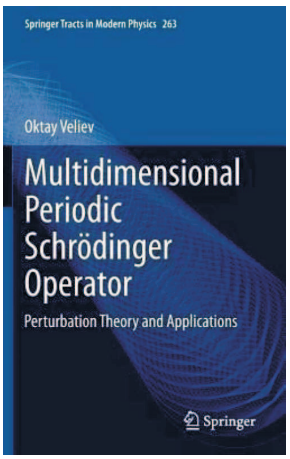
BookReview

Oktay Veliev. Multidimensional Periodic Schrödinger Operator (Perturbation Theory and Applications). ISBN 3319166425. P.242, 2015, Publisher: Springer, Series: Springer Tracts in Modern Physics.



The book is devoted to the spectral theory of the multidimensional Schrödinger operator $L(q)$ with a periodic potential q. This operator is a fundamental operator of the solid state physics and describes the motion of a particle in the bulk matter. The book consists of five chapters. The first chapter presents preliminary defnitions and statements to be used in the next chapters. Besides the author gives a brief discussion of what is known from the literature and what is presented in the book about the perturbation theory of $L(q)$. In the second chapter the asymptotic formulas of arbitrary order for the Bloch eigenvalue and Bloch function of the periodic Schrödinger operator L(q) of arbitrary dimension is obtained. Moreover, the author constructed and estimated the measures of the isoenergetic surfaces in the high energy region which implies the validity of the Bethe-Sommerfeld conjecture for arbitrary dimension and arbitrary lattice. This conjecture was formulated in 1928 and claims that there exist only a finite number of gaps in the spectrum of $L(q)$. Note that the method of this book is the first and unique by which the asymptotic formulas for the Bloch eigenvalues and Bloch functions and the validity of the conjecture for arbitrary lattice and arbitrary dimension were proved. In the third chapter, using the asymptotic formulas being obtained in the second chapter, the author determined constructively a family of the spectral invariants of $L(q)$ from the given Bloch eigenvalues. Some of these invariants are explicitly expressed by the Fourier coefficients of the potential which present the possibility of determining the potential constructively by using the Bloch eigenvalues as the input data. In the forth chapter, is considered the inverse problems of the three-dimensional Schrödinger operator with a periodic potential q by the spectral invariants obtained in the third chapter. The author gives an algorithm and considers the stability of the algorithm for the unique determination of the potential q of the three-dimensional Schrödinger operator with respect to the spectral invariants and Bloch eigenvalues. Finally, the fifth chapter summarizes results from the point of view of both the physicists and mathematicians.

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