TWMS J. App. Eng. Math. V.6, N.1, 2016, pp. 126-134

STOCHASTIC COST EFFICIENCY EVALUATION OF A SUPPLY CHAIN

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Abstract. The main goal of the paper is a consideration of cost efficiency evaluation models related to some supply chain when dealing with imprecise data. Data envelopment analysis (DEA) method is a non-parametric mathematical programming approach to assess the performance. This method is proposed for deterministic data and it can be generalized to inaccurate data, while considering real world applications. Here we consider data as random variables and after reviewing and introducing new models to evaluate cost efficiencies related to the special circumstances of the supply chain using DEA, these models are developed to probabilistic form. Also, deterministic and linear equivalents are proposed using the symmetric error structure of normal distributions. At final, by a numerical example, the proposed models are examined to show relationships of results.

Keywords: Data envelopment analysis, Supply chain management, Performance measures, Symmetric error structure, Cost efficiency.

AMS Subject Classification: 90BXX, 90B15, 90CXX, 90C15.

1. Introduction

Farrell (1957) laid the foundation to measure efficiency and productivity studies at the micro level. His contribution highlighted new insights on two issues: how to define efficiency and productivity, and how to calculate the benchmark technology and efficiency measures. Using Farrell idea, Charnes et al. (1978) by introducing the CCR model, suggested a mathematical technique for evaluating the relative technical efficiency of a set of Decision Making Units (DMUs) with the common set of inputs and outputs. This technique is well known to Data Envelopment Analysis (DEA). Besides technical efficiency, cost efficiency is the other measure of DEA that evaluates the ability of a DMU to produce the current outputs at minimal cost when the prices of inputs are at hand. Cost efficiency was first introduced by Farrell (1957), and then developed by Fre et al. (1985). Tone (2002) pointed out the shortcomings of the cost efficiency measures in the presence of price differences between the DMUs. To overcome this limitation, he relaxed the fixed price assumption and proposed the assessment of the DMUs in the cost space. The assumption of DEA was based on the exact data, but in the real word, because of some conditions such as the financial crisis and social conditions, usually data are imprecise and performance evaluation by usual methods in the presence of inaccurate data may

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§ Manuscript received: May 24,2015, Accepted: February 02, 2016.
TWMS Journal of Applied and Engineering Mathematics, Vol.6, No.1;

lead to errors in decision making process and conventional DEA cannot easily measure the performance. Considering the necessity to use random data in practical applications, several researchers have extended ordinal DEA models to stochastic concepts. Thore (1987) initiated a series of efforts directed to chance-constrained programming with DEA as a method for dealing with random data in DEA. Cooper et al. (1996) extended DEA models with inputs and outputs as normal random variables. Also, they defined stochastic efficient DMU. Li (1998) and Huang and Li (2001) defined the efficiency dominance of a DMU via probabilistic comparisons of inputs and outputs with other DMUs which are assessed by solving a chance constrained programming problem. After that Olesen (2002), Cooper et al. (2004), Khodabakhshi and Asgarian (2008) and Hosseinzadeh Lotfi et al. (2012) have provided some DEA models with random data. In conventional Data Envelopment Analysis, DMUs are generally treated as a black-box in the sense that internal structures are ignored, and the performance of a DMU is assumed to be a function of a set of chosen inputs and outputs. A significant body of work has been directed at problem settings where the DMU is characterized by a multistage process; supply chains and many manufacturing processes take this form. Recent DEA literature on serial processes has tended to concentrate on closed systems, that is, where the outputs from one stage become the inputs to the next stage, and where no other inputs enter the process at any intermediate stage. To estimate the efficiency of such systems, several authors proposed network DEA models Fre and Grosskopf (2000) proposed a Network DEA model for measuring efficiency for DMUs with multiple production stage. Seiford and Zhu (1999) and Chen and Zhu (2004), provide two approaches in modeling efficiency as a two-stage process. Liang et al. (2006) identified the efficiency of supply chain and its members through one DEA model. Chen and Yan (2011) proposed the Network DEA models for measuring efficiency of supply chain, which is based on a radial network DEA model under three mechanisms: centralized, decentralized and mixed.

