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Decompositions of Soft $\alpha-$continuity and Soft $A-$continuity

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Abstract − In this paper, we introduce the concepts of soft $\alpha A-$set, soft $\alpha B-$set, soft $\alpha C-$set and soft $\alpha LC-$set in soft topological spaces which are defined over an initial universe with a fixed set of parameters and discuss their relationships with each other and other weaker forms of soft open sets with counterexamples. We also investigate soft $\alpha A-$continuity, soft $\alpha B-$continuity, soft $\alpha C-$continuity and soft $\alpha LC-$continuity. Finally, we obtain two decompositions of soft α-continuity and a decomposition of soft A-continuity.

Keywords − *Soft set, Soft topological space, Soft $\alpha A-$set, Soft $\alpha B-$set, Soft $\alpha C-$set, Soft $\alpha LC-$set, Soft $\alpha A-$continuous function, Soft $\alpha B-$continuous function, Soft $\alpha C-$continuous function, Soft $\alpha LC-$continuous function.*

1. Introduction

The concept of soft sets was initiated by Molodtsov [1] in 1999 as a completely new approach for modeling vagueness and uncertainty. He has shown several applications of this theory in solving many practical problems in economics, engineering, social science, medical science, etc. Later Maji et al. [2] presented some new definitions on soft sets such as a subset, the complement of a soft set. Research works on soft sets are progressing rapidly in recent years.

Shabir and Naz [3] introduced the soft topological spaces which are defined over an initial universe with a fixed set of parameters. Later Aygünoğlu and Aygün [4], Min [5], Zorlutuna et al. [6] and Hussain and Ahmad [7] continued to study the properties of soft topological spaces. They got many important results in soft topological spaces. Recently, weak forms of soft open sets have been studied [8–19] in soft topological spaces.

The purpose of this paper is to introduce the notions of soft $\alpha A-$set, soft $\alpha B-$set, soft $\alpha C-$set and soft $\alpha LC-$set in soft topological spaces. We study the relations between these different types of subsets in soft topological spaces. We also introduce soft $\alpha A-$continuous, soft $\alpha B-$continuous, soft $\alpha C-$continuous and soft $\alpha LC-$continuous functions. Finally, we obtain some decompositions.

2. Preliminary

In this section, we present the basic definitions and results of soft set theory which may be found in earlier studies.

Definition 2.1. [1] Let X be an initial universe set and E be the set of all possible parameters with respect to X. Let $P(X)$ denote the power set of X. A pair (F, A) is called a soft set over X where $A \subseteq E$ and $F : A \to P(X)$ is a set valued mapping.

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The set of all soft sets over X is denoted by $SS(X)_E$.

Definition 2.2. [2] A soft set (F,A) over X is said to be a null soft set denoted by Φ if for all $e \in A$, $F(e)=\emptyset$. A soft set (F,A) over X is said to be an absolute soft set denoted by $\widetilde{A}$ if for all $e \in A$, $F(e)=X$.

Definition 2.3. [3] Let Y be a nonempty subset of X, then $\widetilde{Y}$ denotes the soft set (Y,E) over X for which $Y(e)=Y$, for all $e \in E$. In particular, (X,E) will be denoted by $\widetilde{X}$.

Definition 2.4. [2] For two soft sets (F,A) and (G,B) over X, we say that (F,A) is a soft subset of (G,B) if $A \subseteq B$ and $F(e) \subseteq G(e)$ for all $e \in A$. We write $(F,A) \sqsubseteq (G,B)$. (F,A) is said to be a soft super set of (G,B), if (G,B) is a soft subset of (F,A). We denote it by $(G,B) \sqsubseteq (F,A)$. Then (F,A) and (G,B) are said to be soft equal if (F,A) is a soft subset of (G,B) and (G,B) is a soft subset of (F,A).

Definition 2.5. [2] The union of two soft sets (F,A) and (G,B) over X is the soft set (H,C), where $C=A\cup B$ and for all $e \in C$, $H(e)=F(e)$ if $e \in A\backslash B$, $H(e)=G(e)$ if $e \in B\backslash A$, $H(e)=F(e)\cup G(e)$ if $e \in A\cap B$. We write $(F,A)\sqcup(G,B)=(H,C)$.

