUCTION

Oubina [8] initiated a new class of almost contract manifold, called trans-Sasakian manifold, which is of type (0, 0), (α, 0) and (0, β) are respectively, familiar as the cosymplectic, α-Sasakian and β-Kenmotsu manifold, α, β are the scalar smooth functions. In fact if $\alpha = 0,\ \beta = 1$ and $\alpha = 1,\ \beta = 0$, then a trans-Sasakian manifold will enhance a Kenmotsu and Sasakian manifold, respectively.

In 1971, Pokhariyal and Mishra[9] established a new curvature known as m-projectively curvature tensor on Riemannian manifold. Followed that many researcher such as Ojha [6, 7], Singh [12], Choubey and Ojha [3] studied properties of m-projective curvature in different manifolds. We say that a Riemannian manifold is flat if its curvature vanishes at each point. Following this sense Ojha [7] and Zengin [15] consider the m-projective flat in the Sasakian and LP-Sasakian manifold, respectively. The idea of local symmetry of a Riemannian manifold studied by Cartan [2] and mild version of local symmetry, Takahashi [13] introduced the notion of ϕ-symmetry on a Sasakian manifold. In this series, we investigate some results about flatness, symmetry and space time with m-projective curvature in trans-Sasakian structure.

The paper classified as follows: In part 2, we put some basic formulae and definition of trans-Sasakian manifold. In the next part, we confer about m-projectively flat trans-Sasakian manifold and mentioned a sufficient condition for such a manifold to be Einstein.

[1] Department of Mathematics, Maulana Azad National Institute of Technology, Bhopal, M. P. India.
e-mail: asstprofjpmanit@gmail.com, ORCID[1]: http://orcid.org/0000-0003-4308-2280;

[2] Department of Mathematics, Maulana Azad National Institute of Technology, Bhopal, M. P. India.
e-mail: arjunsinghyadav7@gmail.com, ORCID[2]: http://orcid.org/0000-0003-3594-7846;

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Then, we found the condition such that the m-projective conservative trans-Sasakian manifold is of constant curvature. Successive that, we find the condition for ϕ-m-projectively flat trans-Sasakian manifold to be η-Einstein and quasi m-projectively flat is of constant curvature. In the last, we examine the m-projective and ϕ-m-projective symmetric trans-Sasakian manifolds.

2. Preliminaries

In this section, we mention some basic formulae and definitions, which will be used later.

Let M^m be an $m = (2n+1)$ dimensional almost contact metric manifold [1], consisting of a $(1,1)$ tensor field ϕ, a characteristic vector field ξ, a 1-form η and a Riemannian metric g. Then

$$\phi^2 X = -X + \eta(X)\xi,\ \ \eta(\xi) = 1,\ \ \eta(\phi X) = 0,\ \ \phi\xi = 0, \tag{1}$$

$$g(\phi X, \phi Y) = g(X, Y) - \eta(X)\eta(Y), \tag{2}$$

$$g(\xi, \xi) = 1,\ \ \phi \circ \xi = 0,\ \ \eta \circ \phi = 0, \tag{3}$$

for any X, Y in TM. From (1) and (2), it can be easily seen that

$$g(X, \phi Y) = -g(\phi X, Y), g(X, \xi) = \eta(X). \tag{4}$$

For an almost contact metric structure (ϕ, ξ, η, g) on M, we put

$$\Phi(X, Y) = g(X, \phi Y). \tag{5}$$

Let M^{2n+1} be almost contact manifold and consider the structure $(M \times \mathcal{R},\ \mathcal{J},\ \mathcal{G})$ belongs to the class W_4 of the Hermitian manifolds, we denote a vector field on $M \times \mathcal{R}$ by $(X, f\frac{d}{dt})$, where X is tangent to M, t is the co-ordinates of $\mathcal{R}$ and f as C^∞ function on $M \times \mathcal{R}$. Define an almost complex structure [4]

