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Another Decomposition of Nano Continuity Using Ng^*-Closed Sets

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Abstract — There are various types of nano generalization of continuous function in the development of nano topology. In this paper, we obtain a decomposition of nano continuity using a nano generalized continuity called nano g^*-continuity in nano topology.

Keywords — *Ng^*-closed set, $Nglc^*$-set, nano g^*-continuous function, $Nglc^*$-continuous function*

1. Introduction

Different types of nano generalizations of continuous function were introduced and studied by various authors in the recent development of nano topology. The decomposition of nano continuity is one of the many problems in nano topology. Recently, Ganesan et. al. [2] obtained on some decomposition of nano continuity. In this paper, we obtain a decomposition of nano continuity in nano topological spaces using nano g^*-continuity in nano topological spaces.

2. Preliminary

Definition 2.1. [3] Let U be a non-empty finite set of objects called the universe and R be an equivalence relation on U named as the indiscernibility relation. Elements belonging to the same equivalence class are said to be indiscernible with one another. The pair (U, R) is said to be the approximation space. Let $X \subseteq U$.

1. The lower approximation of X with respect to R is the set of all objects, which can be for certain classified as X with respect to R and it is denoted by $L_R(X)$.
   i.e., $L_R(X) = \bigcup_{x\in U}\{R(X) : R(X) \subseteq X\}$ where $R(x)$ denotes the equivalence class determined by X.

2. The upper approximation of X with respect to R is the set of all objects, which can be possibly classified as X with respect to R and it is denoted by $U_R(X)$.
   i.e., $U_R(X) = \bigcup_{x\in U}\{R(X) : R(X)\cap \mathrm{X} \neq \emptyset\}$

3. The boundary region of X with respect to R is the set of all objects, which can be neither in nor as not-X with respect to R and it is denoted by $B_R(X)$.
   i.e., $B_R(X) = U_R(X) - L_R(X)$.

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Proposition 2.2. [3] If (U, R) is an approximation space and $X, Y \subseteq U$, then

1. $L_R(X) \subseteq$X$\subseteq U_R(X)$.
2. $L_R(\emptyset) = U_R(\emptyset) = \emptyset, L_R(U) = U_R(U) = U$.
3. $U_R(X \cup Y) = U_R(X) \cup U_R(Y)$.
4. $U_R(X \cap Y) \subseteq U_R(X) \cap U_R(Y)$.
5. $L_R(X \cup Y) \supseteq L_R(X) \cup L_R(Y)$.
6. $L_R(X \cap Y) = L_R(X) \cap L_R(Y)$.
7. $L_R(X) \subseteq L_R(Y)$ and $U_R(X) \subseteq U_R(Y)$ whenever $X \subseteq Y$.
8. $U_R(X^c) = [L_R((X)]^c$ and $L_R(X^c = [U_R(X)]^c$.
9. $U_R(U_R(X)) = L_R(U_R(X)) = U_R(X)$.
10. $L_R(L_R(X)) = U_R(L_R(X)) = L_R(X)$.

Definition 2.3. [3] Let U be an universe, R be an equivalence relation on U and $\tau_R(X) = \{U, \emptyset, L_R(X), U_R(X), B_R(X)\}$, where $X \subseteq U$. Then, by proposition 2.2, $\tau_R(X)$ satisfies the following axioms

1. $U, \emptyset \in \tau_R(X)$.
2. The union of the elements of any sub-collection of $\tau_R(X)$ is in $\tau_R(X)$.
3. The intersection of the elements of any finite sub collection of $\tau_R(X)$ is in $\tau_R(X)$.

   Then, $\tau_R(X)$ is called the nano topology on U with respect to X.

   The space $(K, \tau_R(X))$ is the nano topological space. The elements of are called nano open sets.

Definition 2.4. [3] If $(U, \tau_R(X))$ is the nano topological space with respect to X where $X \subseteq U$ and if $A \subseteq U$, then

1. The nano interior of the set M is defined as the union of all nano open subsets contained in A and it is denoted by $NInt(A)$. That is, $NInt(A)$ is the largest nano open subset of A.
2. The nano closure of the set A is defined as the intersection of all nano closed sets containing A and it is denoted by $NCl(A)$. That is, $NCl(A)$ is the smallest nano closed set containing A.

