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A New Class of Closed Set in Digital Topology

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Abstract — The purpose of this paper is to introduce a new class of closed set called g*$\omega\alpha$-closed sets in digital topology. We establish a relationship between closed and g*$\omega\alpha$-closed sets in digital topology. Also, we obtained the properties of g*$\omega\alpha$-closed sets in digital plane.

Keywords — *g*$\omega\alpha$-closed sets, g*$\omega\alpha$-open sets, digital plane.*

1. Introduction

In the literature, the concept of Digital Topology was first introduced and studied in the late 60's by the computer image analysis researcher Azriel Rosenfeld [1]. The digital line, the digital plane and the three dimensional digital spaces are of great importance in the study of applications of point set topology to computer graphics. Digital Topology consist in providing algorithmic tools for pattern recognition, image analysis and image processing using a discrete formalism for geometrical objects and it is applied in image processing.

First we recall the related definitions and some properties of the digital plane. The digital line or called Khalimsky Line is the set of integers Z, equipped with the topology K having 2n+1, 2n, 2n-1 : n $\in$ Z as a subbas and is denoted by (Z, K). Thus, a subset U is open in (Z, K) if and only if, whenever x $\in$ U is an even integer, then x-1, x+1 $\in$ U. Let (Z^2, K^2) be the topological product of two digital lines (Z, K), where Z^2 = Z $\times$ Z and K^2 = K $\times$ K. This space is called the digital plane ([2], [3], [4], [5], [6], [7]). For each point x $\in$ Z^2, there exists a smallest open set containing x say U(x). For the case of x = (2n+1, 2m+1), U(x) = 2n+1 $\times$ 2m+1 ; for the case of x = (2n, 2m), U(x) = 2n-1, 2n, 2n+1 $\times$ 2m-1, 2m, 2m+1 ; for the case of x = (2n, 2m+1), U(x) = 2n-1, 2n, 2n+1 $\times$ 2m+1 ; for the case of x = (2n+1, 2m), U(x) = 2n+1 $\times$ 2m-1, 2m, 2m+1 where n, m $\in$ Z.

For a subset E of (Z^2, K^2), we have the following three subsets as follows:

E_F = x $\in$ E: x is closed in (Z^2, K^2) ; E_{K^2} = x $\in$ E: x is open in (Z^2, K^2) ; E_{mix} = E $\setminus$ (E_F $\cup$ E_{K^2}). Then it is shown that E_F = (2n, 2m) $\in$ E: n, m $\in$ Z ; E_{K^2} = (2n+1, 2m+1) $\in$ E: n, m $\in$ Z and E_{mix} = (2n, 2m+1) $\in$ E: n, m $\in$ Z $\cup$ (2n+1, 2m) $\in$ E : n, m $\in$ Z.

In the digital plane if the corner points of a digital plane are even then it is called closed set. If the corner points of a digital plane are odd it is called an open set.

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2. Preliminaries

Definition 2.1. A subset A of a topological space X is called a

(i) g*$\omega\alpha$-closed [8] if cl(A) $\subseteq$ U whenever A $\subseteq$ U and U is $\omega\alpha$-open in X.

(ii) g*$\omega\alpha$-open [8] if U $\subseteq$ int(A) whenever U $\subseteq$ A and U is $\omega\alpha$-closed in X.

(iii) $\omega\alpha$-closed [9] if αcl(A) $\subseteq$ U whenever A $\subseteq$ U and U is ω-open in X.

Definition 2.2. A subset A of a topological space X is called a

(i) T_g*$\omega\alpha$-space [10] if every g*$\omega\alpha$-closed set is closed.

(ii) g$\omega\alpha T_g\omega\alpha$-space [10] if every g$\omega\alpha$-closed set is g*$\omega\alpha$-closed.

(iii) $T_{g\omega\alpha}$-space [11] if every g$\omega\alpha$-closed set is closed.

3. g*$\omega\alpha$-Closed Sets in Digital Plane

Lemma 3.1. [4] Let (Z^2, K^2) be a digital plane. Then the following properties hold:

(i) if m is even point, that is m = (2n, 2m), then cl(2n, 2m) = 2n, 2m

(ii) if m is odd point, that is m = (2n+1, 2m+1), then
cl(2n+1, 2m+1) = 2n, 2n+1, 2n+2 $\times$ 2m, 2m+1, 2m+2

(iii) if m is mixed point, that is m = (2n+1, 2m) or (2n, 2m+1), then
cl(2n, 2m+1) = 2n $\times$ 2m ,2m+1, 2m+2
cl(2n+1, 2m) = 2n, 2n+1, 2n+2 $\times$ 2m

Theorem 3.2. Every closed in (Z^2, K^2) is g*$\omega\alpha$-closed in (Z^2, K^2).