The main goal of this paper is a consideration of cost efficiency evaluation models related to centralized and decentralized supply chain models when dealing with imprecise data. Here we consider data as random variables and after reviewing and introducing new models to evaluate cost efficiency related to the special circumstances of the supply chain using DEA, these models are developed to probabilistic form. Also, deterministic and linear equivalents are proposed using the symmetric error structure of normal distributions. At final, by a numerical example, the proposed models are examined to show relationships of results.

The remainder of the paper is organized as follows. The next section briefly introduces the method of DEA and efficiency evaluation of supply chain. Then proposed models in stochastic area are presented in section3. The model is then illustrated by an example in this section. Also, Conclusions are provided in the last section.

2. Preliminaries

In this section, first, CCR model in evaluating the technical efficiency and the cost efficiency model is introduced. After that, the centralized and decentralized models of Chen and Yan (2011) in technical supply chain efficiency evaluation are presented.

2.1. **cost efficiency.** Data Envelopment Analysis (DEA) was introduced by Charnes et al. (1978), CCR henceforth for short. They developed the piece-wise-linear convex hull approach to frontier estimation proposed by Farrell (1957) in a model which has an input orientation and assumes constant returns to scale, in the following CCR model. Subsequent papers have considered alternative sets of assumptions, such as variable return to scale (VRS) and output orientation (Banker et al., 1984). The originally suggested input oriented CCR is formulated as:

$$\begin{array}{lll} \min & \theta & \\ s.t. & \sum_{j=1}^{n} \lambda_j x_{ij} \leq \theta\, x_{io}, & i = 1, ..., m, \\ & \sum_{j=1}^{n} \lambda_j y_{rj} \geq y_{ro}, & r = 1, ..., s, \\ & \lambda_j \geq 0, & j = 1, ..., n. \end{array} \tag{1}$$

where θ is a scalar, $\lambda = (\lambda_1, \lambda_2, ..., \lambda_n)$is a $n \times 1$ vector of constants, $y_{rj}$is the rth output for a DMU_j, $r = 1, ..., s$, $j = 1, ..., n$. Also, $x_{ij}$is the ith input for the DMU_j, $i = 1, ..., m$. The value of θobtained will be the technical efficiency score for the oth DMU where $0 \leq \theta \leq 1$. A DMU is called efficient if the related optimal θ equals 1.

Besides technical efficiency, cost efficiency is the other measure of DEA that evaluates the ability of a DMU to produce the current outputs at minimal cost when the prices of inputs are at hand. Farrell first introduced the concept of cost efficiency underlying a DEA assessment (1957). Fare et al. (1985) operationalized cost efficiency measures based on the Farrell concept in the DEA literature. Their model is the following linear problem:

$$\begin{array}{lll} \min & \sum_{i=1}^{m} c_i x_i & \\ s.t. & \sum_{j=1}^{n} \lambda_j x_{ij} \leq x_i, & i = 1, ..., m, \\ & \sum_{j=1}^{n} \lambda_j y_{rj} \geq y_{ro} & r = 1, ..., s, \\ & \lambda_j \geq 0, & j = 1, \ldots, n. \end{array} \tag{2}$$

where c_i is the unit cost of the input $i$which is the benchmark projection that can be different from one DMU to another. The minimization problem is calculated for each DMU of the sample, thus identifying for each a benchmark combination of inputs and cost.

Based on an optimal solution (x^*, λ^*) of model (2), the cost efficiency of DMU_o is defined as

$$CE_o = \frac{cx^*}{cx_o} \tag{3}$$

where CE_o is the ratio of minimum cost to observed cost to the oth firm.

2.2. **Supply chain efficiency evaluation.** A supply chain is in the form of a network with multiple divisions and relationships. The supply chain performance measurement that only considers the initial inputs and the final outputs is generally inadequate since it ignores the interactions among the divisions. To measure the supply chain performance properly, it is necessary to explore the complex internal structure and emphasize the interrelated nature in a supply chain. Here the centralized and decentralized models of Chen and Yan (2011) are introduced. They incorporated the interactions in supply chain and developed different DEA models under the concept of centralized, decentralized and mixed organization mechanism respectively for supply chain performance evaluation. Supply chain efficiency is referring to technical efficiency.

Now consider a set of two stage supply chain with one supplier and two manufacturers, as DMUs. The structure of DMU_o is portrayed in Fig. 1.