Definition 2.6. [20] The intersection (H,C) of two soft sets (F,A) and (G,B) over X, denoted by $(F,A)\sqcap(G,B)$, is defined as $C=A\cap B$, and $H(e)=F(e)\cap G(e)$ for all $e\in C$.

Definition 2.7. [3] The difference (H,E) of two soft sets (F,E) and (G,E) over X, denoted by $(F,E)\backslash(G,E)$, is defined as $H(e)=F(e)\backslash G(e)$ for all $e\in E$.

Definition 2.8. [3] The relative complement of a soft set (F,E) is denoted by $(F,E)^c$ and is defined by $(F,E)^c=(F^c,E)$ where $F^c: E \longrightarrow P(X)$ is a mapping given by $F^c(e)=X\backslash F(e)$ for all $e\in E$.

Definition 2.9. [3] Let τ be the collection of soft sets over X, then τ is said to be a soft topology on X if

(1) Φ, $\widetilde{X} \in \tau$
(2) If (F,E), $(G,E)\in\tau$, then $(F,E)\sqcap(G,E)\in\tau$
(3) If $\{(F_i,E)\}_{i\in I}\in\tau$, $\forall i\in I$, then $\sqcup_{i\in I}(F_i,E)\in\tau$.

The triplet (X,τ,E) is called a soft topological space over X. Every member of τ is called a soft open set in X. A soft set (F,E) over X is called a soft closed set in X if its relative complement $(F,E)^c$ belongs to τ. We will denote the family of all soft open sets (resp., soft closed sets) of a soft topological space (X,τ,E) by SOS(X,τ,E) (resp., SCS(X,τ,E)).

Definition 2.10. Let (X,τ,E) be a soft topological space and (F,E) be a soft set over X.

(1) [3] The soft closure of (F,E) is the soft set $cl(F,E)=\sqcap\{(G,E):\ (G,E)$ is soft closed and $(F,E)\sqsubseteq(G,E)\}$.
(2) [6] The soft interior of (F,E) is the soft set $int(F,E)=\sqcup\{(H,E):(H,E)$ is soft open and $(H,E)\sqsubseteq(F,E)\}$.

Clearly, $cl(F,E)$ is the smallest soft closed set over X which contains (F,E) and $int(F,E)$ is the largest soft open set over X which is contained in (F,E).

Throughout the paper, the spaces X and Y (or (X,τ,E) and (Y,ν,K)) stand for soft topological spaces assumed unless stated otherwise.

Definition 2.11. A soft set (F,E) is called

(i) soft semi-open [8] in a soft topological space X if $(F,E)\sqsubseteq cl(int(F,E))$.
(ii) soft pre-open [9] in a soft topological space X if $(F,E)\sqsubseteq int(cl(F,E))$.
(iii) soft α-open [10] in a soft topological space X if $(F,E)\sqsubseteq int(cl(int(F,E)))$.

The relative complement of a soft semi-open (resp., soft pre-open, soft α-open) set is called a soft semi-closed (resp., soft pre-closed, soft α-closed) set.

We will denote the family of all soft semi-open (resp., soft pre-open and soft α-open) sets of a soft topological space (X,τ,E) by SSOS(X,τ,E) (resp., SPOS(X,τ,E) and SαOS(X,τ,E)).

Definition 2.12. [8] Let (X, τ, E) be a soft topological space and (F, E) be a soft set over X. The soft semi-closure of (F, E) is the soft set $cl_s(F, E) = \sqcap\{(H, E) : (H, E)$ is soft semi-closed and $(F, E) \sqsubseteq (H, E)\}$ and $cl_s(F, E)$ is soft semi-closed.

Theorem 2.13. [8] Let (X, τ, E) be a soft topological space and (F, E) be a soft set over X. We have $(F, E) \sqsubseteq cl_s(F, E) \sqsubseteq cl(F, E)$.

Definition 2.14. *Let (X, τ, E) be a soft topological space. A soft set (F, E) is called*

(1) *a soft regular closed set [19] in X if $(F, E) = cl(int(F, E))$.*
(2) *a soft $A-$set [17] in X if $(F, E) = (G, E) \sqcap (H, E)$, where (G, E) is a soft open set and (H, E) is a soft regular closed set in X.*
(3) a soft t-set [17] in X if $int(cl(F, E)) = int(F, E)$.
(4) a soft $B-$set [17] in X if $(F, E) = (G, E) \sqcap (H, E)$, where (G, E) is a soft open set and (H, E) is a soft t-set in X.
(5) a soft α^*-set [16] in X if $int(cl(int(F, E))) \sqsubseteq (F, E)$.
(6) a soft $C-$set [16] in X if $(F, E) = (G, E) \sqcap (H, E)$ where (G, E) is soft open and (H, E) is a soft α^*-set in X.
(7) a soft locally closed set (briefly; soft $LC-$set) [14] in X if $(F, E) = (G, E) \sqcap (H, E)$, where (G, E) is soft open and (H, E) is soft closed in X.