Proof. Let A be a subset of (Z^2, K^2). Let us consider the following three cases:

(i) The set A contains all even points ($E_F \subseteq$ A)
that is A = (2n, 2m), then U(A) = { 2n-1, 2n, 2n+1 } $\times$ { 2m-1, 2m, 2m+1 }. Let A $\subseteq$ U and U is $\omega\alpha$-open (Z^2, K^2). Then by lemma 3.1, cl(A) = cl({ 2n, 2m }) = { 2n, 2m } = A, that is cl(A) = A.
This implies cl(A) = A $\subseteq$ U. Hence A is g*$\omega\alpha$-closed in (Z^2, K^2).

(ii) The set A contains all even, odd and mixed points ($E_{mix} \cup E_F \cup E_{K^2} \subseteq$ A)
Let A ={ 2n, 2n$\pm$1, 2n$\pm$2 ... 2n$\pm$q $\pm$ 2m, 2m$\pm$1, 2m$\pm$2 ... 2m$\pm$q} where n, m and q are even integer.
Then U = {2n, 2n$\pm$1, 2n$\pm$2 . . . 2n$\pm$q, 2n$\pm$q$\pm$1 } $\times$ { 2m, 2m$\pm$1, 2m$\pm$2 ... 2m$\pm$q, 2m$\pm$q$\pm$1}.
Let A $\subseteq$ U, where U is $\omega\alpha$-open in (Z^2, K^2). Since A is closed, cl(A) = A. Therefore cl(A) = A $\subseteq$ U. Therefore A is g*$\omega\alpha$-closed in (Z^2, K^2).

(iii) A contains all even and mixed points ($E_F \cup E_{mix} \subseteq$ A) Let A = { 2n } $\times$ { 2m, 2m$\pm$1, 2m$\pm$2 ... 2m$\pm$q }, where n, m, q are even integers. Then U = { 2n, 2n$\pm$1, 2n$\pm$2 ... 2n$\pm$q, 2n$\pm$q$\pm$1 } $\times$ { 2m, 2m$\pm$1, 2m$\pm$2 ... 2m$\pm$q, 2m$\pm$q$\pm$1 } for any n, m, q are even integers. Let A $\subseteq$ U, where U is $\omega\alpha$-open in (Z^2, K^2). Since A is closed, cl(A) = A $\subseteq$ U, that is cl(A) $\subseteq$ U. Therefore A is g*$\omega\alpha$-closed set in (Z^2, K^2). Similarly, A is g*$\omega\alpha$-closed by considering A = { 2n, 2n$\pm$1, 2n$\pm$2 ... 2n$\pm$q } $\times$ {2m }.

□

Remark 3.3. The following example shows that the converse is not true in general.

Example 3.4. Let A = (2,2), (2,3), (2,4), (3,3), (3,4), (4,4) and U = {1, 2, 3, 4, 5} $\times$ {1, 2, 3, 4, 5} Then cl(A) = {2, 3, 4} $\times$ {2, 3, 4} , that is cl(A) $\subseteq$ U. Therefore A is g*$\omega\alpha$-closed in (Z^2, K^2). But cl(A) = A, hence A is not closed in (Z^2, K^2).

Theorem 3.5. Every open set in (Z^2, K^2) is g*$\omega\alpha$-open in (Z^2, K^2).

PROOF. Let us consider the following three cases:

Case (i): A contains all odd points ($E_{K^2} \subseteq$ A). Let A = (2n+1, 2m+1) and U = φ. Assume that U$\subseteq$ A, where A is $\omega\alpha$-closed in (Z^2, K^2). Then U $\subseteq$ A = int(A), as A is open in (Z^2, K^2). Therefore U $\subseteq$ int(A) and U is $\omega\alpha$-closed in (Z^2, K^2). Hence A is g*$\omega\alpha$-open in (Z^2, K^2).

Case (ii): A contains all even, odd and mixed points ($E_{mix} \cup E_{K^2} \cup E_F \subseteq$ A) Let A = { 2n, 2n±1, 2n±2 … 2n±q } × { 2m, 2m±1, 2m±2 … 2m±q }, for any n, m, q is odd integers and U = { 2n, 2n±1, 2n±2 … 2n±q, 2n±(q-1) } × { 2m, 2m±1, 2m±2 … 2m±q, 2m±(q-1) }. Let A $\subseteq$ U, where U is $\omega\alpha$-closed in (Z^2, K^2), that is U $\subseteq$ int(A) = A, as A is open in (Z^2, K^2). Therefore U $\subseteq$ int(A) and hence A is g*$\omega\alpha$-open in (Z^2, K^2).

