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The beech (*Fagus orientalis* Lipsky) diameter frequency fit with probability distribution in Iran's north forests

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Abstract

In order to study the quality of the Beech tree diameter at breast height and its fit through the statistical distribution two one hectare sample pieces were randomly selected in a lozenge shape of *Fagus orientalis* Lipsky of natural stand forests in the northern slope of Iran's north forests. In these pieces all trees with more than 7.5 cm diameter were measured according to full callipering. The Beta, Gamma, Weibull, Normal, Lognormal, Exponential and Power statistical models were used in order to fit the data. The results obtained from chi-square test χ^2 revealed that the Beta distribution in west and east directions and also exponential distribution in the west direction have produced a good fit for trees diameter distribution. But the other distributions didn't show much capability in explaining the studied trees diameter distribution.

Key words: fit, statistical distributions, diameter, beech

1. Introduction

An essential programming on the field of natural resources needs qualitative and quantitative data which is usually obtained by measuring the stand's characteristics (Zobeiri, 2000). Diameter at breast height of trees is one of the most essential characteristics or external sized of forest trees which can be studied from different aspects; for instance the diameter distribution of the trees of a stand or a forest can be determined and considered in which this distribution itself shows the diameter structure of the stand or forest (Mohamed Alizadeh et. al., 2009). In this direction various statistical models are used, because statistical models are among the factors that are used to prepare the growing models for estimating the site's future situation (Mataji et. al., 2000).

The quantity distribution evaluation in a forest's diameter classes by desirable probability theories is not only important for product type evaluation but also it could be beneficial for forest programming (Nanang, 1998). The first effort for modeling the diameter data were done by F.de Licourt in 1898 (Bailey, 1980). In order to do this he used geometrical progression terms with the general term of $a_n= aq^{1-n}$. In 1933, also Mayer used the $y= k^{-ax}$ exponential function for modeling the diameter data (Gardiner, 1968). Almost after the sixtieth decade A.D using statistical distributions in forest studies became customary (Namiranian, 1999.). During his studies in 1988 Shiver in order to fit the Weibull distribution to the diameter information of *Pinus elliottii* (Slash pine) used the three following methods maximum likelihood, modified moments and percentile (Shiver,1988). In the study in Ghana by Nanang (1998), used three distributions Weibull, Lognormal, Normal, to fit the data related to *Azadirachta indica*. The results obtained from Kolmogorov-Smirnov experiment proved the preference of lognormal distribution to fit the data (Nanang, 1998). In another study that was done on *Pinus teada* (Loblolly pine) the data were collected from 20 sample pieces of 0.62 hectares. In this survey the trees diameter distribution description was made by the help of three parameter Weibull distribution and for forecasting distribution parameters variables such as number of trees in a hectare, prevailing height, stand age and the tree's pertaining distances were used (Cao, 2004). In another study, a model was made for *Fagus* trees diameter distribution by the use of Weibull distribution. This model was based on distribution function parameters and it was made by the use of non-linear least square method (Fallah et al, 2005). In Iran the first studies that were done in this case refer to the tree diameter distribution studies in Noshahr's Khairoud Kenar forest (Namiranian, 1999). In this study the three Beta, Weibull and negative binomial distributions were used. The results obtained from accomplishing goodness fit and Kolmogorov-Smirnov test revealed that the two Weibull and Beta distributions have the ability to describe the trees diameter distribution. Also, Mataji et al. (2000) studied the trees diameter distribution in stands with

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different ages in Khairoud forest near Noshahr and got to the result that the tree Beta, Weibull and normal distributions have exciding capability in explaing trees diameter distribution (Mataji et al., 2000). In another study by Fallah et al. (2015) used various regression models to study *Fagus* trees' diameter structure in un-even aged stands. Also, in this study in order to consider the trees' diameter data distribution Beta, Gamma, Weibull, Power Exponential, Normal and Lognormal statistical models were used so that the capability and similarity of these distributions would be revealed for the fit of the diameter data (Nord-Larson Cao, 2006).