We will denote the family of all *soft regular closed sets* (resp., soft $A-$sets, soft $B-$sets, soft $C-$sets and soft $LC-$sets) of a soft topological space X by SRCS(X) (resp., SAS(X), SBS(X), SCS(X) and SLCS(X)).

Remark 2.15. In a soft topological space (X, τ, E);

(1) every soft open set is soft α-open [10],
(2) every soft α-open set is soft pre-open and soft semi-open [10],
(3) every soft regular closed set is soft closed [19],
(4) every soft open set is a soft $A-$set [17],
(5) every soft $A-$set is soft semi-open [17],
(6) every soft $A-$set is a soft $LC-$set [14],
(7) every soft $LC-$set is a soft $B-$set [14],
(8) every soft $B-$set is a soft $C-$set [16].

Definition 2.16. [21] Let (X, τ, E) be a soft topological space and (F, E) be a soft set over X. (F, E) is called (1) a soft dense set if $cl(F, E) = \widetilde{X}$, (2) a soft nowhere dense set if $int(cl(F, E)) = \Phi$.

Definition 2.17. [22] Let $SS(X)_E$ and $SS(Y)_K$ be families of soft sets, $u : X \longrightarrow Y$ and $p : E \longrightarrow K$ be mappings. Then the mapping $f_{pu} : SS(X)_E \longrightarrow SS(Y)_K$ is defined as:

(1) Let $(F, E) \in SS(X)_E$. The image of (F, E) under f_{pu}, written as $f_{pu}(F, E) = (f_{pu}(F), p(E))$, is a soft set in $SS(Y)_K$ such that

$$f_{pu}(F)(y) = \begin{cases} \cup_{x \in p^{-1}(y) \cap A} u(F(x)) & , p^{-1}(y) \cap A \neq \emptyset \\ \emptyset & , otherwise \end{cases}$$

for all $y \in K$.

(2) Let $(G, K) \in SS(Y)_K$. The inverse image of (G, K) under f_{pu}, written as $f_{pu}^{-1}(G, K) = (f_{pu}^{-1}(G), p^{-1}(K))$, is a soft set in $SS(X)_E$ such that

$$f_{pu}^{-1}(G)(x) = \begin{cases} u^{-1}(G(p(x))) & , p(x) \in K \\ \emptyset & , otherwise \end{cases}$$

for all $x \in E$.

Definition 2.18. [6] Let (X, τ, E) and (Y, υ, K) be soft topological spaces and $f_{pu} : SS(X)_E \longrightarrow SS(Y)_K$ be a function. Then f_{pu} is called a soft continuous function if for each $(G, K) \in \upsilon$ we have $f_{pu}^{-1}(G, K) \in \tau$.

Definition 2.19. Let (X, τ, E) and (Y, υ, K) be soft topological spaces and $f_{pu} : SS(X)_E \longrightarrow SS(Y)_K$ be a function. Then f_{pu} is called

(1) a soft semi-continuous function [15] if for each $(G, K) \in SOS(Y)$ we have $f_{pu}^{-1}(G, K) \in SSOS(X)$.

(2) a soft α-continuous function [10] if for each $(G, K) \in SOS(Y)$ we have $f_{pu}^{-1}(G, K) \in S\alpha OS(X)$.

(3) a soft pre-continuous function [10] if for each $(G, K) \in SOS(Y)$ we have $f_{pu}^{-1}(G, K) \in SPOS(X)$.

(4) a soft $A-$continuous function [17] if for each $(G, K) \in SOS(Y)$, $f_{pu}^{-1}(G, K)$ is a soft $A-$set in X.

(5) a soft $B-$continuous function [17] if for each $(G, K) \in SOS(Y)$, $f_{pu}^{-1}(G, K)$ is a soft $B-$set in X.

