Mathematical Definition of Structure and Design of Woven Fabrics

Dokuma Kumaşların Matematiksel Yapı Tanımı ve Tasarımı

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MATHEMATICAL DEFINITION OF STRUCTURE AND DESIGN OF WOVEN FABRICS

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*ABSTRAC**T**: A general mathematical definition of woven fabric structures is developed, as based on their structural characteristics, which is expressed in the form of a fabric weight function. This function is composed of the terms representing weights of constituent yarns with their structural characteristics and frequency of use, including also variables denoting weave and design features in single or multi layer fabric structure. In some fancy and figured fabrics extra yarns are used to obtain certain aesthetic features and the general function developed should take into account the manner in which these yarns are used, according to the shedding arrangement of the loom to be employed. A method of structural and aesthetic design is, then, developed using this function, in accord with weaving and aesthetic design requirements, the structural aim being a required unit weight, the aesthetic aim being a desired figure size. The method of design is developed to secure the weavability of fabric on the loom and the dimensional stability of fabric in subsequent use.

Keywords: Fabric unit weight, extra yarns, structural fabric parameters, yarn parameters, design parameters, design capacity, surface design, weavability, dimensional stability.

DOKUMA KUMAŞLARIN MATEMATİKSEL YAPI TANIMI VE TASARIMI

ÖZET- Dokuma kumaş yapılarının yapısal özelliklerine dayalı genel bir matematiksel tanımı tek bir ağırlık fonksiyonu biçiminde geliştirilmiştir. Bu fonksiyon tek ya da çok katlı yapılarda bileşen ipliklerin yapısal özellikleri ile kullanım sıklıklarını belirleyen, aynı zamanda örgü ve desen özelliklerini simgeleyen değişkenleri de içeren ağırlık terimlerinden oluşmaktadır. Bazı fantezi ve motifli kumaşlarda belirli estetik ögeler oluşturmak için ekstra iplikler kullanılır ve geliştirilen genel fonksiyon, bu ipliklerin kullanılacak tezgahın ağızlık açma düzenine bağlı olarak kullanım biçimini hesaba katacak biçimde geliştirilmelidir. Daha sonra, bu fonksiyon kullanılarak, yapısal amacı istenen birim ağırlık, estetik amacı arzu edilen motif büyüklüğü olan bir yapısal ve estetik tasarım metodu geliştirilmiştir. Tasarım metodu kumaşın tezgahta dokunabilirliğini ve sonraki kullanımında boyutsal dayanıklılığını güvenceye alacak biçimde geliştirilmiştir.

Anahtar Sözcükler: Kumaş birim ağırlığı, ekstra iplikler, yapısal kumaş parametreleri, iplik parametreleri, tasarım parametreleri, tasarım kapasitesi, yüzey tasarımı, dokunabilirlik, boyutsal dayanıklılık.

1. INTRODUCTION

A woven fabric is formed by two sets of yarns, at right angles to each other, intersecting and interweaving to form a planar structure. The unit weight of a woven fabric is the sum of the weights of constituent yarns used in unit fabric area. Thus, fabric weight function is a linear combination of weights of yarns of different types and different properties used in certain frequencies according to fabric structure.

Woven fabrics can be defined verbally by the number of layers, the weave definitions of each layer, the frequency of use of constituent yarns with their type and raw material definitions together with their structural features.

In some fancy and figured fabrics extra yarns are used to obtain certain aesthetic features and the general function developed should include terms to take into account the extra weights of these yarns with their different properties and the manner they are used in accord with the shedding arrangement of the loom to be employed in actual weaving.

In figured fabrics a different weave may be used in the figured area, which necessitates the use of a certain number of different warp movements other than those needed for the main weave, or weaves in the case of double fabrics. In some cases extra yarns may have been used either in warp or weft or in both directions to form a figured area of different structure or colour on top of the main fabric. All these aspects should be taken into account in the design work in accord with the shedding arrangement of the actual loom to be employed.

As most of all these aspects can somehow be expressed by numbers, they may and must be included in the weight function arranged in a certain manner to define fabric structure. Having thus defined the structure exactly, the general fabric weight function can then be used to develop a general design method and calculation procedure to prepare the production order. A general design method for complex and figured woven fabrics was reported in earlier publications [1-3]. The calculations to prepare production order are given in various textbooks such as that of Baser [4]. The aim of this paper is to work out a generalised algorithm to enable the design of almost all woven fabric structures as a step to computer aided design programming.

