

Research Article

A folding back analysis on elementary students' growth in mathematical understanding

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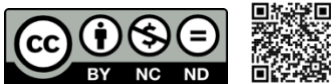
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Abstract

The purpose of this study is to analyze folding back characteristics on elementary students' growth in mathematical understanding with relational and instrumental understanding. This research used a qualitative approach with a case study. The growth in mathematical understanding is analyzed in layers of the Pirie-Kieren model, namely primitive knowing, image making, image having, property noticing, formalizing, observing, structuring, and inventizing. The results of this study are folding back on students with relational understanding, from the property noticing stage to image making stage. In addition, there was also a folding back from image making stage to primitive knowing stage. Folding back that occurs in students with relational understanding has a trajectory to observing stage. In students with instrumental understanding, there is a folding back from the property noticing stage to image making stage, and has a path to formalizing stage. The conclusion of this research is that there is a two folding back on students with relational understanding and one folding back on students with instrumental understanding. In addition, students with relational understanding have a folding back trajectory that is longer than students with instrumental understanding.

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Introduction

Growth in mathematical understanding with the Pirie-Kieren model is recognized as an appropriate theory in analyzing students' mathematical understanding (Kieren, Pirie & Gordon Calvert, 1999). Folding back can occur when students encounter a dead-end in facing a problem, students will return to the steps or knowledge they have before to get a better understanding in order to solve the problem being faced. There are categories of folding back results, which are effective folding back and ineffective folding back (Martin, LaCroix & Fownes, 2005). Folding back in the growth in mathematical understanding the Pirie-Kieren model which has eight layers, namely primitive knowing, image making, image having, property noticing, formalizing, observing, structuring, and inventizing (Kieren, 1989).

The Pirie-Kieren model develops simultaneously with the elaboration of students' mathematical understanding (Gülkılık, Uğurlu, & Yürük, 2015; Towers, 2004). The role of a teacher is very important in improving the development of students' mathematical understanding. Some things that can be done to improve it are teacher preparation, meaningful teacher knowledge, teacher actions during learning (Borgen, 2006; Ramadhani, Huda & Umam, 2019; Wright, 2014). Furthermore, the development of good student understanding causes the effectiveness of students in folding back. This is due to the formation of connected, flexible and integrated understanding (Martin & Towers, 2016a).

Folding back has three categories, which include the source, shape, and results of folding back (Martin, 2008). Folding back of each student is different according to the type of understanding. There are two types of understanding, relational and instrumental understanding (Skemp, 2006). Students who have a relational understanding or instrumental understanding have certainly experienced a dead end or are unable to solve the problem at hand. So

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students will return to the previous step to gain understanding in order to solve the problem at hand (Amin et al. 2020; Debrenti, 2015; Indrawatiningsih et al. 2020). Activities returning to the previous step or returning to deeper layers can be indicated that students experience folding back. Folding back experienced by students must occur in the layers of growth in mathematical understanding, in this case it occurs in the layers of growth in mathematical understanding of the Pirie-Kieren model.

A preliminary study of 16th grade V elementary school students showed that when they were given a math problem, they could not solve it. Students stop working, but they go back to the previous step by recalling the knowledge they had obtained previously, rereading material related to the problem, or asking their friends about material related to the problem. The results of this preliminary study indicate that grade V students of primary schools do folding backs, but it is not yet known where the folding backs of the students come from, what kind of folding back students look like, and how the results of folding back students.

Problem of Research

Researchers analyzed in depth the folding back of students in the layers of growth in mathematical understanding using the Pirie-Kieren growth in mathematical understanding model. Folding back students who were excavated included the source of the occurrence of folding back students, such as what form of folding back students, and how the results of folding back students. This includes folding back students who have a relational understanding and students who have instrumental understanding. So that later can provide convenience in identifying the potential when there are disturbances or difficulties faced by students, the disconnection of the understanding of students who are developing at the elementary school level. Even anticipating or overcoming difficulties faced by students, and folding back can be a stimulus in the development and progress of understanding of elementary school students. Based on the preliminary study, thus the problem of research in this study are followed.

- How the folding back characteristics of students who have a relational understanding of the growth in mathematical understanding using the Pirie-Kieren model. ?
- How the folding back characteristics of students who have an instrumental understanding of the growth in mathematical understanding using the Pirie-Kieren model. ?

