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Research Article

Addendum to "Ideal Rothberger spaces" [Hacet. J. Math. Stat. 47(1), 69-75, 2018]

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Abstract

In this addendum we give an example to show that there is an error in Theorem 3.7 in "Ideal Rothberger spaces" [Hacet. J. Math. Stat. 47(1), 69-75, 2018]. We also prove the theorem with different hypothesis.

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We use notation and terminology from [2]. In [2], the author gave the following theorem for inverse invariant.

A function f from a topological space X to a space Y is said to be perfect map [1] if

(1) f is onto
(2) f is continuous
(3) f is closed map
(4) $f^{-1}(y)$ is compact in X for each $y \in Y$.

Theorem 1 ([2])**.** *Let $f : X \to Y$ be a perfect map and $\mathcal{I}$ be an ideal in Y. If Y is Rothberger modulo $\mathcal{I}$, then X is Rothberger modulo $f^{-1}(\mathcal{I})$.*

Here we give an example which contradicts the Theorem 1 given in [2].

Example 2. Let $\mathbb{R}$ be set of real numbers with usual topology and $\mathcal{I} = \{\phi\}$ be an ideal in $\{a\}$. Take a constant function f from $[0,1]$ to one point Rothberger space or $\{a\}$, where $[0,1]$ is compact closed subspace of $\mathbb{R}$. Then f is closed, open, onto and continuous map. Also $f^{-1}(a) = [0,1]$ is compact but $[0,1]$ is not Rothberger [3] since $\{a\}$ is Rothberger.

Now we give positive result regarding this and provide maps under which Rothberger modulo an ideal spaces are inverse invariant.

Theorem 3. *Let f be an open bijective map from a space X to Y and $\mathcal{I}$ be an ideal in Y. If Y is Rothberger modulo $\mathcal{I}$, then X is Rothberger modulo $f^{-1}(\mathcal{I})$.*

Proof. Let $< \mathcal{U}_n : n \in \omega >$ be a sequence of open covers of X. Then for each n,

$$\mathcal{V}_n = \{f(U) : U \in \mathcal{U}_n\}$$

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is an open cover of Y. Since Y is Rothberger modulo $\mathcal{I}$, there are $J \in \mathcal{I}$ and a sequence $< \mathcal{W}_n : n \in \omega >$ such that for each n, $\mathcal{W}_n$ is a singleton subset of $\mathcal{U}_n$ and for each $y \in Y \setminus J$, belongs to $\bigcup \mathcal{W}_n$ for some n. Now assume that for each n,

$$\mathcal{W}_n = \{f(U_{n,1})\} \text{ and } \mathcal{G}_n = \{U_{n,1}\}.$$

Then $f^{-1}(J) \in f^{-1}(\mathcal{I})$ and sequence $< \mathcal{G}_n : n \in \omega >$ witnesses Rothberger modulo $f^{-1}(\mathcal{I})$ property of X for the sequence $< \mathcal{U}_n : n \in \omega >$. Let $x \in X \setminus f^{-1}(J)$. Then

$$y = f(x) \in Y \setminus J \text{ and } y \in \bigcup \mathcal{W}_n \text{ for some } n.$$

This implies that $y \in f(U_{n,1})$. Since f is one-to-one, $x \in U_{n,1}$. So $x \in \bigcup \mathcal{G}_n$ for some n. This completes the proof. □

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