2. THEORY

The structural definition of the following 21 different types of woven fabrics can be given by a single weight function including structural parameters as given in Table 1 with their symbols used in Equation 1.

Table 1. List of symbols used in the woven fabric weight function

Symbol	Definition of the variable
i	Layer definition: $i = 1$: Single layer or backed fabric, $i = 2$: Double layer fabric, $i = 3$: Treble fabric
u	Figure or wadding indicator (warp): $u = 1$: Figure or wadding exists. $u = 0$: No figure or wadding
v	" " " (weft): $v = 1$: " " " $v = 0$: " "
$k_{F/1}$	k_1 : Warp crimp factor for single fabrics, k_F : Face yarn crimp factor for double fabrics
$k_{B/2}$	k_2 : Weft crimp factor for single fabrics, k_B : Back yarn crimp factor for double fabrics
$S_{F/1}$	S_1 : Warp density for single fabrics, S_F : Face yarn density for double fabrics
$S_{B/2}$	S_2 : Weft density for single fabrics, S_B : Back yarn density for double fabrics
$N_{F/1}$	N_1 : Warp yarn metric count for single fabrics, N_F : Face yarn metric count for single fabrics
$N_{B/2}$	N_2 : Weft yarn metric count for single fabrics, N_B : Back yarn metric count for double fabrics
F_{wF}	Weave factor of the face weave
F_{wB}	Weave factor of the back weave
V_F	Firmness factor of the face weave as a fraction of 1
V_B	Firmness factor of the back weave as a fraction of 1
a	Figure width in single fabrics or figure width in double fabrics with extra warp figuring yarns
e_1	Extra warp yarn metric count / ground yarn metric count in single or double fabrics with figuring
p	Ground warp yarn density / extra warp yarn density in single or double fabrics with figuring
R	Figure width in cm. in single or double fabrics
b	Figure height in cm. in single or extra weft single or extra weft double fabrics
e_2	Extra weft yarn metric count / ground yarn metric count in single or double fabrics with figuring
q	Ground weft yarn density / extra weft yarn density in single or double fabrics with figuring
Q	Figure height in cm. in single or double fabrics
t	Yarn density of the weft or of face yarns to that of the warp or of the back yarns
z	Yarn density reduction for backed or double fabrics (%)/100, z = 0 for single fabrics

Weight per m^2 of fabric in grams is given by the function

$$w = \frac{100ik_{F/1}S_{F/1}}{N_{F/1}} + \frac{100ik_{B/2}S_{B/2}}{N_{B/2}} + \frac{100k_{F/1}S_{Y/1}}{N_{Y/1}} \cdot \frac{au}{e_1 pR} + \frac{100k_{B/2}S_{B/2}}{N_{B/2}} \cdot \frac{bv}{e_2 qQ} \tag{1}$$

There is, also, a relation between fabric weave, yarn counts and yarn densities to be taken into account to obtain good fabric cover and stability of structure expressed by various setting theories such as that of Ashenhurst's [5] and Brierley's [6]. The application of setting theories to complex woven structures was explained by Snowden [7]. Accordingly, it can be generalized by the formulae below, for thread density of warp and weft or of face and back of fabric, by Equations 2 and 3.

$$S_{F/1} = k_{F/2}F_{wF}K_{F/1}V_F\sqrt{N_{F/1}}(1-z) \tag{2}$$

$$S_{B/2} = k_{B/1}F_{wB}K_{B/1}V_B\sqrt{N_{B/2}}(1-z) \tag{3}$$

where

$F_{wF/1}, F_{wB/2}$: Weave factors given by setting theory authors for face and back weave or for warp and weft

  settings respectively (as a fraction of 1) in single fabrics.