$$\mathcal{J}\left(X, f\frac{d}{dt}\right) = \left(\phi X - f\xi, \eta(X)\frac{d}{dt}\right),$$

for any vector field X on $M \times \mathcal{R}$ and $\mathcal{G}$ is Hermitian metric on the product $M \times \mathcal{R}$.This may be expressed by the condition

$$(\nabla_X \phi)Y = \alpha(g(X, Y)\xi - \eta(Y)X) + \beta(g(\phi X, Y)\xi - \eta(Y)\phi X), \tag{6}$$

where ∇ is a Levi-civita connection and α, β are some smooth functions on M^{2n+1} and we say that trans-Sasakian structure is type (α, β). From the above, it is follows that

$$(\nabla_X \eta)Y = -\alpha g(\phi X, Y) + \beta g(\phi X, \phi Y), \tag{7}$$

$$(\nabla_X \xi) = -\alpha\phi X + \beta(X - \eta(X)\xi). \tag{8}$$

On a trans-Sasakian manifold M^{2n+1} with structure $(\phi,\ \xi,\ \eta,\ g)$, the following relations hold [11]

$$\begin{aligned} R(X, Y, \xi) &= (\alpha^2 - \beta^2)[\eta(Y)X - \eta(X)Y] + 2\alpha\beta[\eta(Y)\phi X - \eta(X)\phi Y] \\ &\quad + (Y\alpha)\phi X - (X\alpha)\phi Y + (Y\beta)\phi^2 X - (X\beta)\phi^2 Y, \end{aligned} \tag{9}$$

$$R(\xi, X, \xi) = (\alpha^2 - \beta^2 - \xi\beta)[\eta(X)\xi - X], \tag{10}$$

$$2\alpha\beta + \xi\alpha = 0, \tag{11}$$

$$\eta(R(X, Y, \xi)) = \eta(R(\xi, Y, \xi)) = 0, \tag{12}$$

$$\begin{aligned} R(\xi, Y, Z) &= (\alpha^2 - \beta^2)[g(Z, Y)\xi - \eta(Z)Y] + 2\alpha\beta[g(\phi Z, Y)\xi + \eta(Z)\phi Y] + (Z\alpha)\phi Y \\ &\quad + g(\phi Z, Y)grad\alpha + (Z\beta)[Y - \eta(Y)\xi] - g(\phi Z, \phi Y)grad\beta, \end{aligned} \tag{13}$$

$$S(X, \xi) = [2n(\alpha^2 - \beta^2) - \xi\beta]\eta(X) - (2n-1)X\beta - (\phi X)\alpha, \tag{14}$$

$$S(\xi,\xi) = 2n(\alpha^2 - \beta^2 - \xi\beta), \tag{15}$$
$$S(\phi X, \phi Y) = S(X,Y) - 2n(\alpha^2 - \beta^2 - \xi\beta)\eta(X)\eta(Y), \tag{16}$$
$$Q\xi = (2n(\alpha^2 - \beta^2) - \xi\beta)\xi - (2n-1)grad\beta + \phi(grad\alpha), \tag{17}$$
$$S(X,Y) = g(QX,Y), \tag{18}$$

where R is the curvature tensor, S is the Ricci tensor, r is scalar curvature and Q being the symmetric endomorphism of the tangent space at each point corresponding to Ricci-tensor S. Now, we assume that

$$\phi(grad\alpha) = (2n-1)grad\beta, \tag{19}$$

then [11]

$$S(X,\xi) = 2n(\alpha^2 - \beta^2)\eta(X), \tag{20}$$
$$S(\phi X, \phi Y) = S(X,Y) - 2n(\alpha^2 - \beta^2)\eta(X)\eta(Y), \tag{21}$$
$$Q\xi = 2n(\alpha^2 - \beta^2)\xi, \tag{22}$$

$$\begin{aligned}(\nabla_W S)(Y,\xi) &= 2n(\alpha^2 - \beta^2)[-\alpha g(Y,\phi W) + \beta g(Y,W)] \\ &\quad + \alpha S(Y,\phi W) - \beta S(Y,W).\end{aligned} \tag{23}$$