2. Materials and methods

1.1. Study area

Optimum growth conditions for oriental beech are on the north-facing slopes in natural distribution areas (Ertekin et. al., 2015). The under study area is located in the north slope of Dorfak peak in İran's north forests and it is situated in 36°, 54', 22" latitude till 49°, 36', 58" longitude with a minimum 350 meters and maximum 2720 meters height above the sea level. Its entire measurement is 5383 hectares and its central parts have a relatively excessive slope and it is covered with pure Fagus forests along with other species. Geologically it belongs to the second and the early third geology periods. Pedagogically it contains forest brown soil type with an origin of calcareous mother rock which P_H is neutral to weak acidic with a fluctuation between 6.4-6.9. The average annual rain fall of the under study area is 903.7 millimeters and its annual temperature is 16.2 degree centigrade. The relative humidity of this area fluctuates between 74-86 percent. Its climate type according to Emberger's method is temperate humid.

1.2. Study method

In order to study the quality of Beech tree diameter at breast height distribution and its fit by means of statistical distributions two one hectare sample pieces with 100×100 meters dimensions in a lozenge shape were randomly selected in the under study area in 1200 meters height above the sea level. These areas are pedagogically and geologically similar and their forests are un-even aged. In these pieces all trees with 7.5 centimeters diameters were one hundred percent measured.

1.3. *Statistical distributions*

In this study Beta, Gamma, Weibull, Normal, Lognormal, Exponential, and Power statistical models (distributions) were used that their probability density function would be introduced here.
Beta distribution:
This distribution is a continuous distribution and it is represented as an integral and its formula is as follows:

$$B(m, n) = \int_0^1 x^{m-1}(1-x)^{n-1}dx$$

In this formula X is the considered feature and m and n are the distribution parameters.

Gamma distribution:
Even this distribution is a continuous distribution and it almost has a good flexibility and its frequency curve in all modes has a lean towards right (Zwillinger and Kokoska, 2000). Its mathematical process is as follows:

Γ: Gamma function symbol

$$F(x) = \frac{x^{\alpha-1}}{\beta^{\alpha}T(\alpha)}\exp\left(\frac{-x}{\beta}\right) \qquad 0 \le x < +\infty \qquad \alpha, \beta > 0$$

Weibull distribution:
Weibull distribution is also presented as an cumulative frequency in which a is the starting point and b presents the curve's concavity degree, and c is the curve's factor form or shape index and its mathematical process is as follows:

F (x) = 1-exp $\left[-\left(\frac{x-a}{b}\right)^{c}\right]$ a, b, c > 0

Normal distribution:
Another continuous distribution is Normal distribution which many natural phenomenon's if measured have a close distribution to it as the trees quantity distribution in different diameter classes in a even-age forest stand which has almost a normal distribution. The density function of this distribution is as follows:

F (x) = $\frac{1}{\delta\sqrt{2\pi}}\exp^{-\frac{1}{2}\left[\frac{x-\mu}{\delta}\right]^{2}}$ $-\infty < x < +\infty$, $\mu \in R$, $\delta > 0$

Lognormal distribution:
As the other distribution this is also a continuous distribution and its natural logarithm has a normal distribution. This distribution is in traduced through the following density function:

F (x) = $\frac{1}{\sqrt{2\pi\delta x}}\exp\left(-\frac{1}{2\delta^{2}}(\ln x-\mu)^{2}\right)$ $\delta > 0$, $x > 0$, $0 < \mu < +\infty$

Exponential distribution:
This distribution is also known as the time expecting distribution which is a continuous distribution and its mathematical process is as follows:

F (x) = $\lambda\exp(-\lambda x)$ $\lambda > 0$, $0 < x < +\infty$

Power distribution:
This is another continuous distribution which density function is as follows:

F (x) = $b.x_i^{b_i}$ $0 < x < +\infty$

Goodness of fit study:
In order to choose the best fit it is essential that each of the Beta, Normal, Exponential, Power, Gamma, Weibull and lognormal models to be tested by goodness of fit that in this study chi-square was used as goodness of fit and its mathematical Process is as follows:

$\chi^2 = \frac{(O-E)^2}{E}$ O: Observe Frequency E: Expected Frequency

3. Results

Also the tables 1 and 2 show the distribution's accounts results of 411 single trees in 5 cm diameter classes in west direction, and 462 single trees in the east direction of the under study area along with the evaluations obtained from the utilized probability distributions.

Figures 1 and 2 also show the comparison of the observed frequencies and the evaluated frequencies of the probability distributions in the east and west geographical directions.