$K_{F/1}, K_{B/2}$: Yarn constants given by setting theory authors for yarns of face and back weave or for warp and

A useful relationship is given by a ratio between face and back fabric densities or a ratio of warp and weft yarn densities as

$$\frac{S_{F/1}}{S_{B/2}} = t = \frac{k_{F/2}F_{wF}K_{F/1}V_F\sqrt{N_{F/1}}}{k_{B/1}F_{wB}K_{B/1}V_B\sqrt{N_{B/2}}} = c\frac{\sqrt{N_{F/1}}}{\sqrt{N_{B/2}}}, \tag{4}$$

where

$$c = \frac{k_{F/2}F_{wF}K_{F/1}V_F}{k_{B/1}F_{wB}K_{B/1}V_B}$$

2.1 Design of a Woven Fabric of Required Unit Weight

In design work it is usually aimed to obtain a fabric of a given weight with required properties related to the area or purpose of use. As most of the physical properties are related to the unit weight or thickness of the fabric, unit fabric weight may be an appropriate aim of design. In fancy fabrics with some aesthetic features the dimensions of figured area may be considered as the aim of design rather the unit fabric weight. Thus the design problem should be tackled with these two different approaches.

If fabric unit weight is aimed then yarn counts and yarn densities will be the basic variables. Therefore from Equation (4) we have, for yarn counts,

$$N_{B/2} = \frac{c^2}{t^2}N_{F/1} \tag{5}$$

$$S_{B/2} = \frac{1}{t}S_{F/1} \tag{6}$$

Thus if a value is assigned to warp count in single fabrics or a value for yarn count on the face fabric in double fabrics, the respective values for weft in single fabrics or those of back layer in double fabrics can be calculated.

In the case when fabric unit weight is considered as the principal aim of design, substituting equations (2) and (3) into Equation (1) we obtain for unit weight in g/m^2

Here i will take the value 2 in double fabrics, 1 in single fabrics. In double fabrics the crimp factors of face and back weave and also those of warp and weft may be assumed to be the same for simplicity.

From Equations (7), the yarn count of the warp in single fabrics or that of the face warp in double fabrics can be obtained by the equation

$$w = \left[\frac{100ik_{F/1}S_{F/1}}{N_{F/1}} + \frac{100ik_{B/2}S_{B/2}}{t.c^2/t^2 N_{F/1}} + \frac{100k_{F/1}S_{F/1}}{N_{F/1}}\frac{au}{e_1 pR} + \frac{100k_{B/2}S_{F/1}}{N_{F/1}t.c^2/t^2}\frac{bv}{e_2 qQ}\right]$$

$$= \frac{100k_{F/1}k_{F/2}F_{wF}K_{F/1}V_F}{\sqrt{N_{F/1}}}\left[i\left(1 + \frac{k_{B/1}}{k_{F/1}}\cdot\frac{t}{c^2}\right) + \frac{au}{e_1 pR} + \frac{k_{B/2}}{k_{F/1}}\cdot\frac{t}{c^2}\cdot\frac{bv}{e_2 qQ}\right](1-z) \tag{7}$$

$$N_{F/1} = \left\{\frac{100k_{F/1}k_{F/2}F_{wF}K_{F/1}V_F}{w}\left[i\left(1 + \frac{k_{B/2}}{k_{F/1}}\cdot\frac{t}{c^2}\right) + \frac{au}{e_1 pR} + \frac{k_{B/2}}{k_{F/1}}\cdot\frac{btv}{c^2 e_2 qQ}\right](1-z)\right\}^2 \tag{8}$$

For convenience, warp yarn density in single fabrics or face yarn density in double fabrics will be obtained by Equation (2) and weft yarn density in single fabrics or back yarn density in double fabrics will be obtained by Equation (6). Weft yarn count in single fabrics or back yarn count in double fabrics will be obtained by Equation (5).

2.2 Design of a Woven Fabric of Required Figure Size

In designing fabrics for which a figure effect of a certain size is required, by either weave arrangements or by using extra yarns, a different approach is made to take into account the problems arising from the surface design of the fabric. As the number of different warp movements will be needed in such cases, this may lead to restrictions in tappet and dobby looms, since the number of heald shafts available is limited.