Now we are going to mention the following definition, which will be considered in the later results:

Definition 2.1. [4] *A trans-Sasakian manifold M^{2n+1} is said to be η-Einstein, if the Ricci tensor S satisfies the relation*

$$S(X,Y) = ag(X,Y) + b\eta(X)\eta(Y), \tag{24}$$

for all X and $Z \in TM$, where a and b are smooth functions on M^{2n+1}.

In particular, if $b = 0$ then it reduce to the Einstein manifold.

3. m-projectively flat Trans-Sasakian Manifold

Definition 3.1. [10] *A trans-Sasakian manifold M^{2n+1} is said to be m-projectively flat, if the m-projective curvature tensor M satisfies the relation*

$$M(X,Y,Z) = 0, for\ all\ X, Y and\ Z, \tag{25}$$

where m-projective curvature tensor M is given by [9]

$$\begin{aligned}M(X,Y,Z) &= R(X,Y,Z) - \frac{1}{4n}\Big[S(Y,Z)X - S(X,Z)Y \\ &\quad + g(Y,Z)QX - g(X,Z)QY\Big].\end{aligned} \tag{26}$$

Theorem 3.1. *An m-projectively flat trans-Sasakian manifold M^{2n+1} is an Einstein manifold.*

Proof. Let M^{2n+1} be m-projectively flat trans-Sasakian manifold, then the equation (26) becomes

$$\begin{aligned}R(X,Y,Z) &= \frac{1}{4n}\Big[S(Y,Z)X - S(X,Z)Y \\ &\quad + g(Y,Z)QX - g(X,Z)QY\Big].\end{aligned} \tag{27}$$

Proceeds the inner product in above equation both side with respect to U, then we obtain

$$\begin{aligned} g(R(X,Y,Z),U) &= \frac{1}{4n}\Big[S(Y,Z)g(X,U) - S(X,Z)g(Y,U) \\ &\quad + g(Y,Z)S(X,U) - g(X,Z)S(Y,U)\Big]. \end{aligned} \tag{28}$$

Taking the contraction over X and U, we get

$$S(Y,Z) = \frac{r}{(2n+1)}g(Y,Z). \tag{29}$$

□

Theorem 3.2. *An m-projectively flat trans-Sasakian manifold M^{2n+1} is of constant curvature.*

Proof. Let M^{2n+1} be m-projectively flat trans-Sasakian manifold. Then by existence of the relation (27) and after using the equations (29), we can find

$$R(X,Y,Z) = \frac{r}{2n(2n+1)}[g(Y,Z)X - g(X,Z)Y]. \tag{30}$$

□

By virtue of the Theorem (3.1) and Theorem (3.2), we state the following corollary:

Corollary 3.1. *An m-projectively flat trans-Sasakian manifold M^{2n+1}, is of constant curvature iff it is Einstein.*

4. m-projective conservative Trans- Sasakian Manifold

Definition 4.1. [5] *A trans-Sasakian manifold M^{2n+1} is said to be m-projective conservative, if the m-projective curvature tensor M satisfies the relation*

$$div(M(X,Y,Z)) = 0, for\ all\ X, Y and\ Z, \tag{31}$$

where div denotes the divergence.