Table 1. The quantity distribution in the observed diameter levels and their evaluation with probability distribution in west direction

10	99	91.47	83.13	91.23	174.20	62.45	102.79	30.24
15	65	63.23	74.10	71.27	84.54	77.52	93.24	38.87
20	50	50.32	60.52	55.67	50.61	68.62	61.46	45.7
25	40	41.53	47.85	43.49	34	54.95	41.05	49.19
30	37	34.69	37.11	33.97	24.56	41.91	28.42	48.38
35	31	29.04	28.42	26.53	18.66	31.03	20.32	43.52
40	25	24.21	21.55	20.73	14.70	22.55	14.97	35.77
45	16	19.99	16.17	16.19	11.92	16.15	11.28	26.9
50	11	16.25	12.11	12.65	9.88	11.45	8.68	18.49
55	7	12.94	8.98	9.88	8.33	8.05	6.79	11.62
60	8	9.99	6.68	7.72	7.13	5.63	6.79	6.69
65	9	7.37	4.88	6.03	6.19	3.92	4.34	3.53
70	4	5.08	3.59	4.71	5.42	2.70	3.53	1.68
75	4	3.11	2.62	3.68	4.79	1.87	2.90	0.74
80	3	1.49	1.90	2.87	4.27	1.29	2.41	0.30
85	2	0.31	1.38	2.24	3.83	0.88	2.02	0.11

Table 2. The quantity distribution in the observed diameter classes and their evaluation with the probability distributions in the east direction

10	152	129.12	122.79	92.86	215.79	84.31	154.41	35.99
15	88	70.17	86.65	72.58	97.17	93.16	102.81	43
20	50	50.80	63.46	56.72	55.17	77.71	61.53	47.85
25	29	39.92	47.09	44.33	35.56	59.63	39.20	49.51
30	22	32.54	35.22	34.64	24.84	43.95	26.43	47.66
35	28	27.02	26.48	27.08	18.34	30.60	18.60	42.70
40	19	22.67	20	21.16	14.10	22.36	13.56	35.59
45	10	19.07	15.13	16.54	11.19	15.65	10.15	27.58
50	12	16.02	11.49	12.92	9.09	10.85	7.78	19.91
55	11	13.39	8.74	10.10	7.54	7.48	6.09	13.36
60	11	11.07	6.65	7.89	6.35	5.12	4.84	8.36
65	8	9.02	5.08	6.17	5.42	3.50	3.90	4.85
70	8	7.18	3.88	4.82	4.69	2.39	3.19	2.62
75	6	5.51	2.96	3.77	4.09	1.61	2.63	1.32
80	3	4.02	2.27	2.95	3.60	1.09	2.18	0.616
85	3	2.66	1.74	2.30	3.20	0.436	1.83	0.27
90	1	1.46	1.34	1.80	2.86	0.496	1.55	0.107
95	1	0.41	1.03	1.40	2.57	0.333	1.32	0.04

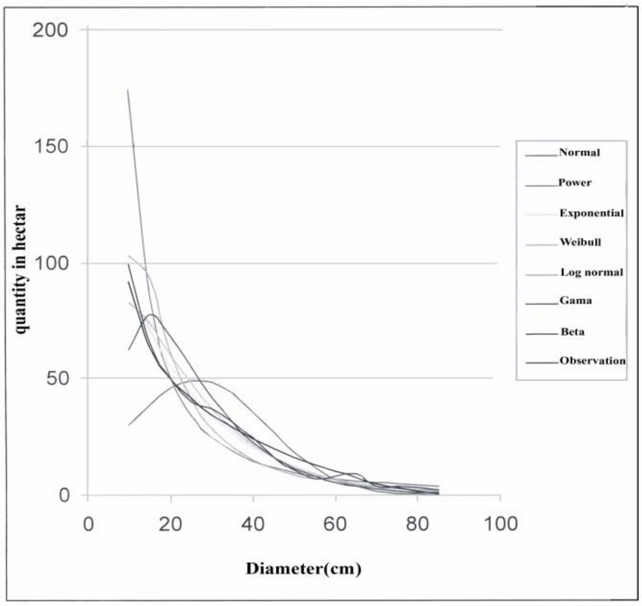

Figure 1**.** The comparison of observed frequencies and evaluated frequencies of probability distributions in west direction