Case (iii): A contains odd and mixed points ($E_{mix} \cup E_{K^2} \cup E_F \subseteq$ A) A contains odd and mixed points ($E_{K^2} \cup E_{mix} \subseteq$ A) Let A = {2n+1} × { 2m, 2m±1, 2m±2 … 2m±q} and U = φ. Let U $\subseteq$ A, where U is $\omega\alpha$-closed in (Z^2, K^2). That is U $\subseteq$ A = int(A), because A is open in (Z^2, K^2). Therefore U $\subseteq$ int(A) and hence A is g*$\omega\alpha$-open in (Z^2, K^2).

□

Example 3.6. The converse of the above theorem is not true follows from the example 3.4.

Theorem 3.7. If A is g*$\omega\alpha$-closed in (Z^2, K^2), then it does not contain all odd points ($E_{K^2} \not\subset$ A).

PROOF. Let A be any set in (Z^2, K^2) which contains all odd points. Let A = (2n+1, 2m+1) and U = (2n+1, 2m+1) be $\omega\alpha$-open set in (Z^2, K^2). Then cl(A) = {2n, 2n+1, 2n+2} × {2m, 2m+1, 2m+2} , we get cl(A) U, which is contradiction to the assumption. Hence A does not contain all odd points. □

Theorem 3.8. A g*$\omega\alpha$-open set A in (Z^2, K^2) does not contain all even points ($E_F \not\subset$ A).

PROOF. Let A be any set in (Z^2, K^2) which contain all even points. Let A = (2n, 2m) and U = (2n, 2m) be any $\omega\alpha$-closed in (Z^2, K^2). Let us assume that U $\subseteq$ A, where U is $\omega\alpha$-closed in (Z^2, K^2). Then U int(A), since int(A) = φ, if A is even, which is contradiction to the fact that A contains all even points. Hence A does not contain all even points. □

Remark 3.9. Union of an open and g*$\omega\alpha$-open set is again a g*$\omega\alpha$-open.

Theorem 3.10. Let A and E be subsets of (Z^2, K^2)

(i) if E is non empty g*$\omega\alpha$-closed, then $E_F \neq \varphi$.

(ii) if E is $\omega\alpha$-closed and E $\subseteq B_{mix} \cup B_{K^2}$ holds for some subset B of (Z^2, K^2), then E = φ.

(iii) The set U(A_F) $\cup A_{mi} \cup A_{k^2}$ is g*$\omega\alpha$-open containing A.

PROOF. .

Case (i) Let y be any point in E, then y ∈ cl(E) = E, as E is closed. Let us consider the following three cases: Let y ∈ E_F , then $E_F \neq \varphi$. Let y ∈ E_{K^2} , that is y = (2n+1, 2m+1) where n, m ∈ Z. Then cl({ y }) = { 2n, 2n+1, 2n+2 } × { 2m, 2m+1, 2m+2 } $\subseteq$ E. Thus, there exists a point x = (2n, 2m) such that x ∈ E_F. Therefore $E_F \neq \varphi$. Let y ∈ E_{mix}, that is y = (2n+1, 2m). Then cl({y}) = { 2n, 2n+1, 2n+2 } × { 2m} $\subseteq$ E. Thus there exists a point x = (2n, 2m) $\subseteq E_F$ such that $E_F \neq \varphi$. Similarly, $E_F \neq \varphi$ for x = (2n, 2m + 1). Therefore in all the three cases we have $E_F \neq \varphi$.

Case (ii) Suppose on the contrary E $\neq \varphi$. From case (i), $E_F \neq \varphi$. By hypothesis $E_F \subseteq (B_{mix} \cup B_{K^2})_F$ = φ, which is contradiction to the assumption. Hence E = φ.

Case (iii) We have to prove that U(A_F) ∪ A_{mix} ∪ A_{K^2} is g*$\omega\alpha$-open containing A. We know that U(A_F) is an open set in (Z^2, K^2). Then we have to show that A_{mix} ∪ A_{K^2} is g*$\omega\alpha$-open in (Z^2,K^2). Let F be any non-empty $\omega\alpha$-closed set such that F ⊆ A_{mix} ∪ A_{K^2} . But from case (ii), F = φ, implies F ⊆ (A_{mix} ∪ A_{K^2}). Thus, A_{mix} ∪ A_{K^2} is g*$\omega\alpha$-open in (Z^2, K^2) and U(A_F) is an open set in (Z^2, K^2). Therefore U(A_F) ∪ Amix ∪ A_{K^2} is g*$\omega\alpha$-open set in (Z^2, K^2) by remark 3.9. Therefore A ⊆ U(A_F) ∪ Amix ∪ A_{K^2}. Therefore U(A_F) ∪ A_{mix} ∪ A_{K^2} is g*$\omega\alpha$-open set containing A.