Theorem 4.1. *An Einstein trans-Sasakian manifold M^{2n+1} with constant scalar curvature is m-projective conservative iff it is conservative.*

Proof. We assume that M^{2n+1} be Einstein M-projective trans-Sasakian manifold then by virtue of relation (26), we obtain

$$M(X,Y,Z) = R(X,Y,Z) - \frac{1}{4n}[g(Y,Z)X - g(X,Z)Y]. \tag{32}$$

By taking covariant derivative both side with respect to W in above equation, we obtain

$$(\nabla_W)M(X,Y,Z) = (\nabla_W)R(X,Y,Z). \tag{33}$$

Contracting the above relation with W, we can find

$$div(M(X,Y,Z)) = div(R(X,Y,Z)). \tag{34}$$

If manifold is m-projective conservative, then

$$div(R(X,Y,Z)) = 0. \tag{35}$$

Then the converse part is trivial. □

5. ϕ-m-projectively flat Trans- Sasakian Manifold

Definition 5.1. [11] *A trans-Sasakian manifold M^{2n+1} is said to be ϕ-m-projectively flat, if the m-projective curvature tensor M satisfies the relation*

$$\phi^2(M(\phi X, \phi Y, \phi Z)) = 0, for\ all\ X, Y and\ Z. \tag{36}$$

Theorem 5.1. *A ϕ-m-projectively flat trans-Sasakian manifold M^{2n+1} is an η-Einstein manifold.*

Proof. Let us we assume that M^{2n+1} be ϕ-M-projectively flat trans-Sasakian manifold. Then by virtue of the relations (36) and (1), we have

$$M(\phi X, \phi Y, \phi Z) = \eta(M(\phi X, \phi Y, \phi Z))\xi, \tag{37}$$

which implies

$$g(M(\phi X, \phi Y, \phi Z), \phi U) = \eta(M(\phi X, \phi Y, \phi Z))g(\xi, \phi U). \tag{38}$$

By the relation (1), the above equation becomes

$$g(M(\phi X, \phi Y, \phi Z), \phi U) = 0. \tag{39}$$

Now, by virtue of the relation (26), we obtain

$$\begin{aligned} g(R(\phi X, \phi Y, \phi Z), \phi U) &= \frac{1}{4n}[S(\phi Y, \phi Z)g(\phi X, \phi U) - S(\phi X, \phi Z)g(\phi Y, \phi U) \\ &\quad + g(\phi Y, \phi Z)S(\phi X, \phi U) - g(\phi X, \phi Z)S(\phi Y, \phi U)]. \end{aligned} \tag{40}$$

Let $\{e_1, e_2, ..., e_{2n}, \xi\}$ be a local orthonormal basis of vector field in M^{2n+1} by using the fact that $\{\phi e_1, \phi e_2, ..., \phi e_{2n}, \xi\}$ is also a orthonormal basis, if we put X=U=e_i in above relation and taking summation with respect to i, then we have

$$\begin{aligned} &\sum_{i=1}^{2n} g(R(\phi e_i, \phi Y, \phi Z), \phi e_i) \\ &= \frac{1}{4n}\Bigg[\sum_{i=1}^{2n} S(\phi Y, \phi Z)g(\phi e_i, \phi e_i) - \sum_{i=1}^{2n} S(\phi e_i, \phi Z)g(\phi Y, \phi e_i) \\ &+ \sum_{i=1}^{2n} g(\phi Y, \phi Z)S(\phi e_i, \phi e_i) - \sum_{i=1}^{2n} g(\phi e_i, \phi Z)S(\phi Y, \phi e_i)\Bigg]. \end{aligned} \tag{41}$$

Now, we find that

$$\sum_{i=1}^{2n} g(R(\phi e_i, \phi Y, \phi Z), \phi e_i) = S(\phi Y, \phi Z) - (\alpha^2 - \beta^2 - \xi\beta)g(\phi Y, \phi Z),$$

$$\sum_{i=1}^{2n} g(\phi e_i, \phi e_i) = 2n, \tag{42}$$

$$\sum_{i=1}^{2n} S(\phi e_i, \phi Z)g(\phi Y, \phi e_i) = S(\phi Y, \phi Z), \tag{43}$$

$$\sum_{i=1}^{2n} S(\phi e_i, \phi e_i) = r - 2n(\alpha^2 - \beta^2 - \xi\beta), \tag{44}$$

$$(2n+2)S(\phi Y,\phi Z) = [r + 2n(\alpha^2 - \beta^2 - \xi\beta)]g(\phi Y,\phi Z). \tag{45}$$