If all divisions are controlled by a single decision maker with access to available information, this refers to a centralized supply chain control. If each division has its own incentive and strategy, and there does not exist such a "super decision maker" to control all divisions, this is characterized as decentralized control supply chain. Then by Chen and Yan (2011), the technical efficiency evaluation models in centralized and decentralized

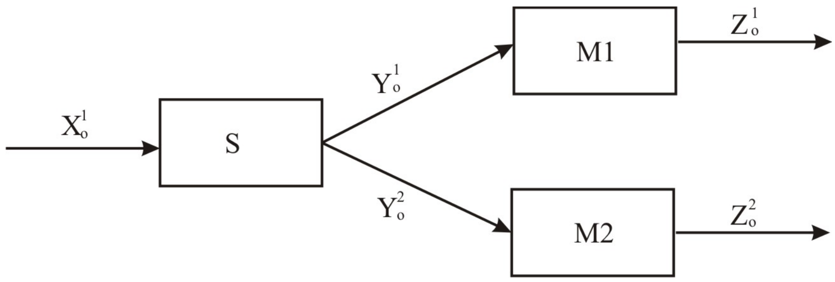

Figure 1. Supply chain

mechanisms are as follows:

$$(D_{central})\begin{cases} \min & \theta_{central} \\ s.t. & \sum_{j=1}^{n}\lambda_j^1 X_j \le \theta_{central}X_o \\ & \sum_{j=1}^{n}\lambda_j^1 Y_j^1 \ge \sum_{j=1}^{n}\lambda_j^2 Y_j^1 \\ & \sum_{j=1}^{n}\lambda_j^1 Y_j^2 \ge \sum_{j=1}^{n}\lambda_j^3 Y_j^2 \\ & \sum_{j=1}^{n}\lambda_j^2 Z_j^1 \ge Z_o^1 \\ & \sum_{j=1}^{n}\lambda_j^3 Z_j^2 \ge Z_o^2 \\ & \lambda_j^1, \lambda_j^2, \lambda_j^3 \ge 0 \end{cases} \tag{4}$$

and

$$(D_{decentral})\begin{cases} \min & \theta_{central} \\ s.t. & \sum_{j=1}^{n}\lambda_j^1 X_j \le \theta_{decentral}X_o \\ & \sum_{j=1}^{n}\lambda_j^1 Y_j^1 \ge \sum_{j=1}^{n}\lambda_j^2 Y_j^1 \\ & \sum_{j=1}^{n}\lambda_j^1 Y_j^2 \ge \sum_{j=1}^{n}\lambda_j^3 Y_j^2 \\ & \sum_{j=1}^{n}\lambda_j^2 Y_j^1 \le Y_o^1 \\ & \sum_{j=1}^{n}\lambda_j^2 Z_j^1 \ge Z_o^1 \\ & \sum_{j=1}^{n}\lambda_j^3 Y_j^2 \le Y_o^2 \\ & \sum_{j=1}^{n}\lambda_j^3 Z_j^2 \ge Z_o^2 \\ & \lambda_j^1, \lambda_j^2, \lambda_j^3 \ge 0 \end{cases} \tag{5}$$

3. Stochastic cost efficiency of the supply chain

In this section we first review the chance constrained stochastic CCR model and its deterministic and linear programming equivalent. Next, the stochastic cost efficiency of the multiple stage supply chain in centralized and decentralized mechanisms is presented.

3.1. Stochastic CCR model. Let us assume that $\tilde{x}_j = (\tilde{x}_{1j}, ..., \tilde{x}_{mj})$ and $\tilde{y}_j = (\tilde{y}_{1j}, ..., \tilde{y}_{sj})$ are the stochastic input and output vectors for DMU_j, in a way that each element of these vectors is a random variable with normal distribution and the following specified or estimated parameters:

$$\begin{aligned} \tilde{x}_{ij} &= x_{ij} + a_{ij}\tilde{\varepsilon}_{ij}, \qquad i = 1, ..., m, \\ \tilde{y}_{rj} &= y_{rj} + b_{rj}\tilde{\xi}_{rj}, \qquad r = 1, ..., s. \end{aligned} \tag{6}$$

where $a_{ij}$and $b_{rj}$are nonnegative real values. Also, $\tilde{\varepsilon}_{ij}$ and $\tilde{\xi}_{rj}$ are the error terms which are random variables with standard normal distributions. Since normal distribution is symmetric, then the structure in expression (6) is named symmetric error structure.