Methods

Research Model

This study aims to analyse the folding back characteristics of students who have a relational and instrumental understanding of the growth in mathematical understanding using the Pirie-Kieren model. The research approach used is qualitative with the type of case study. The case study design is a qualitative research procedure that explores in depth a bounded system (eg, activities, events, processes, or individuals) based on extensive data collection (Creswell, 2012).

Participants

This research was conducted in one school in East Java Indonesia, namely SDN Gamping 2 Krian in June-August 2019. This type of subject taking used snowball sampling to expand research information (Huberman, 1994). After snowball throwing on 40 fifth grade students at SDN Gamping 2 Krian, two subject students were chosen who had a relational and instrumental understanding.

Data Collection Tools

This study uses video recordings as a data collection tool, and uses recordings as data to be analyzed. Video recording is used because it is very effective in conducting detailed analysis (Powell, Francisco & Maher, 2003). Data collection techniques using the task, observation, and documentation in the form of video recordings. The research instruments used included Task of Understanding Type (TuT), Task of Problem Solving (TPS), interview guidelines, and video recording devices.

Task of Understanding Type (TuT)

TuT is used to determine students' initial understanding of the addition and subtraction operations for administrators, the results of giving TuT are grouping types of students' relational and instrumental understanding. The construct validity of TuT statement items was calculated used product moment with the result r count of 1,258 and alpha 0.05, so each item statement was stated empirically valid. The TuT reliability value was calculated using Cronbach Alpha with a result of 0.727, so TuT is classified as having a good reliability.

Task of Problem Solving (TPS)

While TPS is used to analyze folding back of students in solving problems. The problems given represent the daily context of students. There are two types of contexts in TPS, namely: the context of elevators in TPS 1 and the context of shops in TPS 2 (See Appendix). The validity of the TPS construct was calculated used product moment with a result of 1288 and alpha 0.05, so that the TPS could be said to be valid. TPS reliability value is calculated using Cronbach alpha with a result of 0.886, so TPS is classified as having a good reliability.

Results

Folding Back Students who have a Relational Understanding

The researcher grouped students into relational and instrumental understanding in accordance with the concept of folding back. Following are the TPS 1 results of students with relational understanding illustrated in Figure 1.