In fabrics where figuring is required the figured area is obtained either by using a different weave or by introducing extra yarns in warp or weft direction. When a different weave is used for figuring additional heald shafts are needed in the number of warp yarns within the figured area, due to the interaction of ground weave floats with figure contours. The usage of extra yarns will, also, increase the number of heald shafts to be used. When extra warp yarns are used for figuring additional heald shafts are needed for extra warp yarns and if extra weft yarns are used for figuring, the number of heald shafts to control them will be as many as they cut the warp yarns whether ground or extra. If there is symmetry in the figure, the additional heald shafts may be half in number.

Using extra yarns will increase the unit weight. On the other hand, this may lead to a limitation from the point of view of design capacity in tappet (or eccentric) and dobby looms. Thus unit weight requirement and figure size requirement- if there also exists- may not be fulfilled together. In such cases the two problems should be tackled with in turn according to priorities. This problem was discussed in a previous paper by Baser [3].

The number of heald shafts to be used for figuring should be determined as it affects the width of the figured area. Also, the number of heald shafts needed for ground weave (or weaves in double fabrics) plus that to be used for figuring should not exceed the total shafts available. One important aspect is the number of extra yarns used with respect to that of the ground yarns in figured sections. These aspects are reflected in the analysis by ratios.

Figure 1 shows the way extra yarns may be used for figuring and additional parameters are needed to include in the calculations to take into account these surface design alternatives.

New parameters will have to be introduced to cope with these alternative ways of obtaining figures. These are given in Table 2.

In fabrics with figure effects the yarn density of warp in single fabrics and that of face yarns in double fabrics is related to various design parameters as given in equation

$$S_{F/1} = \frac{n[p(A_1 + A_W) + A_2]}{a} \tag{9}$$

Substituting Equation (2) in Equation (9) we have

$$a = \frac{n[p(A_1 + A_W) + A_2]}{k_{F/2} F_{wF} K_{F/1} V_F \sqrt{N_{F/1}} (1 - z)} \tag{10}$$

Table 2: Parameters defining aesthetic design

Symbol	Parameter
n	Symmetry coefficient (n =1: There is no symmetry in the figure, n =2: There is symmetry in the figure)
A	Design capacity of the loom (total number of heald shafts in dobby looms)
A_1	Number of heald shafts to be used for extra warp yarns
A_2	Number of heald shafts to be used for extra weft yarns
A_R	Number of heald shafts to be used for extra weft yarns
A_W	Number of heald shafts to be used for extra weft yarns in a structure with both extra warp and weft.
R	Width of the design unit in cm.
Q	Height of the design unit in cm.

This is an equation which relates figure size represented by the figure width a , yarn count, represented by $N_{F/1}$ and weave type represented by F_{wF} . Equation (10) can be written in a more compact form as

$$a = \frac{n[p(A_1 + A_W) + A_2]}{CF_{wF} \sqrt{N_{F/1}}}, \quad C = k_{F/2} K_{F/1} V_F (1 - z) \tag{11}$$

As yarn count is one of the basic factors affecting unit weight and putting Equation (7) into a simpler form we have

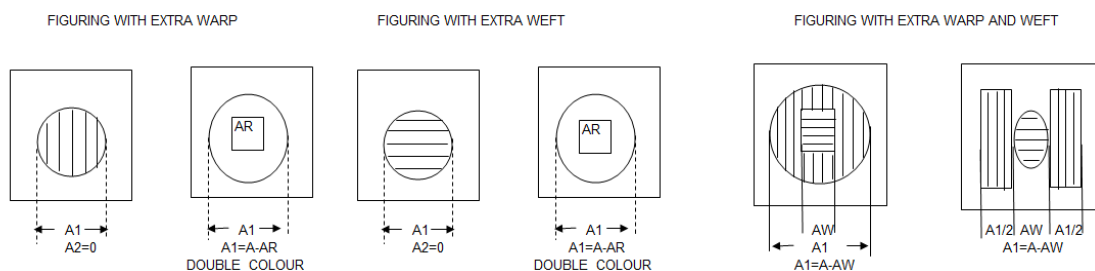$$N_{F/1} = \frac{F_{wF}{}^2 M}{w^2} \tag{12}$$

Figure 1. Figuring by using extra yarns in various ways

where

$$M = 100 k_{F/1} k_{F/2} K_{F/1} V_F \left[i \left(1 + \frac{k_{B/1}}{k_{F/1}} \cdot \frac{t}{c^2} \right) + \frac{au}{e_1 pR} + \frac{k_{B/2}}{k_{F/1}} \cdot \frac{t}{c^2} \cdot \frac{bv}{e_2 qQ} \right] (1-z)$$