Using the relations (42)-(45), the equation (41) becomes

$$\begin{aligned} S(Y,Z) &= \frac{1}{(2n+2)}\Big[r + 2n(\alpha^2 - \beta^2 - \xi\beta)\Big]g(Y,Z) \\ &\quad + \frac{1}{(2n+2)}\Big[2n(2n-1)(\alpha^2 - \beta^2 - \xi\beta)\Big]\eta(Y)\eta(Z). \end{aligned} \tag{46}$$

Hence the manifold is η-Einstein. □

6. Quasi m-projectively flat Trans-Sasakian Manifold

Definition 6.1. [10] *A trans-Sasakian manifold M^{2n+1} is said to be quasi m-projectively flat, if the m-projective curvature tensor M satisfies the relation*

$$g(M(X,Y,Z),\phi U) = 0, for\ all\ X, Y\ Z\ and\ U. \tag{47}$$

Theorem 6.1. *A quasi m-projectively flat trans-Sasakian manifold M^{2n+1} satisfying $\phi(grad\alpha) = (2n-1)grad\beta$ is m-projectively flat if it is of constant curvature .*

Proof. Let M^{2n+1} be a quasi m-projectively flat trans-Sasakian manifold. Then by the relations (47) and (26), we obtain

$$\begin{aligned} g(R(X,Y,Z),\phi U) = \frac{1}{4n}[&S(Y,Z)g(X,\phi U) - S(X,Z)g(Y,\phi U) \\ &+ g(Y,Z)S(X,\phi U) - g(X,Z)S(Y,\phi U)]. \end{aligned} \tag{48}$$

Putting $X = \phi X$ in the above relation, we get

$$\begin{aligned} g(R(\phi X,Y,Z),\phi U) = \frac{1}{4n}[&S(Y,Z)g(\phi X,\phi U) - S(\phi X,Z)g(Y,\phi U) \\ &+ g(Y,Z)S(\phi X,\phi U) - g(\phi X,Z)S(Y,\phi U)]. \end{aligned} \tag{49}$$

After putting X=U=e_i in above relation and taking summation with respect to i, we attain

$$\begin{aligned} &\sum_{i=1}^{2n} g(R(\phi e_i,Y,Z),\phi e_i) \\ &= \frac{1}{4n}\Big[\sum_{i=1}^{2n} S(Y,Z)g(\phi e_i,\phi e_i) - \sum_{i=1}^{2n} S(\phi e_i,Z)g(Y,\phi e_i) \\ &+ \sum_{i=1}^{2n} g(Y,Z)S(\phi e_i,\phi e_i) - \sum_{i=1}^{2n} g(\phi e_i,Z)S(Y,\phi e_i)\Big]. \end{aligned} \tag{50}$$

If M^{2n+1} satisfies $\phi(grad\alpha) = (2n-1)grad\beta$, we have the following relation

$$\sum_{i=1}^{2n} g(R(\phi e_i,Y,Z),\phi e_i) = S(Y,Z) - (\alpha^2 - \beta^2)g(\phi Y,\phi Z), \tag{51}$$

$$\sum_{i=1}^{2n} S(\phi e_i,Z)g(Y,\phi e_i) = S(Y,Z) - 2n(\alpha^2 - \beta^2)\eta(Y)\eta(Z), \tag{52}$$

$$\sum_{i=1}^{2n} S(\phi e_i,\phi e_i) = r - 2n(\alpha^2 - \beta^2). \tag{53}$$

After using the relations (42), (51), (52) and (53) in the equation (50), we obtain

$$S(Y,Z) = \left[\frac{r + 2n(\alpha^2 - \beta^2)}{(2n+2)}\right] g(Y,Z). \tag{54}$$