□

Remark 3.11. [10] If X is a T_g*$\omega\alpha$-space, then every singleton set {x} is either open or $\omega\alpha$-closed.

Theorem 3.12. The digital plane (Z , K) is a T_g*$\omega\alpha$-space.

Proof. Let {x} be any point in (Z^2, K^2).
Let us consider the following three cases:

Case (i) if {x} is odd, that is x = (2n+1, 2m+1), then {x} is open in (Z^2, K^2).

Case (ii) if {x} is even, that is x = (2n, 2m). Then {x} is closed in (Z^2, K^2).

Case (iii) if {x} is a mixed point, that is x = (2n, 2m+1) or x = (2n+1, 2m).
Let U be any α-open set containing { x }. Then αcl({ x }) = { x } ∪ cl(int(cl({ x }))) ={ x } ∪ cl(int([{ 2n } × { 2m, 2m+1, 2m+2 }])) = { x } ∪ cl(φ) = { x } ⊆ U. Therefore { x } is $\omega\alpha$-closed in (Z^2, K^2). Similarly, {x} is $\omega\alpha$-closed in (Z^2, K^2) for x = (2n+1, 2m). Thus, we have, {x} is either open or $\omega\alpha$-closed in all the cases. Therefore from remark 3.11, we have (Z^2, K^2) is T_g*$\omega\alpha$-space. Hence (Z^3, K^2) is T_g*$\omega\alpha$-space.

□

Corollary 3.13. The digital plane (Z^2, K^2) is a $T_{g\omega\alpha}$-space.

Theorem 3.14. The digital plane (Z^2, K^2) is ${}_{g\omega\alpha}T_g{}^*{}_{\omega\alpha}$-space.

Proof. Let A be g$\omega\alpha$-closed in (Z^2, K^2). From corollary 3.13, (Z^2, K^2) is $T_{g\omega\alpha}$-space, so A is closed. From [8], every closed set in g*$\omega\alpha$-closed. Hence A is g*$\omega\alpha$-closed in (Z^2, K^2). Hence (Z^2, K^2) is ${}_{g\omega\alpha}T_g$*$\omega\alpha$-space. □

Theorem 3.15. Let B be a non empty subset of (Z^2, K^2). If B = φ, then B is g*$\omega\alpha$-open in (Z^2,K^2).

Proof. Let F be an $\omega\alpha$-closed in (Z^2, K^2) such that F ⊆ B. From hypothesis, B_E = φ, then B = B_mix ∪ B_{K^2}. Then from Theorem 3.10 (ii), we have F = φ. Thus, we say that, whenever F is $\omega\alpha$-closed and F ⊆ B, F = φ ⊆ int(B). This implies F ⊆ int(B). Thus B is g*$\omega\alpha$-open in (Z^2, K^2). □

Remark 3.16. [8] A topological space X is said to be g*$\omega\alpha$-closed if and only if cl(A) \ A does not contain any non empty $\omega\alpha$-closed sets.

Theorem 3.17. Let A be a subset of (Z^2, K^2) and x be a point of (Z^2, K^2). If A is g*$\omega\alpha$-closed in (Z^2, K^2) and x ∈ A_{mix}, then cl({x}) \ {x} ⊆ A and hence cl({x}) ⊆ A.

Proof. From thy hypothesis, we have x ∈ Amix, that is x = (2n, 2m+1) or x = (2n+1, 2m).
Let x = (2n, 2m+1). Then cl({ x }) = cl({ 2n, 2m+1 }) = { 2n } × { 2m, 2m+1, 2m+2 } = {(2n, 2m) (2n, 2m+1) (2n, 2m+2)} = { x_1, x , x_2 }, where x_1 = (2n, 2m), x = (2n, 2m+1) and x_2 = (2n, 2m+2).
Let x = (2n+1, 2m), then cl({ x }) = cl({ 2n+1, 2m }) = { 2n, 2n+1, 2n+2 } × {2m } = { (2n, 2m), (2n+1, 2m), (2n+2, 2m) } = { x_1, x, x_2 }, where x_1 = (2n, 2m), x = (2n+1, 2m) and x_2 = (2n+2, 2m).
Thus, cl({ x }) \ { x } = { x_1, x, x_2 } \ { x } = { x_1, x_2 }. It should be noted that {x_1} and {x_2} are $\omega\alpha$-closed singleton sets in (Z^2, K^2).
Let us prove: x_1 ∈ A or x_2 ∈ A.
Consider x_1 ∉ A, then x_1 ∈ cl({ x }) ⊆ cl(A), implies that x_1 ∈ cl(A) \ A. Thus cl(A) \ A contains a $\omega\alpha$-closed set { x_1 }, which is contradiction to the remark 3.16.