Definition 2.5. Let $(U, \tau_R(X))$ be a nano topological space. A subset A of $(U, \tau_R(X))$ is called

1. Nano generalised closed (briefly, Ng-closed) set [1] if $Ncl(A) \subseteq V$ whenever $A \subseteq V$ and V is nano open in $(U, \tau_R(X))$. The complement of Ng-closed set is called Ng-open.
2. Nano generalised star closed (briefly, Ng^*-closed) set [5] Ncl(A) $\subseteq V$ whenever $A \subseteq V$ and V is Ng-open in $(U, \tau_R(X))$. The complement of Ng^*-closed set is called Ng^*-open.

Definition 2.6. A function $f : (U, \tau_R(X)) \to (V, \tau'_R(Y))$ is called:

1. nano continuous [4] if the inverse image of every nano closed set in V is nano closed in U.
2. nano g^*-continuous [6] if the inverse image of every nano closed set in V is Ng^*-closed in U.

Proposition 2.7. [5] Every nano closed set is Ng^*-closed set but not conversely.

Proposition 2.8. [6] Every nano continuous function is nano g^*-continuous but not conversely.

3. Decomposition of nano continuity

In this section, we obtain a decomposition of nano continuity in nano topological spaces by using nano g^*-continuity.

To obtain a decomposition of nano continuity, we first introduce the notion of $Nglc^*$-continuous function in nano topological spaces and prove that a function is nano continuous if and only if it is both nano g^*-continuous and $Nglc^*$-continuous.

Definition 3.1. A subset A of a space $(U, \tau_R(X))$ is said to be $Nglc^*$-set if $A = M \cap O$, where M is Ng-open set and O is nano closed in $(U, \tau_R(X))$.

Example 3.2. Let $U = \{a, b, c\}$, with $U/R = \{\{a\}, \{b, c\}\}$ and $X = \{a\}$. Then, the nano topology $\tau_R(X) = \{U, \emptyset, \{a\}\}$. Then, nano closed are U, $\emptyset$, and $\{b, c\}$. Then, $Nglc^*(U, \tau_R(X)) = \{U, \emptyset, \{a\}, \{b\}, \{c\}, \{a, b\}, \{a, c\}, \{b, c\}\}$. Here, the set $\{c\}$ is $Nglc^*$-set in $(U, \tau_R(X))$.

Proposition 3.3. Every nano closed set is $Nglc^*$-set but not conversely.

PROOF. It is follows from Definition 3.1.

Example 3.4. Let $U = \{a, b, c\}$ with $U/R = \{\{b\}, \{a, c\}\}$ and $X = \{b\}$. Then, nano topology $\tau_R(X) = \{U, \emptyset, \{b\}\}$. The nano closed sets are U, $\emptyset$, and $\{a, c\}$. Then, $Nglc^*(U, \tau_R(X)) = \{U, \emptyset, \{a\}, \{b\}, \{c\}, \{a, b\}, \{a, c\}, \{b, c\}\}$. Here, the set $\{a, b\}$ is $Nglc^*$-set but not nano closed in $(U, \tau_R(X))$.

Remark 3.5. Ng^*-closed sets and $Nglc^*$-sets are independent of each other.

Example 3.6. Let $U = \{a, b, c\}$ with $U/R = \{\{c\}, \{a, b\}, \{b, a\}\}$ and $X = \{a, b\}$. Then, nano topology $\tau_R(X) = \{U, \emptyset, \{a, b\}\}$. The nano closed sets are U, $\emptyset$, and $\{c\}$. Then, $Ng^*(U, \tau_R(X)) = \{U, \emptyset, \{c\}, \{a, c\}, \{b, c\}\}$ and $Nglc^*(U, \tau_R(X)) = \{U, \emptyset, \{a\}, \{b\}, \{c\}, \{a, b\}\}$. Here, the set $\{a, c\}$ is an Ng^*-closed but not $Nglc^*$-set in $(U, \tau_R(X))$.