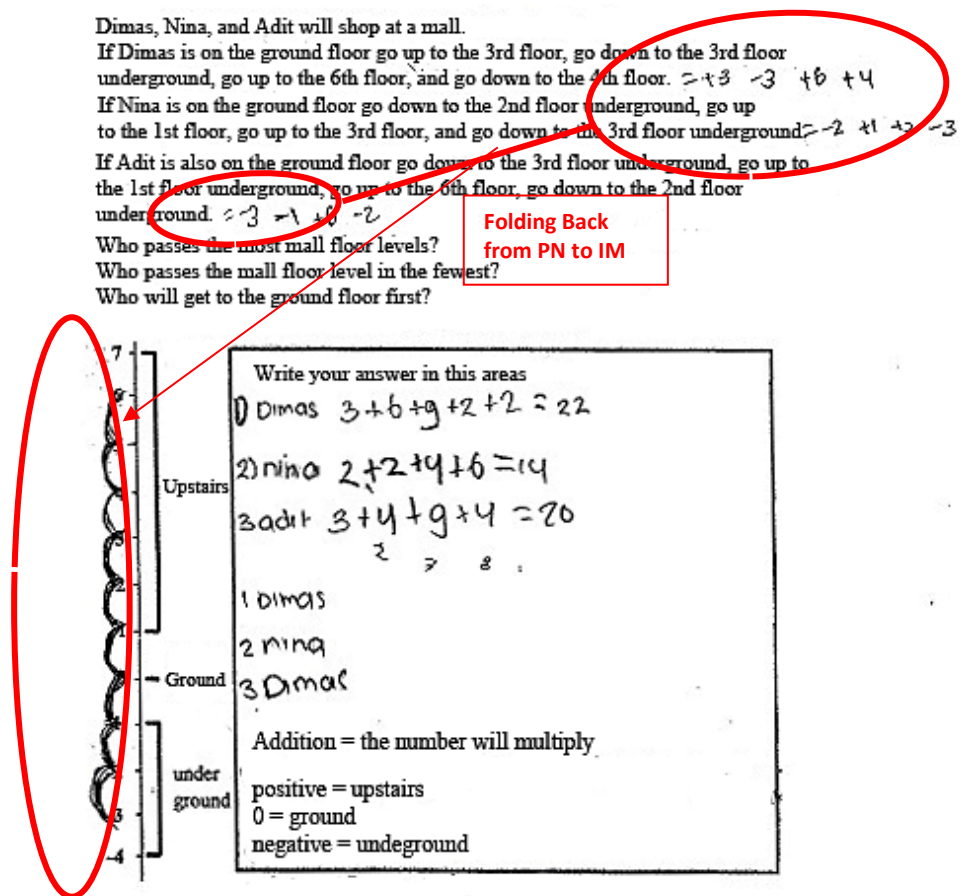


Figure 1.
Folding Back Subject A from Property Noticing (PN) to Image Making (IM)

In Figure 1 it can be seen that students have an understanding of folding back from the process of PN to IM. Following are the results of recap interviews with subject A.

- Researcher : Explain the solution you used in answering this problem!
- Subject A : The numbers are added.
- Researcher : What numbers?
- Subject A : 3, -3, 6, 4
- Researcher : Why are there negative numbers?
- Subject A : Because the underground floor is negative.
- Researcher : If it's not an underground floor?
- Subject A : Yes, a positive number.
- Researcher : Try to explain which positive and negative numbers are in the figure!
- Subject A : From the ground floor up to the 3rd floor (point to the number 0 then move to point to the number +3 in the figure in the problem)
- Researcher : Who from the ground floor goes up to the 3rd floor?

Subject A : Dimas. Go down to the 3rd floor underground (point number -3). Go up to the 6th floor and go down to the 4th floor (point to the number +6 and switch to point to the number +4 in the figure in the problem).

Based on the results of the tasks and interviews it can be seen that subject A understands the positive and negative numbers contained in the questions. Subject A mentions the underground floor as a negative number which is on the third floor underground so it includes the number -3. While non-underground floors are positive numbers, namely floors 3, 6, and 4. Subject A's answers indicate that in this study, subject A was in the fourth layer, property noticing. Next, subject A folds back from the noticing property layer to the image making layer. Subject A folded back to the second layer, image making), which is shown from the results of the interview subject A understands that the top floor is a positive number consisting of numbers 1 to 7. On the ground floor consists of floor 0 which is zero, while the floor underground is a negative number consisting of numbers -1 to -3. Positive numbers, negative numbers, and zeros are identified through the image by pointing and observing the images in the TPS 1. So it can be seen that subject A understands that the top floor is a positive number, the ground floor is a zero number, and the underground floor is a negative number through the figure in the TPS 1. Subject A folded back a second time (Figure 2), from the previous layer, IM to the PK layer.

Dimas, Nina, and Adit will shop at a mall.

If Dimas is on the ground floor go up to the 3rd floor, go down to the 3rd floor underground, go up to the 6th floor, and go down to the 4th floor. $-3 -3 +6 +4$

If Nina is on the ground floor go down to the 2nd floor underground, go up to the 1st floor, go up to the 3rd floor, and go down to the 3rd floor underground $-2 +1 +3 -3$

If Adit is also on the ground floor go down to the 3rd floor underground, go up to the 1st floor underground, go up to the 6th floor, go down to the 2nd floor underground. $-3 -1 +6 -2$

Who passes the most mall floor levels?
 Who passes the mall floor level in the fewest?
 Who will get to the ground floor first?

Write your answer in this areas

1) Dimas $3+6+9+2+2=22$

2) Nina $2+2+4+6=14$

3) Adit $3+4+9+4=20$

Folding Back from IM to PK

1) Dimas
2) Nina
3) Dimas

Addition = the number will multiply

positive = upstairs
 0 = ground
 negative = underground

Figure 2.
 Folding Back Subject A from Image Making (IM) to Primitive Knowing (PK)

The following are the results of the interview.

- Researcher : Try to explain how you counted.
- Subject A : Dimas on the 0th floor then go up to the 3rd floor ie1 2 3, go down to the 3rd floor underground means 4 5 6 7 8 9. Then go up to the 6th floor means 10 11 12 13 14 15 16 17 18. Continue to go down to the 4th floor means the count is 19 20.
- Researcher : Is the method of calculation continued from the previous calculation?
- Subject A : Yes, continued from the previous count.
- Researcher : What is the purpose of addition or addition?
- Subject A : Increased number of floors passed.
- Researcher : So the sum is increasing in number?