Assume that the ith input of every DMUs is interrelated. Similarly, assume rth output of every DMUs is interrelated, too. i.e. for every $j \neq k$,

$$\begin{array}{ll} Cov(\tilde{x}_{ij}, \tilde{x}_{ik}) = 0, & i = 1, ..., m, \\ Cov(\tilde{y}_{rj}, \tilde{y}_{rk}) = 0, & r = 1, ..., s. \end{array} \quad (7)$$

Now consider the chance constraint CCR model as:

$$\begin{array}{lll} \min & \theta & \\ s.t. & P\left(\sum_{j=1}^{n} \lambda_j \tilde{x}_{ij} \leq \theta \tilde{x}_{io}\right) \geq 1-\alpha, & i = 1, ..., m, \\ & P\left(\sum_{j=1}^{n} \lambda_j \tilde{y}_{rj} \geq \tilde{y}_{ro}\right) \geq 1-\alpha, & r = 1, ..., s, \\ & \lambda_j \geq 0, & j = 1, ..., n. \end{array} \quad (8)$$

where in the above model, P means "probability" and α is a level of error between 0 and 1. Model (8) can be converted to a deterministic and linear programming applying symmetric error structure and independent properties (6) and (7) as the following model (refer to Behzadi and Mirbolouki, 2012):

$$\begin{array}{lll} \min & \theta & \\ s.t. & \sum_{j=1}^{n} \lambda_j x_{ij} - \Phi^{-1}(\alpha)\bar{\sigma}(p_i^+ + p_i^-) \leq \theta x_{io} & i = 1, ..., m, \\ & \sum_{j=1}^{n} \lambda_j y_{rj} + \Phi^{-1}(\alpha)\bar{\sigma}(q_r^+ + q_r^-) \geq y_{ro}, & r = 1, ..., s, \\ & \sum_{j=1}^{n} \lambda_j a_{ij} - \theta a_{io} = p_i^+ - p_i^-, & i = 1, ..., m, \\ & \sum_{j=1}^{n} \lambda_j b_{rj} - b_{ro} = q_r^+ - q_r^-, & r = 1, ..., s, \\ & \lambda_j, p_i^+, p_i^-, q_r^+, q_r^- \geq 0, & \forall j, \forall i, \forall r \end{array} \quad (9)$$

In the above model, Φ is the cumulative distribution function of the standard normal distribution.

3.2. **Stochastic cost efficiency of supply chain.** First, using symmetric error structure, the stochastic cost efficiency model related to the stochastic version of the model (2) is presented to compare with the efficiency of the supply chain. This model is:

$$\begin{array}{lll} \min & \sum_{i=1}^{m} c_i x_i & \\ s.t. & \sum_{j=1}^{n} \lambda_j x_{ij} - \Phi^{-1}(\alpha)(p_i^+ + p_i^-) \leq x_i, & i = 1, ..., m, \\ & \sum_{j=1}^{n} \lambda_j y_{rj} + \Phi^{-1}(\alpha)(q_i^+ + q_i^-) \geq y_{ro}, & r = 1, ..., s, \\ & p_i^+ - p_i^- = \sum_{j=1}^{n} \lambda_j a_{ij}, & i = 1, ..., m, \\ & q_i^+ - q_i^- = \sum_{j=1}^{n} \lambda_j b_{rj} - b_{ro}, & r = 1, ..., s, \\ & \lambda_j \geq 0, & j = 1, \ldots, n. \end{array} \quad (10)$$

Here, the stochastic cost efficiency is defined by the below formulation:

$$\tilde{E}_c^o = \frac{\sum_{i=1}^{m} c_i x_i^*}{\sum_{i=1}^{m} c_i (x_{io} + 3a_{io})} \quad (11)$$

where $(x_{io}+3a_{io})$ is the upper bound of 6σ interval and it is used to increase the probability of $\tilde{E}_c^o \leq 1$. It must be noted that by this definition, almost always $\tilde{E}_c^o < 1$. Therefore, there is not any number to show a stochastic efficient DMU, similar in deterministic form. Now consider a multiple stage supply chain, including one supplier and k manufacturer with the following stochastic symmetric structure data:

$$\begin{array}{ll} \tilde{x}_{ij} = x_{ij} + a_{ij}\tilde{\varepsilon}_{ij}, & i = 1, ..., m, \\ \tilde{y}_{lj}^t = y_{lj}^t + b_{lj}^t \tilde{\xi}_{lj}^t, & l = 1, ..., L_k, t = 1, ..., k, \\ \tilde{z}_{rj}^t = z_{rj}^t + f_{rj}^k \tilde{\zeta}_{rj}^t, & r = 1, ..., S_k, t = 1, ..., k. \end{array}$$