(6) a soft $C-$continuous function [16] if for each $(G, K) \in SOS(Y)$, $f_{pu}^{-1}(G, K)$ is a soft $C-$set in X.

(7) a soft $LC-$continuous function [14] if for each $(G, K) \in SOS(Y)$, $f_{pu}^{-1}(G, K)$ is a soft $LC-$set in X.

Remark 2.20. Let (X, τ, E) and (Y, υ, K) be soft topological spaces and $f_{pu} : SS(X)_E \longrightarrow SS(Y)_K$ be a function. Then,

(1) every soft continuous function is soft α-continuous [10],
(2) every soft α-continuous function is soft semi-continuous and soft pre-continuous [10],
(3) every soft continuous function is soft $A-$continuous [17],
(4) every soft $A-$continuous function is soft semi-continuous [17],
(5) every soft $A-$continuous function is soft $LC-$continuous [14],
(6) every soft $LC-$continuous function is soft $B-$continuous [14],
(6) every soft $B-$continuous function is soft $C-$continuous [16].

3. Soft $\alpha A-$sets, Soft $\alpha B-$sets, Soft $\alpha C-$sets and Soft $\alpha LC-$sets

Definition 3.1. A soft set (F, E) in a soft topological space (X, τ, E) is called

1) a soft $\alpha A-$set if $(F, E) = (G, E) \sqcap (H, E)$ where (G, E) is soft $\alpha-$open and (H, E) is soft regular closed.

2) a soft $\alpha B-$set if $(F, E) = (G, E) \sqcap (H, E)$ where (G, E) is soft $\alpha-$open and (H, E) is a soft $t-$set in X.

3) a soft $\alpha C-$set if $(F, E) = (G, E) \sqcap (H, E)$ where (G, E) is soft $\alpha-$open and $int(cl(int(H, E))) \sqsubseteq (H, E)$.

4) a soft $\alpha LC-$set if $(F, E) = (G, E) \sqcap (H, E)$ where (G, E) is soft $\alpha-$open and (H, E) is soft closed.

We will denote the family of all soft $\alpha A-$sets (resp.,soft $\alpha B-$sets, soft $\alpha C-$sets and soft $\alpha LC-$sets) of (X, τ, E) by $S\alpha AS(X)$ (resp., $S\alpha BS(X)$, $S\alpha CS(X)$ and $S\alpha LCS(X)$).

Theorem 3.2. For a soft topological space (X, τ, E), the following hold:

1) Every soft $A-$set is a soft $\alpha A-$set.
2) Every soft $B-$set is a soft $\alpha B-$set.
3) Every soft $C-$set is a soft $\alpha C-$set.
4) Every soft $LC-$set is a soft $\alpha LC-$set.

PROOF. The proofs are obvious since every soft open set is soft $\alpha-$open. □

Example 3.3. Let $X = \{x_1, x_2, x_3\}$, $E = \{e_1, e_2\}$ and $\tau = \{\Phi, \widetilde{X}, (F, E)\}$ such that

$$(F, E) = \{(e_1, \{x_1\}),\ (e_2, \{x_2\})\}.$$

Then τ defines a soft topology on X and thus (X, τ, E) is a soft topological space over X [17]. Then $(G, E) = \{(e_1, \{x_1, x_2\}), (e_2, \{x_2\})\}$ is a soft α-open set in X but not soft open. Since $(G, E) = (G, E) \sqcap \tilde{X}$ and $\tilde{X}$ is a soft $t-$set, (G, E) is a soft $\alpha B-$set but not a soft $B-$set.

Example 3.4. Let $X = \{x_1, x_2, x_3\}$ and $E = \{e_1, e_2\}$. Let us take the soft topology τ on X and the soft set $(G, E) = \{(e_1, \{x_1, x_2\}), (e_2, \{x_2\})\})\}$ in Example 3.3. (G, E) is a soft $\alpha C-$set but not a soft $C-$set.

Example 3.5. Let $X = \{x_1, x_2, x_3\}$ and $E = \{e_1, e_2\}$. Let us take the soft topology τ on X and the soft set $(G, E) = \{(e_1, \{x_1, x_2\}), (e_2, \{x_2\})\})\}$ in Example 3.3. (G, E) is a soft $\alpha LC-$set but not a soft $LC-$set.