Example 3.7. Let $U = \{a, b, c\}$ with $U/R = \{\{a\}, \{b, c\}\}$ and $X = \{a, c\}$. Then, nano topology $\tau_R(X) = \{U, \emptyset, \{a\}, \{b, c\}\}$. The nano closed sets are U, $\emptyset$, $\{a\}$, and $\{b, c\}$. Then, $Ng^*(U, \tau_R(X)) = \{U, \emptyset, \{a\}, \{b, c\}\}$ and $Nglc^*(U, \tau_R(X)) = \{U, \emptyset, \{a\}, \{b\}, \{c\}, \{a, b\}, \{a, c\}, \{b, c\}\}$. Here, the set $\{a, b\}$ is an $Nglc^*$-set but not Ng^*-closed in $(U, \tau_R(X))$.

Proposition 3.8. Let $(U, \tau_R(X))$ be a nano topological space. Then, a subset A of $(U, \tau_R(X))$ is nano closed if and only if it is both Ng^*-closed and $Nglc^*$-set.

PROOF. Necessity is trivial. To prove the sufficiency, assume that A is both Ng^*-closed and $Nglc^*$-set. Then, $A = M \cap O$, where M is Ng-open set and O is nano closed set in $(U, \tau_R(X)))$. Therefore, $A \subseteq M$ and $A \subseteq O$ and so by hypothesis, $N_{cl}(A) \subseteq M$ and $N_{cl}(A) \subseteq O$. Thus, $N_{cl}(A) \subseteq M \cap O = A$ and hence $Ncl(A) = A$ i.e., A is nano closed set in $(U, \tau_R(X))$.

Definition 3.9. Let $f : (U, \tau_R(X)) \to (V, \tau'_R(Y))$ is called $Nglc^*$-continuous if for each nano closed set B of $(V, \tau'_R(Y))$, $f^{-1}(B)$ is $Nglc^*$-set of $(U, \tau_R(X))$.

Example 3.10. Let $U = \{a, b, c\}$ with $U/R = \{\{c\}, \{a, b\}\}$ and $X = \{c\}$. Then, nano topology $\tau_R(X) = \{U, \emptyset, \{c\}\}$. Then, nano closed sets are U, $\emptyset$, and $\{a, b\}$. Then, $Nglc^*(U, \tau_R(X)) = \{U, \emptyset, \{a\}, \{b\}, \{c\}, \{a, b\}, \{a, c\}, \{b, c\}\}$. Let $V = \{a, b, v\}$ with $V/R' = \{\{b\}, \{a, c\}\}$ and $Y = \{a, b\}$. Then, the nano topology $\tau'_R(Y) = \{V, \emptyset, \{b\}, \{a, c\}\}$. The nano closed sets are V, $\emptyset$, $\{b\}$, and $\{a, c\}$. Let $f : (U, \tau_R(X)) \to (V, \tau'_R(Y))$ be the identity function. Then, f is $Nglc^*$-continuous function. Since for the nano closed set $\{a, c\}$ in $(V, \tau'_R(Y))$, $f^{-1}(\{a, c\}) = \{a, c\}$, which is $Nglc^*$ set in $(U, \tau_R(X))$.

Proposition 3.11. Every nano continuous function is $Nglc^*$-continuous but not conversely..

Example 3.12. Let $U = \{a, b, c\}$ with $U/R = \{\{a\}, \{b, c\}\}$ and $X = \{a\}$. Then, nano topology $\tau_R(X) = \{U, \emptyset, \{a\}\}$. The nano closed sets are U, $\emptyset$, and $\{b, c\}$. Then, $Nglc^*(U, \tau_R(X)) = \{U, \emptyset, \{a\}, \{b\}, \{c\}, \{a, b\}, \{a, c\}, \{b, c\}\}$. Let $V = \{a, b, c\}$ with $V/R' = \{\{a\}, \{b, c\}\}$ and $Y = \{a, c\}$. Then, nano topology $\tau'_R(Y) = \{V, \emptyset, \{a\}, \{b, c\}\}$. The nano closed sets are V, $\emptyset$, $\{a\}$, and $\{b, c\}$. Let $f : (U, \tau_R(X)) \to (V, \tau'_R(Y))$ be the identity function. Then, f is $Nglc^*$-continuous function. Since for the nano closed set $\{a\}$ in $(V, \tau'_R(Y))$, $f^{-1}(\{a\}) = \{a\}$, which is not nano closed in $(U, \tau_R(X))$, f is not nano continuous.