Thus, the chance constrained model related to cost efficiency of supply chain within the centralized mechanism can be obtained by the following model:

$$\begin{array}{lll} \min & \sum_{i=1}^{m} c_i x_i & \\ s.t. & P\left\{\sum_{j=1}^{n} \mu_j \tilde{x}_{ij} \leq x_i\right\} \geq 1-\alpha, & i=1,...,m, \\ & P\left\{\sum_{j=1}^{n} \mu_j \tilde{y}_{lj}^t \geq \sum_{j=1}^{n} \lambda_j^t \tilde{y}_{lj}^t\right\} \geq 1-\alpha, & t=1,...,k, l=1,...,L_k, \\ & P\left\{\sum_{j=1}^{n} \lambda_j^t \tilde{z}_{rj}^t \geq \tilde{z}_{ro}^t\right\} \geq 1-\alpha, & t=1,...,k, r=1,....,S_k, \\ & \mu_j \geq 0, & j=1,...,n, \\ & \lambda_j^t \geq 0, & j=1,...,n,\ t=1,...,k. \end{array} \tag{12}$$

The model (12), is transformed to deterministic form using the following model:

$$\begin{array}{lll} \min & \sum_{i=1}^{m} c_i x_i & \\ s.t. & \sum_{j=1}^{n} \mu_j x_{ij} - \Phi^{-1}(\alpha)(v_i + \bar{v}_i) \leq x_i, & i=1,...,m, \\ & \sum_{j=1}^{n} \mu_j y_{lj}^t + \Phi^{-1}(\alpha)(w_l^t + \bar{w}_l^t) \geq \sum_{j=1}^{n} \lambda_j^t y_{lj}^t, & t=1,...,k, l=1,...,L_k, \\ & \sum_{j=1}^{n} \lambda_j^t z_{rj}^t + \Phi^{-1}(\alpha)(u_r^t + \bar{u}_r^t) \geq z_{ro}^t, & t=1,...,k, r=1,....,S_k, \\ & v_i - \bar{v}_i = \sum_{j=1}^{n} \mu_j a_{ij}, & i=1,...,m, \\ & w_l^t - \bar{w}_l^t = \sum_{j=1}^{n} \mu_j b_{lj}^t - \sum_{j=1}^{n} \lambda_j^t b_{lj}^t, & t=1,...,k, l=1,...,L_k \\ & u_r^t - \bar{u}_r^t = \sum_{j=1}^{n} \lambda_j^t f_{rj}^t - f_{ro}^t, & t=1,...,k, r=1,....,S_k, \\ & \mu_j \geq 0, & j=1,...,n, \\ & \lambda_j^t \geq 0, & j=1,...,n,\ t=1,...,k, \\ & v_i, \bar{v}_i, w_l^t, \bar{w}_l^t, u_r^t, \bar{u}_r^t \geq 0, & \forall i, l, k, r. \end{array} \tag{13}$$

While efficiency can be calculated by the expression (11). If we consider decentralized mechanism, the following constraints must be added to the model (13):

$$\begin{array}{ll} \sum_{j=1}^{n} \mu_j y_{lj}^t - \Phi^{-1}(\alpha)(g_l^t + \bar{g}_l^t) \leq y_{lo}^t, & t=1,...,k, l=1,...,L_k, \\ g_l^t - \bar{g}_l^t = \sum_{j=1}^{n} \mu_j b_{lj}^t - b_{lo}^t, & t=1,...,k, l=1,...,L_k, \\ g_l^t \geq 0,\ \bar{g}_l^t \geq 0, & t=1,...,k, l=1,...,L_k. \end{array} \tag{14}$$

The stochastic cost efficiency is a function of the level of error α. Theorem 3.1 shows that this function is a non-decreasing function.

Theorem 3.1. *The optimal objective function of model (3-8) will be decreased by increasing*α.