Subject A : Yes

The task and interview results show that subject A folded back to the PK layer. Subject A mentioned that the calculation carried out in solving TPS 1 problems was by the addition operation. Subject A provides an explanation of the definition of the concept of addition, which is as an addition. That is, every person who has passed the mall floor before being added to the mall floor that is being passed so that the results multiply. Subject A developed his understanding obtained through folding back twice towards the fourth layer (PN). Starting from the PK layer to the PN layer. Figure 3 showed this process.

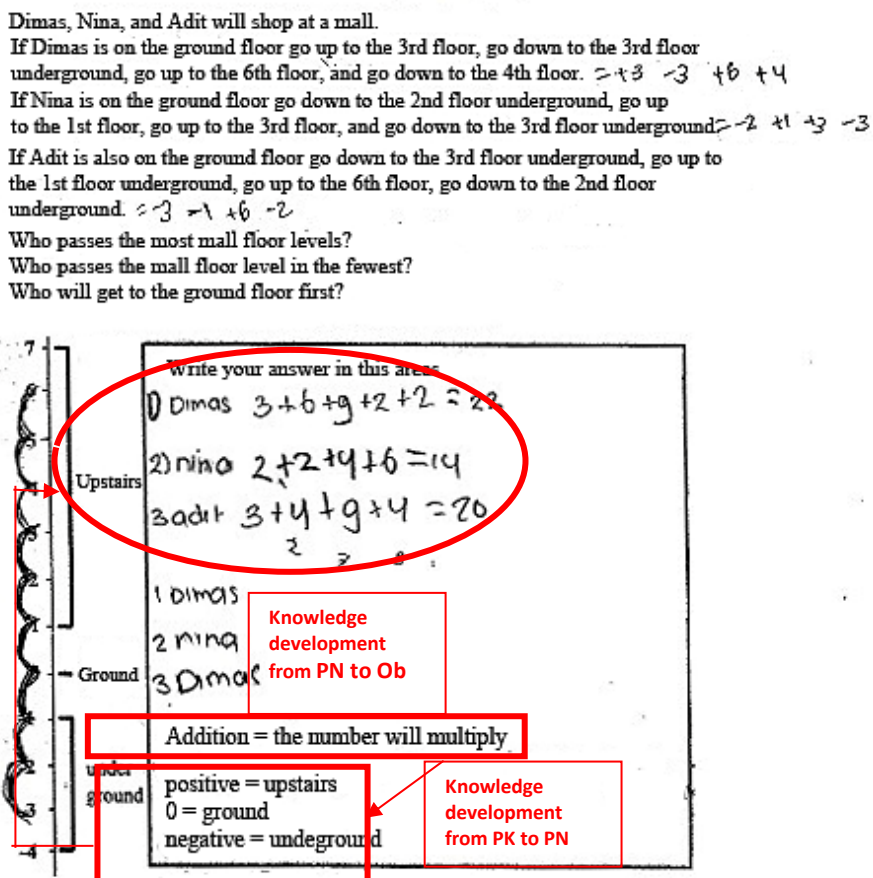


Figure 3.

Growth in mathematical understanding of Subject A from Primitive Knowing (PK) to Property Noticing (PN) and from Property Noticing (PN) to Observing (Ob)

The following are the results of the interview.

Researcher : Why do you count while raising your fingers on the image?

Subject A : Because it also goes up and down.

Researcher : Then what about the numbers?

Subject A : Yes, as before, if the top floor is a positive number, if the underground floor is negative.

Based on the results of the tasks and interviews show that subject A is in PN layer. This can be known through understanding subject A to the positive, zero and negative numbers in the TPS 1. Subject A explains that the positive numbers consist of numbers on the top floor, zero on the ground floor, whereas negative numbers consist of numbers that are on the underground floor. Next subject A moves and arrives at the sixth layer (Ob). Subject A coordinates the problem with the concept of addition that has been understood, and is able to determine the algorithm or systematic procedure that is appropriate in working on the TPS 1. Here is a figure of the trajectory of the growth in mathematical understanding subject A from the explanation above.