Proposition 3.6. In a soft topological space (X, τ, E), every soft $\alpha A-$set is a soft $\alpha LC-$set.

PROOF. The proof is obvious since every soft reguler closed set is soft closed. □

Example 3.7. Let $X = \{x_1, x_2, x_3, x_4\}$, $E = \{e_1, e_2\}$ and $\tau = \{\Phi, \tilde{X}, (F_1, E), ..., (F_{11}, E)\}$ such that

$$\begin{aligned}
(F_1, E) &= \{(e_1, \{x_1\}), (e_2, \{x_1\})\}, \\
(F_2, E) &= \{(e_1, \{x_2\}), (e_2, \{x_2\})\}, \\
(F_3, E) &= \{(e_1, \{x_1, x_2\}), (e_2, \{x_1, x_2\})\}, \\
(F_4, E) &= \{(e_1, \{x_1, x_2, x_3\}), (e_2, \{x_1, x_3\})\}, \\
(F_5, E) &= \{(e_1, \{x_1, x_2, x_4\}), (e_2, \{x_1, x_2, x_3\})\}, \\
(F_6, E) &= \{(e_1, \{x_2\}), (e_2, \varnothing)\}, \\
(F_7, E) &= \{(e_1, \{x_1, x_2\}), (e_2, \{x_1\})\}, \\
(F_8, E) &= \{(e_1, \{x_1, x_2, x_3\}), (e_2, \{x_1, x_2, x_3\})\}, \\
(F_9, E) &= \{(e_1, X), (e_2, \{x_1, x_2, x_3\})\}, \\
(F_{10}, E) &= \{(e_1, \{x_1, x_2\}), (e_2, \{x_1, x_2, x_3\})\}, \\
(F_{11}, E) &= \{(e_1, \{x_1, x_2\}), (e_2, \{x_1, x_3\})\}.
\end{aligned}$$

Then τ defines a soft topology on X and thus (X, τ, E) is a soft topological space over X [19]. Let us take a soft set $(G, E) = (F_9, E) \sqcap (F_{11}, E)^c = \{(e_1, \{x_3, x_4\}), (e_2, \{x_2\})\}$ on X. Since (G, E) is a soft $LC-$set, (G, E) is a soft $\alpha LC-$set. But (G, E) is not a soft $\alpha A-$ set.

Proposition 3.8. In a soft topological space (X, τ, E), every soft $\alpha B-$set is a soft $\alpha C-$set.

PROOF. Let (F, E) be a soft $\alpha B-$set, so $(F, E) = (G, E) \sqcap (H, E)$ where (G, E) is soft $\alpha-$open and (H, E) is a soft $t-$set. Since (H, E) is a soft $t-$set, $int(cl(H, E)) = int(H, E)$. Then $int(cl(int(H, E))) \sqsubseteq int(cl(H, E)) = int(H, E) \sqsubseteq (H, E)$. Hence we obtain (F, E) is a soft $\alpha C-$set. □

Example 3.9. Let $X = \{x_1, x_2, x_3\}$, $E = \{e_1, e_2\}$ and $\tau = \{\Phi, \tilde{X}, (F_1, E), (F_2, E), (F_3, E)\}$ such that

$$(F_1, E) = \{(e_1, \{x_1\}), (e_2, \{x_1\})\},$$

$$(F_2, E) = \{(e_1, \{x_2\}), (e_2, \{x_2\})\},$$

$$(F_3, E) = \{(e_1, \{x_1, x_2\}), (e_2, \{x_1, x_2\})\}.$$

Then τ defines a soft topology on X and thus (X, τ, E) is a soft topological space over X. $(G, E) = \{(e_1, \{x_3\}), (e_2, \{x_1, x_3\})\}$ is a soft α^*-set since $int(cl(int(G, E))) = int(G, E)$. So it is a soft $C-$set and a soft $\alpha C-$set. But (G, E) is not a soft $\alpha B-$set.

Proposition 3.10. In a soft topological space (X, τ, E), every soft $\alpha LC-$set is a soft $\alpha B-$set.