Substituting $N_{F/1}$ into Equation (11) we have for figure width

$$a = \frac{n \left[p(A_1 + A_W) + A_2 \right]}{CF_{wF}^2 M} w$$

(13)

$$w = \frac{a CF_{wF}^2 M}{n \left[p(A_1 + A_W) + A_2 \right]}$$

(14)

These two equations show the design objects, namely figure size and unit weight, as the function of each other, including various design parameters, physical or aesthetical.

The structural design parameters belonging to 21 different woven structures may be given as shown in Figure 2 in a matrix form. Here term parameters are inserted in the appropriate cells of the structural design matrix. The question marks inserted in other cells represent the numerical values of the design parameter to be inserted according to design requirements. This matrix form will help in applying a general method of design for woven fabrics of complex structures.

3. CONCLUSION

The study presented here shows that woven fabrics of various structures can be represented mathematically by a single function. A method of structural design is developed, using this function, according to design objectives and securing, at the same time, requirements of weavability, which means the geometrical and mechanical requirements for the weaving process to be practically realised, and of dimensional stability of fabric in subsequent usage.

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No.	Fabric definitions	Sym. n	No.of heald shafts A1	A2	AR	AW	Figure width a cm.	Figure height b cm.	Yarn Ratio S1/S2 or SF/SB p	Width Rt. Fig./Des. q	Height Rt. Fig./Des.	EXTRA YARN warp e_1	Nm weft e_2	Density reduction z	Term coefficient u	v	Layer coeff. i
1	Single layer fabric	1	?	0	0	0	0	0	?	0	0	0	0	?	0	0	1
2	Single layer figured fabric	?	?	0	?	0	?	?	?	?	?	0	0	?	0	0	1
3	Single layer extra warp fabric	?	?	?	0	0	0	0	?	0	0	?	0	?	1	0	1
4	Single layer extra weft fabric	1	?	0	0	?	0	0	?	0	0	0	?	?	?	1	1
5	Single layer extra warp and weft fabric	?	?	?	0	?	0	0	?	0	0	?	?	?	1	1	1
6	Warp backed fabric	1	?	?	0	0	0	0	?	0	0	?	0	?	?	0	1
7	Warp backed weft wadded fabric	1	?	?	0	?	0	0	?	0	0	?	?	?	1	1	1
8	Weft backed fabric	1	?	0	0	?	0	0	?	0	0	0	?	?	0	1	1
9	Weft backed warp wadded fabric	1	?	?	0	?	0	0	?	0	0	?	?	?	1	1	1
10	Weft reversible fabric	1	?	0	0	0	0	0	0	0	0	0	?	?	0	1	1
11	Double layer self stitched fabric	1	?	?	0	0	0	0	?	0	0	0	0	?	0	0	2
12	Double layer self stitched fabric with extra warp	1	?	?	0	0	0	0	?	0	0	?	0	?	?	0	2
13	Double layer self stitched fabric with extra weft	1	?	?	0	?	0	0	?	0	0	0	?	?	0	1	2
14	Double layer self stitched fabric with extra warp and weft	1	?	?	0	?	0	0	?	0	0	?	?	?	1	1	2
15	Double layer centre stitched fabric	1	?	?	0	0	0	0	?	1	0	?	0	?	1	0	2
16	Double faced fabric	1	?	?	0	0	0	0	?	0	0	0	0	?	0	0	2
17	Double faced fabric with figure effect	?	?	0	?	0	?	?	?	?	?	0	0	?	0	0	2
18	Double faced fabric with extra warp	1	?	?	0	0	0	0	?	1	0	?	0	?	1	0	2
19	Double faced fabric with extra weft	1	?	?	0	0	0	0	?	0	1	0	?	?	0	1	2
20	Double faced fabric with extra warp and weft	1	?	?	0	?	0	0	?	1	1	?	?	?	1	1	2
21	Self stitched treble fabric	1	?	?	0	0	0	0	?	0	0	?	?	?	0	0	3

Figure 2. The matrix of structural parameters for different fabric types