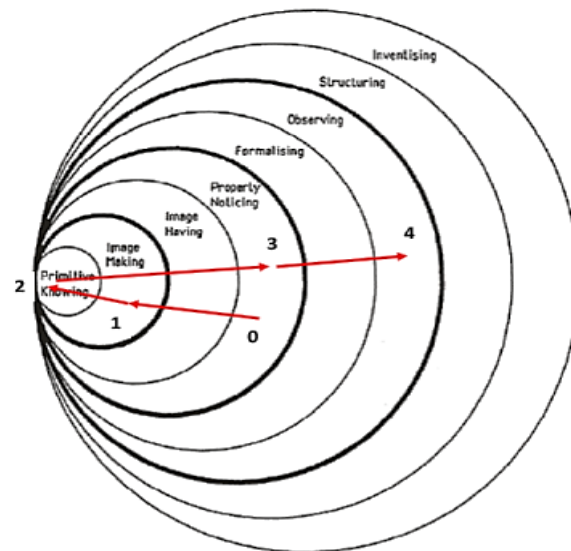


Figure 4.
The Trajectory of Subject A' Growth in Mathematical Understanding for TPS 1

Folding back is done by subject A twice, whereas the first folding back is from the PN layer to the IM layer. The first folding back source came from self intervention. This self-intervention is done intentionally and focused. The second source of folding back, from the IM layer to the PK layer, also comes from deliberate and focused self-intervention. The form of folding back done by subject A is collecting in a deeper layer, both folding back is done the first time and the second time. This form of folding back collects in deeper layers occurs because subject A knows what is needed, but cannot remember directly about the knowledge that he had before. In the first folding back, from the PN layer to the IM layer, the results of the folding back done by subject A are ineffective. The result of the first ineffective folding back, then subject A folded back a second time, namely from the IM layer to the PK layer. The second folding back result is effective (Figure 5).

5	Father and mother	Doll shop	Musical shop	Write your answer in this areas. Father go down 1 floor Mother go up 2 floor Uncle go down 2 floor Upstairs = positive Under ground = negative Ground = zero Addition = go up Substraction = go down 5-4-1+3=3 $5-4-1+3=3$ $3-2+1+2=4$ $5-5=0$ $5-4-1+3=3-1=2$ $3-2+1+2=4-2=2$ $5-5=0+2=2$	
	Uncle	Bookstore	Fruit store		
	Father	Bakery shop	Clothes shop		
		Restaurant	Shoe store		
		Stationery store	Electronic store		
	mother	Toy store	Kitchen shop		
		Toy store			
		Florist			

Figure 5.
Folding Back Subject A from Primitive Knowing (PK) to Image Making (IM)

The following are the results of the interview recap.

- Researcher : What method did you use in doing this problem? Try to explain !
- Subject A : The method is added and subtracted.

Researcher : How can it be added?
 Subject A : Yes because it goes up.
 Researcher : What's going up?
 Subject A : This is the floor.
 Researcher : Then why can something be reduced?
 Subject A : Because the floor is down.
 Researcher : Then what do we do next?
 Subject A : Here ... I think I use this. (pointing to figure)
 Researcher : Yes, just try to continue.
 Subject A : Father and mother are on the 5th floor of a mall. Mother headed for the kitchen shop on the ground floor. (write the numbers 0, 1, 2, 3, 4, 5, 6, -1, -2 in the figure).

Based on the results of the interview it can be seen that subject A identifies addition and subtraction operations. Subject A mentions if the elevator or floor rises, then it uses addition operations. Meanwhile, if the elevator or floor goes down, then use a reduction operation. The answer of subject A indicates that in this study, subject A is in the fourth layer, PN. Furthermore, subject A folds back initially from the PN layer to the IM layer. Subject A folded back to the second layer (IM), can be seen from the results of the interview subject A knowing that the upper floor is a positive number consisting of numbers 1 to 6. On the ground floor consists of floor 0, while the underground floor is negative numbers consisting of numbers -1 to -2. Positive numbers, negative numbers, and zeros are identified by pointing and observing the image in the TPS 2. So it can be seen that subject A understands that the top floor is a positive number, the ground floor is a zero, and the underground floor is a negative number through the image. Subject A developed the understanding gained through folding back to the fourth layer, PN again. Subject A remembers that if the elevator or floor rises using addition operations, whereas if the elevator or floor goes down using subtraction. Subject A also remembers and then writes in the answer column that the top floor is a positive number, the ground floor is a zero, and the underground floor is a negative number. Next subject A is in the fifth layer, formalizing (F). Subject A writes the addition and subtraction operations according to the characteristics of the rise and fall of the elevator in accordance with the matter of TPS 2. The following is a figure of the trajectory of the growth in mathematical understanding subject A according to the explanation above.