PROOF. Let $(G, E) \sqcap (H, E)$ be a soft $\alpha LC-$set such that (G, E) is soft $\alpha-$open and $cl(H, E) = (H, E)$. Since $int(cl(H, E)) = int(H, E)$ then the proof is obvious. □

Example 3.11. Let $X = \{x_1, x_2, x_3\}$, $E = \{e_1, e_2\}$ and $\tau = \{\Phi, \tilde{X}, (F_1, E), (F_2, E), (F_3, E)\}$ such that

$$(F_1, E) = \{(e_1, \{x_2\}), (e_2, \{x_2\})\},$$

$$(F_2, E) = \{(e_1, \{x_3\}), (e_2, \{x_3\})\},$$

$$(F_3, E) = \{(e_1, \{x_2, x_3\}), (e_2, \{x_2, x_3\})\}.$$

Then τ defines a soft topology on X and thus (X, τ, E) is a soft topological space over X. Then $(G, E) = \{(e_1, \{x_2\}), (e_2, \{x_1, x_2\})\}$ is a soft $\alpha B-$set, but it is not a soft $\alpha LC-$set.

Lemma 3.12. [12] Let (X, τ, E) be a soft topological space, $(F, E) \in S\alpha OS(X)$ and $(G, E) \in SSOS(X)$. Then $(F, E) \sqcap (G, E) \in SSOS(X)$.

Proposition 3.13. In a soft topological space (X, τ, E), every soft $\alpha A-$set is soft semi-open.

Proof. Let $(G, E) \sqcap (H, E)$ be a soft $\alpha A-$set, (G, E) is soft $\alpha-$open and $cl(int(H, E)) = (H, E)$. Hence (H, E) is soft semi-open. Using Lemma 3.12 we have that $(G, E) \sqcap (H, E)$ is soft semi-open. □

Theorem 3.14. [14] For a soft topological space (X, τ, E), we have $SAS(X) = SSOS(X) \cap SLCS(X)$.

Theorem 3.15. For a soft topological space (X, τ, E), we have $SAS(X) = S\alpha AS(X) \cap SLCS(X)$.

Proof. Every soft $\alpha A-$set is soft semi-open by Proposition 3.13 and $SAS(X) = SSOS(X) \cap SLCS(X)$ by Theorem 3.14. Hence $S\alpha AS(X) \cap SLCS(X) \subseteq SSOS(X) \cap SLCS(X) = SAS(X)$. So we obtain $S\alpha AS(X) \cap SLCS(X) \subseteq SAS(X)$.
By Theorem 3.2, we have that every soft $A-$set is a soft $\alpha A-$set. Also, since $SAS(X) = SSOS(X) \cap SLCS(X)$ then $SAS(X) \subseteq SLCS(X)$ and thus $SAS(X) \subseteq S\alpha AS(X) \cap SLCS(X)$. □

Proposition 3.16. Let (X, τ, E) be a soft topological space. A soft set (F, E) over X is a soft $\alpha-$open set iff $(F, E) = (G, E) \setminus (H, E)$ where (G, E) is soft open and (H, E) is soft nowhere dense.

Proof. If (F, E) is a soft $\alpha-$open set we have

$$(F, E) = int(cl(int(F, E))) \setminus ((int(cl(int(F, E)))) \setminus (F, E))$$

where $int(cl(int(F, E))) \setminus (F, E)$ clearly is soft nowhere dense.
Conversely, if $(F, E) = (G, E) \sqsubseteq (H, E)$, (G, E) is soft open, (H, E) is soft nowhere dense, we can see that $(G, E) \sqsubset cl(int(F, E))$ and consequently

$$int(cl(int(F, E))) \sqsupseteq (G, E) \sqsupseteq (F, E)$$

So the proof is complete. □

Proposition 3.17. Let (X, τ, E) be a soft topological space. (F, E) is a soft $\alpha B - set$ if and only if $(F, E) = (G, E) \sqcap (H, E)$ where (G, E) is soft $B-$set and $int(H, E)$ is soft dense.