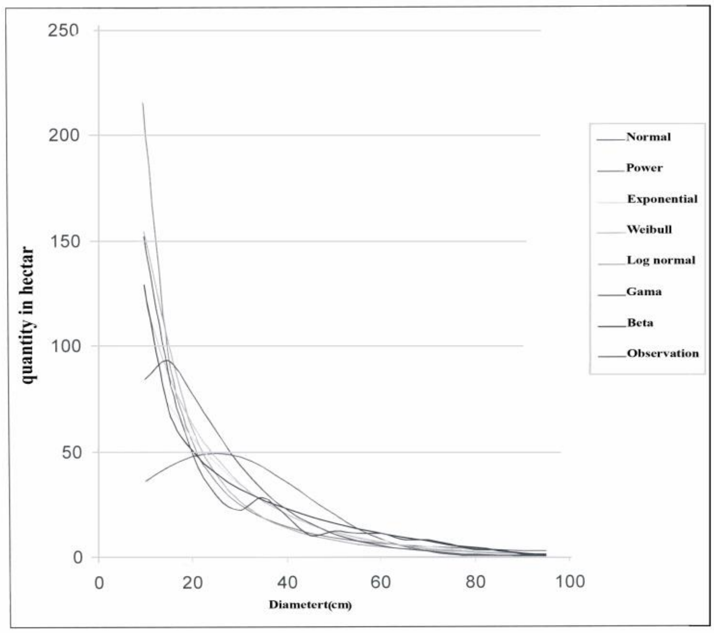

Figure 2. The comparison of the observed frequencies and evaluated frequencies of probability distribution in east direction

As mentioned in order to consider the fit capability of the utilized probability distributions in this study chi-square test was used which the amounts resulting from accomplishing this test are shown in table 3.

Table 3. χ^2 amounts for the utilized probability distribution in

Distribution type / Direction	Beta	Exponenti al	Power	Weibull	Gama	Lognorma l	Normal	χ^2_c
West	ns 18.640	ns 6.899	** 64.330	** 14.413	** 47.889	** 36.231	** 277.915	χ^2 , df_{16}= 26.296
East	ns 21.90	** 63.127	** 46.473	** 37.444	** 145.395	** 45.951	** 561.714	χ^2 , df_{16}= 26.296

* It is significant in 5% level. ns is no significant. ns = non - significant

According to the quantities that were the outcome of this test (Table 3) it was specified that Beta distribution in the two west and east direction hasn't been significant with the observed frequencies. In other words the observed distributions in the under study area has been a random sample from the community which characteristics are explained through Beta distribution. Therefore since Beta distribution has a lesser χ^2 in comparison to the χ^2 of the table it can be considered as a model with a better fit for describing the area's forests. Figures 1 and 2 also show the models for the distribution quantity in different geographical directions. It is observed that Beta model has made a fine fit in distributing the main parts in west and east directions. Of course the Weibull and Exponential models especially exponential model for having lesser χ^2 has made a very appropriate distribution in west direction and this fit isn't seen in the other direction. Concerning the other probability distributions it has been observed that the difference among Power, Gamma, Lognormal and Normal distribution with real quantities is significant and this means that these distributions don't have high accuracy in order to demonstrate the manner of the trees distribution in different diameter classes.

4. Conclusions and discussion

According to the importance of diameter at breast height as the main biometric variable of the forest trees, the studies on it have a great importance. On the other hand the frequency distribution or the distribution of this variable is also mostly used for determining the stand or forest diameter structure but it can also have other usage such as growing models. In this case, Nanang (1998) think that the usage of appropriate probability distributions is important for the forecast of trees distribution state in a forest stand. These studies are the same as Namirian (1999) studies in Iran since they explain that Weibull and Beta probability distributions have a greater capability for showing the manner of the trees distribution in different diameter classes. Mataji et. al. (2000) studies also show that the Normal, Weibull and Beta probability distributions have a greater capability to explain the trees diameter distribution. Cao (2004) studies on *Pinus teada* diameter information also explains that using Weibull probability distribution is appropriate. Nord-Larson & Cao (2006), have also used Weibull distribution model in studying *Fagus* diameter distribution. These comparisons show that for achieving an appropriate model for an un even-aged structure in a forest special attention should be given to the site situations and features. Finally, it is necessary to mention that the results obtained from this study is influenced by its information and necessarily they may be different in order studies.

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