Proof. Let $\alpha' < \alpha$. Since $\Phi^{-1}(\alpha)$ is an increasing function, the optimal solution of model (13) in α' level of error is a feasible solution for α level. Therefore the minimizing objective function proves the theorem. □

Consider $EC_o^{centeral}$and $EC_o^{decenteral}$ as the stochastic cost efficiency related to centralized and decentralize mechanisms respectively. Also, EC_o^{CCR} is the stochastic cost efficiency of considering each DMU as a black-box. Theorem 3.2 shows the relations between these values.

Theorem 3.2. *in every level of error,* $EC_o^{decenteral} \leq EC_o^{centeral} \leq EC_o^{Black-Box}$.

Proof. it is obvious by the constraints of the related models. □

3.3. **Numerical example.** In this subsection, a numerical example of a supply chain, including one supplier and two manufacturers for every DMU is proposed to show the obtained results and theorems in the last subsection. The needed data are illustrated in Table 1. Here, it is assumed that all inputs and outputs of suppliers and manufactures are random variables while distributed by normal distributions with μ and σ^2 parameters. Also, assume that the first and second input of any suppliers cost 200 and 310 unit price respectively. Data in Table 1 relates to 10 DMUs with supply chain structures, which their suppliers have two inputs (x_1 and x_2) and two outputs(y_1 and y_2). Every output of supplier is as input of a manufacturer. The outputs of each DMU are as outputs of manufacturers (z^1 and z^2).

Table 1. Stochastic inputs and outputs.

	x_1		x_2		y_1		y_2		z^1		z^2	
	μ	σ	μ	σ	μ	σ	μ	σ	μ	σ	μ	σ
DMU1	120	18	40	6	70	21	51	3	18	5	41	9
DMU2	170	16	60	7	54	9	48	8	28	6	56	2
DMU3	90	14	30	3	60	8	39	7	19	3	30	5
DMU4	85	11	50	8	27	2	18	2	14	1	27	7
DMU5	40	6	20	2	24	3	21	4	12	2	12	2
DMU6	270	31	80	10	91	11	57	5	38	1	96	17
DMU7	35	8	25	4	17	1	21	3	8	1	9	3
DMU8	118	19	70	13	62	5	31	2	21	2	68	14
DMU9	65	5	40	9	37	4	18	1	17	3	32	7
DMU10	50	7	27	1	29	3	11	1	10	2	21	4

Table 2. Computationally results of stochastic centralized, decentralized and black-box cost efficiencies.

	$EC_j^{black-box}$		$EC_j^{centeral}$		$EC_j^{decenteral}$	
	$\alpha = 0.05$	$\alpha = 0.1$	$\alpha = 0.05$	$\alpha = 0.1$	$\alpha = 0.05$	$\alpha = 0.1$
DMU1	0.75	0.72	0.41	0.39	0.41	0.39
DMU2	0.89	0.87	0.52	0.45	0.54	0.45
DMU3	0.87	0.83	0.53	0.51	0.53	0.51
DMU4	0.65	0.6	0.36	0.34	0.86	0.83
DMU5	0.87	0.84	0.66	0.63	0.66	0.63
DMU6	0.88	0.85	0.47	0.43	0.57	0.46
DMU7	0.58	0.54	0.39	0.37	0.44	0.4
DMU8	0.85	0.8	0.44	0.41	0.54	0.49
DMU9	0.86	0.82	0.5	0.48	0.6	0.57
DMU10	0.88	0.84	0.47	0.45	0.55	0.51

Data in Table 1 are substituted in models (10), (13) and (13) with (14) extended constraints. Results that are gathered in Table 2 for two different levels of confidence, 90% and 95%, confirm Theorems 3.1 and 3.2.

4. Concluding results

The main goal of this paper is a consideration of cost efficiency evaluation models related to centralized and decentralized supply chain models when dealing with imprecise data. Here we consider data as random variables and after reviewing and introducing new models to evaluate cost efficiency related to the special circumstances of the supply chain using

DEA, these models are developed to probabilistic form. Also, deterministic and linear equivalents are proposed using the symmetric error structure of normal distributions. At final, by a numerical example, the results of models are examined.

Acknowledgement The authors would like to extend their gratitude to Young Researchers and Elite Club because of their support.

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