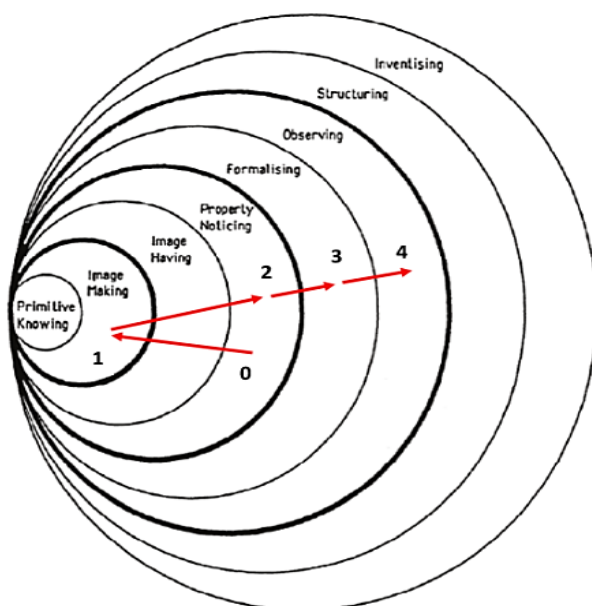


Figure 6.
The Trajectory of Subject A 'Growth in Mathematical Understanding for TPS 2

Subject A folds back from the PN layer to the IM layer. This folding back source comes from self intervention. This self-intervention is done intentionally and focused. The folding back shape performed by subject A is working on deeper layers. The result of folding back is effective.

Folding Back Students with Instrumental Understanding

Following are the TPS 1 results of students with instrumental understanding illustrated in Figure 7.

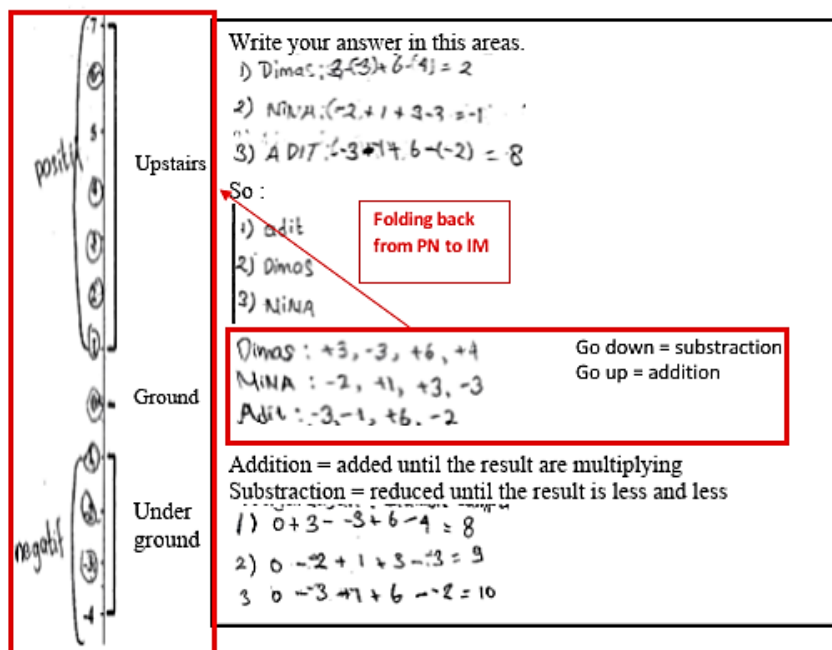


Figure 7.

Folding Back Subject B from Property Noticing (PN) to Image Making (IM)

The following are the results of the interview.

Researcher: Explain how you used this problem!

Subject B: Plus.

Researcher: Which is added?

Subject B: Here, 3 plus -3.