Remark 3.13. Nano g^*-continuity and $Nglc^*$-continuity are independent of each other.

Example 3.14. Let $U = \{a, b, c\}$ with $U/R = \{\{c\}, \{a, b\}, \{b, a\}\}$ and $X = \{a, b\}$. Then, nano topology $\tau_R(X) = \{U, \emptyset, \{a, b\}\}$. The nano closed sets are U, $\emptyset$, and $\{c\}$. Then, $Ng^*(U, \tau_R(X)) = \{U, \emptyset, \{c\}, \{a, c\}, \{b, c\}\}$ and $Nglc^*(U, \tau_R(X)) = \{U, \emptyset, \{a\}, \{b\}, \{c\}, \{a, b\}\}$. Let $V = \{a, b, c\}$ with $V/R' = \{\{a\}, \{b, c\}\}$ and $Y = \{a\}$. Then, nano topology $\tau'_R(Y) = \{V, \emptyset, \{a\}\}$. The nano closed sets are V, $\emptyset$, and $\{b, c\}$. Let $f : (U, \tau_R(X)) \to (V, \tau'_R(Y))$ be the identity function. Then, f is nano g^*-continuous function. Since for the nano closed set $\{b, c\}$ in $(V, \tau'_R(Y))$, $f^{-1}(\{b, c\}) = \{b, c\}$, which is not $Nglc^*$-set in $(U, \tau_R(X))$, f is not $Nglc^*$-continuous.

Example 3.15. Let $U = \{a, b, c\}$ with $U/R = \{\{b\}, \{a, c\}\}$ and $X = \{b\}$. Then, nano topology $\tau_R(X) = \{U, \emptyset, \{b\}\}$. The nano closed sets are U, $\emptyset$, and $\{a, c\}$. Then, $Ng^*(U, \tau_R(X)) = \{U, \emptyset, \{a, c\}\}$ and $Nglc^*(U, \tau_R(X)) = \{U, \emptyset, \{a\}, \{b\}, \{c\}, \{a, b\}, \{a, c\}, \{b, c\}\}$. Let $V = \{a, b, c\}$ with $V/R' = \{\{a\}, \{b, c\}, \{c, b\}\}$ and $Y = \{b, c\}$. Then, nano topology $\tau'_R(Y) = \{V, \emptyset, \{b, c\}\}$. The nano closed sets are V, $\emptyset$, and $\{a\}$. Let $f : (U, \tau_R(X)) \to (V, \tau'_R(Y))$ be the identity function. Then, f is $Nglc^*$-continuous function. Since for the nano closed set $\{a\}$ in $(V, \tau'_R(Y))$, $f^{-1}(\{a\}) = \{a\}$, which is not Ng^*-closed set in $(U, \tau_R(X))$, f is not nano g^*-continuous.

We have the following decomposition for nano continuity

Theorem 3.16. A function $f : (U, \tau_R(X)) \to (V, \tau'_R(Y))$ is nano continuous if and only if it is both nano g^*-continuous and $Nglc^*$-continuous.

Proof. Assume that f is nano continuous. Then, by Proposition 2.8 and Proposition 3.11, f is both nano g^*-continuous and $Nglc^*$-continuous.

Conversely, assume that f is both nano g^*-continuous and $Nglc^*$-continuous. Let B be a nano closed subset of $(V, \tau'_R(Y))$. Then, $f^{-1}(B)$ is both Ng^*-closed and $Nglc^*$-set. By Proposition 3.8, $f^{-1}(B)$ is a nano closed set in $(U, \tau_R(X))$ and so f is nano continuous.

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