By virtue of the equation (26) and using the above relation, we can get

$$\begin{aligned} M(X,Y,Z) &= R(X,Y,Z) \\ &\quad -\left[\frac{r + 2n(\alpha^2 - \beta^2)}{(n+1)}\right][g(Y,Z)X - g(X,Z)Y]. \end{aligned} \tag{55}$$

Which shows the statement. □

7. *m*-projectively symmetric Trans-Sasakian Manifold

Definition 7.1. [4] *A trans-Sasakian manifold M^{2n+1} is said to be m-projectively symmetric, if the m-projective curvature tensor M satisfies the relation*

$$(\nabla_W M)(X,Y,Z) = 0, for\ all\ X, Y\ Z\ and\ W. \tag{56}$$

Theorem 7.1. *A m-projectively symmetric trans-Sasakian M^{2n+1} manifold is Ricci-recurrent.*

Proof. Let M^{2n+1} is a m-projectively symmetric trans-Sasakian manifold. Then by the equations (56) and (26), we find

$$\begin{aligned} g((\nabla_W R)(X,Y,Z),U) &= \frac{1}{4n}[(\nabla_W S)(Y,Z)g(X,U) - (\nabla_W S(X,Z)g(Y,U) \\ &\quad +(\nabla_W S)(X,U)g(Y,Z) - (\nabla_W S)(Y,U)g(X,Z)]. \end{aligned} \tag{57}$$

Taking contraction over X and U, we secure

$$\begin{aligned} (\nabla_W S)(Y,Z) &= \frac{1}{4n}[(2n+1)(\nabla_W S)(Y,Z) - (\nabla_W S(Y,Z) \\ &\quad +dr(W)g(Y,Z) - (\nabla_W S)(Y,Z)], \end{aligned} \tag{58}$$

which implies

$$(\nabla_W S)(Y,Z) = \frac{dr(W)}{(2n+1)} g(Y,Z). \tag{59}$$

Hence the manifold is Ricci-recurrent. □

Suppose the scalar curvature r is constant then we mention the corollary:

Corollary 7.1. *An m-projective symmetric trans-Sasakian manifold M^{2n+1} with constant scalar curvature is Einstein.*

8. ϕ-*m*-projectively symmetric Trans-Sasakian Manifold

Definition 8.1. [4] *A trans-Sasakian manifold M^{2n+1} is said to be $\phi - m$-projectively symmetric, if the m-projective curvature tensor M satisfies the relation*

$$\phi^2(\nabla_W M)(X,Y,Z) = 0, for\ all\ X, Y\ Z\ and\ W. \tag{60}$$

Theorem 8.1. *A $\phi - m$-projectively symmetric trans-Sasakian M^{2n+1} manifold is an Einstein.*

Proof. Let us consider M^{2n+1} is a $\phi - m$-projectively symmetric trans-Sasakian manifold. Then by the equations (60) and (1), we get

$$g((\nabla_W M)(X,Y,Z),U) = \eta((\nabla_W M)(X,Y,Z))g(\xi,U). \tag{61}$$

The existence of the relation (26), the above equation becomes

$$g((\nabla_W R)(X,Y,Z),U) - \frac{1}{4n}\Big[(\nabla_W S)(Y,Z)g(X,U) - (\nabla_W S)(X,Z)g(Y,U)$$
$$+(\nabla_W S)(X,U)g(Y,Z) - (\nabla_W S)(Y,U)g(X,Z)\Big]$$
$$= g((\nabla_W R)(X,Y,Z),\xi)g(\xi,U) - \frac{1}{4n}\Big[(\nabla_W S)(Y,Z)g(X,\xi) - (\nabla_W S)(X,Z)g(Y,\xi)$$
$$+(\nabla_W S)(X,\xi)g(Y,Z) - (\nabla_W S)(Y,\xi)g(X,Z)\Big]g(\xi,U). \tag{62}$$