Consider $x_2 \notin$ A, then $x_2 \in$ cl({ x }) $\subseteq$ cl(A), implies that $x_2 \in$ cl(A) \ A contains a $\omega\alpha$-closed set $\{x_2\}$, which is again contradiction to the remark 3.16. Therefore, $x_1 \in$ A or $x_2 \in$ A. Hence cl({ x }) $\subset$ A, because x $\in A_{mix} \subset$ A. □

Theorem 3.18. The following properties holds for any singleton set {x} in (Z^2, K^2):

(i) if x $\in (Z^2)_{K^2}$, then { x } is g*$\omega\alpha$-open, but not g*$\omega\alpha$-closed in (Z^2, K^2).

(ii) if x $\in (Z^2)_F$, then { x } is g*$\omega\alpha$-closed, but not g*$\omega\alpha$-open in $(Z^2)_{K^2}$.

(iii) if x $\in (Z^2)_{mix}$, then { x } is not g*$\omega\alpha$-closed, it is g*$\omega\alpha$-open in (Z^2, K^2).

Proof. (i) We know that { x } is open in (Z^2, K^2). Then { x } is g*$\omega\alpha$-open in (Z^2,K^2) [8]. Let x = (2n+1, 2m+1) $\in (Z)_{K^2}$, that is x = (2n+1, 2m+1) $\subseteq$ U = (2n+1, 2m+1), where U is $\omega\alpha$ open set in (Z^2, K^2). Then cl({ x }) = { 2n, 2n+1, 2n+2 } × { 2m, 2m+1, 2m+2 }. But cl({x}) U, this implies {x} is not g*$\omega\alpha$-closed in (Z^2, K^2).

(ii) Let x $\in (Z^2)_F$, that is {x}is closed in (Z^2, K^2). From [8], {x} is g*$\omega\alpha$-closed in (Z^2,K^2). Let, F = (2n, 2m) $\subseteq$ {x} =(2n, 2m), where F is $\omega\alpha$-closed in (Z^2, K^2). Then F $\subsetneq$ int({x}) , because int({x}) = φ, if x is even. This shows that {x} is not g*$\omega\alpha$-open in (Z^2, K^2).

(iii) Let x $\in (Z^2)_{mix}$, that is x = (2n+1, 2m) or x = (2n, 2m+1).
The cl({ x }) = { 2n, 2n+1, 2n+2 } × { 2m } $\not\subset$ { x } = U, where U is $\omega\alpha$-open in (Z^2, K^2). Therefore {x} is not g*$\omega\alpha$-closed in (Z^2, K^2).
Similarly, we can prove that {x} is not g*$\omega\alpha$-closed by taking x = (2n, 2m+1).

□

Theorem 3.19. Let B be a non empty subset of (Z^2, K^2). For a subset $B_F \neq \varphi$, if B is g*$\omega\alpha$-open in (Z^2, K^2), then (U({ x }))$K_2 \subset$ B holds for each point x $\in B_F$.

Proof. Let x $\in B_F$. Since {x} is closed, {x} is g*$\omega\alpha$-closed [8] and {x} $\subset$ B. As B is g*$\omega\alpha$-open, {x} $\subset$ int(B). This shows that {x} is the interior point of the set B. Thus, for the smallest open set U(X) containing x, U(X) $\subset$ B. That is int({ x }) = int(int(B)) = int(B) and int({x}) =U(x). Therefore U(x) $\subset$ int(B).
Let us consider x = (2n, 2m). Since U(2n, 2m) = { 2n-1, 2n, 2n+1 } × { 2m-1, 2m, 2m+1 }. Then $(U(\{ x \}))_{K^2}$ = { (x_1, x_2) $\in$ U(x): x_1 and x_2 are odd } = { y_1, y_2, y_3, y_4 }, where y_1 = (2n-1, 2m-1), y_2 = (2n-1, 2m+1), y_3 = (2n+1, 2m-1) and y_4 = (2n+1, 2m+1). Thus for each point P_i ($1 \leq i \leq 4$), we have $P_i \in$ B and $P_i \cap B \neq \varphi$. Therefore $(U(\{ x \}))_{K^2} \subset$ B. □

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