Researcher: Let's try to explain!

Subject B: Being on the 3rd floor down to the 3rd floor the underground was reduced.

Researcher: Why is it reduced?

Subject B: Because it goes down so it's reduced.

Researcher: What if it goes up?

Subject B: Plus.

Researcher: What about the numbers?

Subject B: If the 3rd floor is positive, the 3rd floor underground is negative, if the 6th and 4th floors are also positive.

Researcher: What's next?

Subject B: Mmm ...

Researcher: Let's try to look more at the problem and the figure.

Subject B: Yes. On the ground floor go up to the 3rd floor. The ground floor is 0 and the 3rd floor is +3 (point to the number 0 then move to point to the number +3 in the figure). Then Dimas went down to the 3rd floor underground, which is the number -3 (pointing number -3). Going up to the 6th floor and down to the 4th floor. The 6th and 4th floors are positive numbers (point to +6 and switch to +4).

Based on the results of these tasks and interviews it can be seen that subject B identifies positive and negative numbers, and identifies addition and subtraction operations. Subject B explains that if the elevator or floor rises then it uses addition operations, but if the elevator or floor goes down it uses a subtraction operation. Subject B also mentioned floors 3, 4, and 6 are positive numbers. While underground 3 floors are negative numbers. The answer of subject B indicated that in this study, subject B was in the fourth layer PN. Next, subject B folds back initially from the PN layer to the IM layer. Subject B folded back a second time, from the previous layer, Image Making IM to the PK layer. Here are the results of student tasks.

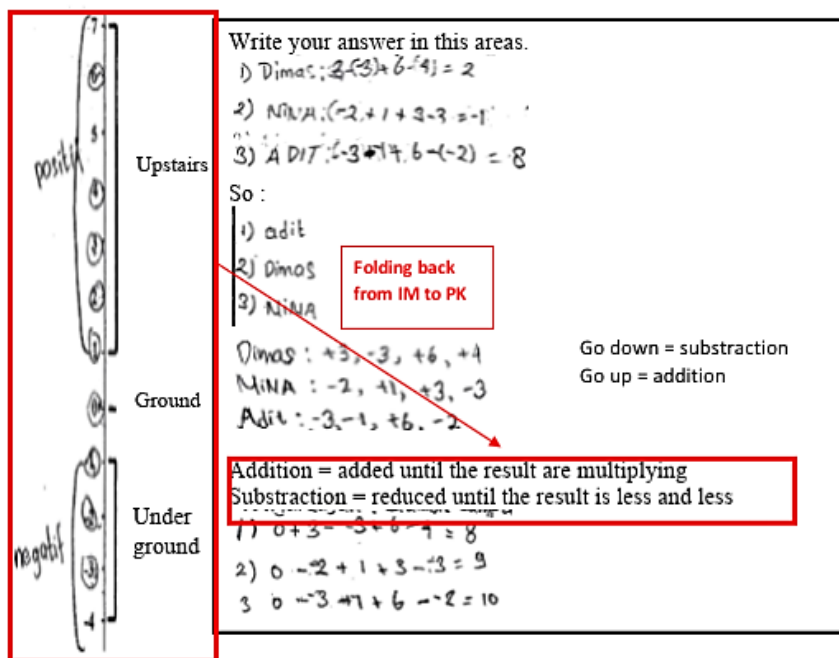


Figure 8.

Folding Back Subject B from Image Making (IM) to Primitive Knowing (PK)

The following are the results of the interview.

Researcher: What do you count next?

Subject B: Being on the 0th floor then going up to the 3rd floor means it's added. Then down to the 3rd floor underground means reduced.

Researcher: Means using addition and subtraction operations?

Subject B: Yes.

Researcher: What is the meaning of the sum?

Subject B: It means that you add it up until you get a lot more results.

Researcher: What does reduction mean?

Subject B: Yes, subtracted, the number is taken with another number, so the result is less.