After contraction over X and Z, we secure

$$(\nabla_W S)(Y,U) - (\nabla_W S)(Y,\xi)\eta(U) = \frac{dr(W)}{(6n-1)}[-g(Y,U) + g(Y,\xi)\eta(U)]. \tag{63}$$

Putting $Y = \xi$, we get

$$(\nabla_W S)(\xi,U) = 0. \tag{64}$$

By virtue of the relation (23) and above equation, we have

$$2n(\alpha^2 - \beta^2)[-\alpha g(U,\phi W) + \beta g(U,W)] + \alpha S(U,\phi W) - \beta S(U,W) = 0. \tag{65}$$

We put $U{=}\phi U$ and$W{=}\phi W$, respectively in the above relation and then using equations $(1),(4),(18),(19)$ and (22), we find that

$$S(U,W) = 2n(\alpha^2 - \beta^2)g(U,W)$$

and

$$S(\phi U,W) = 2n(\alpha^2 - \beta^2)g(\phi U,W). \tag{66}$$

□

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References

[1] Blair,D.E., (1976), Contact manifolds in Riemannian geometry, Lecture Notes in Math. 509, Springer Verlag.
[2] Cartan,E., (1926), Sur une classes remarquable d'espaces de Riemann, Bull. Soc. Math. France, 54, pp.214-26.
[3] Chaubey,S.K. and Ojha,R.H., (2010), On the m-projective curvature tensor of a Kenmotsu manifold, Diff. Geom. Dyn. Sys., 12, pp.52-60.
[4] De,U.C. and Shaikh,A.A., (2009), Complex manifolds and contact manifolds, Narosa Publication, New Delhi, India.
[5] Hicks,N.J., (1969), Notes on Differential Geometry, Affiliated East West Press Pvt. Ltd.
[6] Ojha,R.H., (1975), A note on the mprojective curvature tensor, Indian J. Pure Appl. Math., 8 (12), pp.1531-1534.
[7] Ojha,R.H., (1986), m-projectively flat Sasakian manifolds, Indian J. Pure Appl. Math., 17 (4), pp.1531-1534.

[8] Oubina,J.A., (1985), New classes of almost contact metric structures, Publ. Math. Debrecen, 32, pp.21-38.
[9] Pokhariyal,G.P. and Mishra,R.S., (1971), Curvature tensors and their relativistic significance-ii, Yokohama Math. J., 19, pp.97-103.
[10] Prakash,A., Ahmad,M., and Srivastava,A., (2013), m-projective curvature tensors on a LP-Sasakian manifolds, IOSR J. Math., 6, pp.19-23.
[11] Prasad,R. and Srivastava,V., (2013), Some results on trans-Sasakian manifolds, Math. Vesnik, 65, pp.346-352.
[12] Singh,J.P., (2015), On the M-projective curvature tensor of Sasakian manifold, Sci. Vis., 15 (2), pp.76-79.
[13] Takahashi,T., (1977), Sasakian ϕ-symmetric space, Tohoku Math. J., 29, pp.91-113.
[14] Tamassy,L. and Binh,T.Q., (1989), On weakly symmetric and weakly projective symmetric Riemannian manifold, Colloquia Math. Soc., 50, pp.663-667.
[15] Zengin,F.O., (2013), On m-projectively flat LP-Sasakian manifolds, Ukr. Math. J., 65, pp.1725-1732.

Jai Prakash Jaiswal is currently working as an assistant professor in the Department of Mathematics of Maulana Azad National Institute of Technology, Bhopal, India. He received his Ph. D. degree from Banaras Hindu University, Varanasi, India. His area of interest includes differential geometry of manifolds and numerical functional analysis.

Arjun Singh Yadav worked as junior research fellow in NBHM-DAE, Mumbai, India sponsored research project in the Department of Mathematics of Maulana Azad National Institute of Technology, Bhopal, India. He received his B. Sc. and M. Sc. degrees from Vikram University, Ujjain, India. His area of interest includes differential geometry of manifolds.