Proof. Let (F, E) be a soft $\alpha B-$set, we have $(F, E) = (G, E) \sqcap (H, E)$ where (G, E) is a soft $\alpha-$open set and (H, E) is a soft $t-$set. By Proposition 3.16, we write $(G, E) = (G_1, E) \sqcap (G_2, E)$ where (G_1, E) is a soft open $B-$set and $int(G_2, E)$ is soft dense. Hence $(F, E) = (G, E) \sqcap (H, E) = ((G_1, E) \sqcap (G_2, E)) \sqcap (H, E) = ((G_1, E) \sqcap (H, E)) \sqcap (G_2, E)$ where $(G_1, E) \sqcap (H, E)$ is a soft $B-$set and $int(G_2, E)$ is soft dense.
Conversely, let $(F, E) = (H, E) \sqcap (G_2, E)$ such that (H, E) is a soft $B-$set and $int(G_2, E)$ is soft dense. Then we have $(H, E) = (G_1, E) \sqcap (H_1, E)$ where (G_1, E) is soft open and (H_1, E) is a soft $t-$set. Thus $(F, E) = (H, E) \sqcap (G_2, E) = ((G_1, E) \sqcap (H_1, E)) \sqcap (G_2, E) = ((G_1, E) \sqcap (G_2, E)) \sqcap (H, E)$ is soft $\alpha-$open from Proposition 3.16 and (H, E) is a soft $t-$set. Thus (F, E) is a soft $\alpha B-$set. □

Theorem 3.18. [8] A soft subset (F, E) in a soft topological space (X, τ, E) is soft semi-cosed iff $int(cl(F, E)) \sqsubseteq (F, E)$.

Theorem 3.19. [12] Let (X, τ, E) be a soft topological space and $(F, E) \in SS(X, E)$. Then (F, E) is soft semi-closed iff $(F, E) = (F, E) \sqcup int(cl(F, E))$.

Another description of soft $\alpha B-$sets is given in the next result.

Proposition 3.20. Let (X, τ, E) be a soft topological space. (F, E) is a soft $\alpha B - set$ iff $(F, E) = (G, E) \sqcap cl_s(F, E)$ for some $(G, E) \in S\alpha OS(X)$.

Proof. Let (F,E) be a soft $\alpha B-$set, we have $(F,E) = (G,E) \sqcap (H,E)$ where (G,E) is a soft $\alpha-$open set and (H,E) is a soft $t-$set. Now $(F,E) \sqsubseteq (G,E)$ and $(F,E) \sqsubseteq (H,E)$, so $cl_s(F,E) \sqsubseteq cl_s(H,E) = (H,E) \sqcup int(cl(H,E))$ by Theorem 3.19. Since (H,E) is a soft $t-$set, $int(cl(H,E)) = int(H,E)$. Hence we obtain

$$(F,E) \sqsubseteq (G,E) \sqcap cl_s(F,E) \sqsubseteq (G,E) \sqcap cl_s(H,E) = (G,E) \sqcap ((H,E) \sqcup int(cl((H,E)))) =$$

$$= (G,E) \sqcap ((H,E) \sqcup int(H,E)) = (G,E) \sqcap (H,E) = (F,E).$$

Conversely, assume that $(F,E) = (G,E) \sqcap cl_s(F,E)$ for some $(G,E) \in S\alpha OS(X)$. Put $(H,E) = cl_s(F,E)$. Then (H,E) is soft semi-closed and we have $int(cl(F,E)) \sqsubseteq (H,E)$ by Theorem 3.18. Hence $int(cl(H,E)) = int(H,E)$ and (H,E) is a soft $t-$set. Therefore (F,E) is a soft $\alpha B-$set. □

Theorem 3.21. For a soft topological space (X,τ,E) we have

$$S\alpha OS(X) = SPOS(X) \cap S\alpha BS(X).$$

Proof. It is clear that $S\alpha OS(X) \subseteq SPOS(X) \cap S\alpha BS(X)$. For the converse, let $(F,E) \in SPOS(X) \cap S\alpha BS(X)$. From $(F,E) \in SPOS(X)$ we have $(F,E) \sqsubseteq int(cl(F,E))$. Since $(F,E) \in S\alpha BS(X)$, we have that $(F,E) = (G,E) \sqcap cl_s(F,E)$ for some $(G,E) \in S\alpha OS(X)$ by Proposition 3.20. Also $cl_s(F,E) = (F,E) \sqcup int(cl(F,E)) = int(cl(F,E))$ from Theorem 3.19. Thus $(F,E) = (G,E) \sqcap int(cl(F,E))$ where $(G,E) \in S\alpha OS(X)$ and $int(cl(F,E)) \in SOS(X) \subseteq S\alpha OS(X)$. Therefore $(F,E) = (G,E) \sqcap int(cl(F,E)) \in S\alpha OS(X)$. □

Theorem 3.22. For a soft topological space (X,τ,E) the following hold:

$$S\alpha OS(X) = SPOS(X) \cap S\alpha LCS(X).$$

Proof. Since every soft $\alpha-$open set is soft pre-open and every soft $\alpha-$open set is soft $\alpha LC-$set, we obtain $S\alpha OS(X) \subseteq SPOS(X) \cap S\alpha LCS(X)$.
From Theorem 3.21, $S\alpha OS(X) = SPOS(X) \cap S\alpha BS(X)$. Also, since every soft $\alpha LC-$set is a soft $\alpha B-$ set we obtain $SPOS(X) = SPOS(X) \cap S\alpha LCS(X) \subseteq SPOS(X) = SPOS(X) \cap S\alpha BS(X) = S\alpha OS(X)$. □

4. Decompositions of Soft $\alpha-$continuity and Soft $A-$continuity

In this section, two new decompositions of soft $\alpha-$continuity are given. Also we obtain a decomposition of soft $A-$continuity.

Definition 4.1. Let (X,τ,E) and (Y,ϑ,K) be soft topological spaces. Let $u : X \to Y$ and $p : E \to K$ be mappings and $f_{pu} : SS(X)_E \to SS(Y)_K$ be a function. Then the function f_{pu} is called soft $\alpha A-$continuous (resp., soft $\alpha B-$continuous, soft $\alpha C-$continuous, soft $\alpha LC-$continuous) if for each $(G,K) \in SOS(Y)$, $f_{pu}^{-1}(G,K)$ is a soft $\alpha A-$set (resp., soft $\alpha B-$set, soft $\alpha C-$set, soft $\alpha LC-$set) in X.

Theorem 4.2. Let (X,τ,E) and (Y,ϑ,K) be soft topological spaces and $f_{pu} : SS(X)_E \to SS(Y)_K$ be a function. Then the following hold:

1) If f_{pu} is soft $A-$continuous, then it is soft $\alpha A-$continuous.
2) If f_{pu} is soft $B-$continuous, then it is soft $\alpha B-$continuous.
3) If f_{pu} is soft $C-$continuous, then it is soft $\alpha C-$continuous.
4) If f_{pu} is soft $LC-$continuous, then it is soft $\alpha LC-$continuous.

Proof. The proof is obvious from Theorem 3.2. □

Theorem 4.3. Let (X,τ,E) and (Y,ϑ,K) be soft topological spaces and $f_{pu} : SS(X)_E \to SS(Y)_K$ be a function. Then every soft $\alpha A-$continuous function is soft semi-continuous.

Proof. The proof is obvious from Proposition 3.13. □

Theorem 4.4. Let (X,τ,E) and (Y,ϑ,K) be soft topological spaces and $f_{pu}: SS(X)_E \to SS(Y)_K$ be a function.Then f_{pu} is soft $A-$continuous and soft $\alpha A-$continuous, then it is soft $LC-$continuous.

PROOF. This is a direct consequence of Theorem 3.15. □

Theorem 4.5. Let (X,τ,E) and (Y,ϑ,K) be soft topological spaces and $f_{pu}: SS(X)_E \to SS(Y)_K$ be a function.Then f_{pu} is soft $A-$continuous and soft $\alpha LC-$continuous, then it is soft $\alpha B-$continuous.

PROOF. The proof is obvious from Proposition 3.10. □

Theorem 4.6. Let (X,τ,E) and (Y,ϑ,K) be soft topological spaces and $f_{pu}: SS(X)_E \to SS(Y)_K$ be a function.Then f_{pu} is soft $A-$continuous iff it is soft $\alpha A-$continuous and soft $LC-$continuous.

PROOF. This follows immediately from Theorem 3.15. □

Theorem 4.7. Let (X,τ,E) and (Y,ϑ,K) be soft topological spaces and $f_{pu}: SS(X)_E \to SS(Y)_K$ be a function.Then f_{pu} is soft $\alpha-$continuous iff it is soft pre-continuous and soft $\alpha B-$continuous.

PROOF. This follows immediately from Theorem 3.21. □

Theorem 4.8. Let (X,τ,E) and (Y,ϑ,K) be soft topological spaces and $f_{pu}: SS(X)_E \to SS(Y)_K$ be a function.Then f_{pu} is soft $\alpha-$continuous iff it is soft pre-continuous and soft $\alpha LC-$continuous.

PROOF. This follows immediately from Theorem 3.22. □

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