Task results and interviews above show subject B folding back to the PK layer. Subject B explained that the calculation carried out in solving TPS 1 problems was by using addition and subtraction operations. Subject B explains the definition of the concept of addition as an addition, meaning that the mall floor that is passed by each person will increase more and more because the floor that was previously passed is added to the mall floor that is being passed. So subject B understands that in addition operations, that is, the numbers added by other numbers, the results get more and more. Whereas the subtraction operation is the number that is reduced or taken by another number the results are getting fewer. Subject B developed the understanding obtained through folding back twice towards the fifth layer, Formalizing (F). Starting from the PK layer to the F layer. Furthermore, subject B writes the addition and subtraction operations according to the characteristics of positive, zero, and negative numbers, and up and down lifts according to the problem of TPS 1. The following is a figure of the trajectory of the growth in mathematical understanding of subject B based on explanation above.

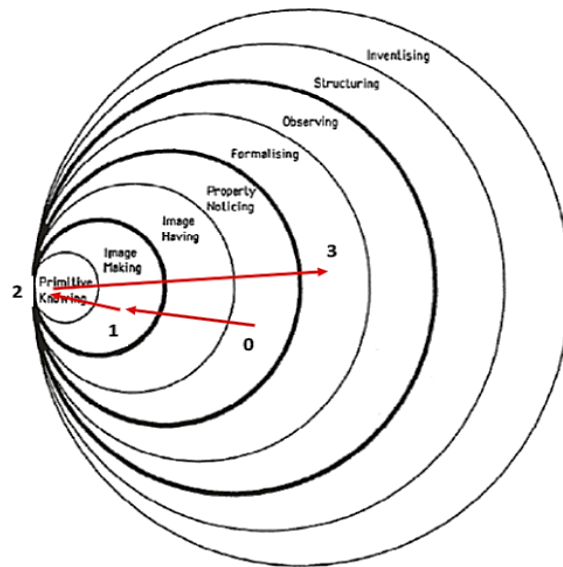


Figure 9.
The Trajectory of Subject B 'Growth in Mathematical Understanding for TPS 1

Folding back is done by subject B twice, whereas the first folding back is from the PN layer to the IM layer. The first folding back source came from self intervention. This self-intervention is done intentionally and focused. The second source of folding back, from the IM layer to the PK layer, also comes from deliberate and focused self-intervention. The form of folding back done by subject B is collecting in the deeper layers, which is what happens to the folding back that is done first. The second folding back is from the IM layer to the PK layer. It has a folding back shape that works on a deeper layer. The first folding back is from the PN layer to the IM layer, the results of folding back done by subject B are ineffective. The result of the first ineffective folding back, then subject B did the second fold back, from the IM layer to the PK layer. The result of folding back the second time is effective. Figure 10 are the results of student tasks.

5		Doll shop	Musical shop	Write your answer in this areas. 1) Mother go up 2 floor 2) Father go down 1 floor 3) Uncle go down 2 floor Addition = go up Substaction = go down Positive = upstairs Negative = under ground Zero = ground Understanding development from PN to Ob 1) $5 - 1 - 1 + 3 = 3 - 1 = 2$ 2) $3 - 2 + 1 + 2 = 4 - 2 = 2$ 3) $5 - 5 = 0 + 2 = 2$
4	uncle	Bookstore	Fruit store	
3	Father	Bakery shop	Clothes shop	
2	Understanding development from IM to PN	Restaurant	Shoe store	
1		Stationery store	Electronic store	
0	Mother	Toy store	Kitchen shop	
-1		Toy store		
-2			Florist	

Figure 10.
Folding Back Subject B from Property Noticing (PN) to Image Making (IM), Growth in Mathematical Understanding from IM to PN, and PN to Observing (Ob)

The following are the results of the interview.
 Researcher: Try to explain how you do this problem!

Subject B: Father and mother are on the 5th floor of a mall. Mother headed for the kitchen shop on the ground floor. Daddy down 4, down 1, up 3. This will be added later.

Researcher: Why added?

Subject B: Because of this the floor is up. Then something is reduced.

Researcher: So why is there a reduction?

Subject B: Because the floor is down.

Researcher: What do you do after that?

Subject B: Er ... what are you doing, ma'am?

Researcher: You already know that increasing is increased and if decreasing is reduced, now try to look and think first. Can it be counted directly?

Subject B: Father and mother are on the 5th floor of a mall. Mother headed for the kitchen shop on the ground floor. (write the numbers 0, 1, 2, 3, 4, 5, 6, -1, -2 in the figure). Right mom?

Researcher: Yes.

Based on the results of the tasks and interviews it can be seen that subject B identifies the addition and subtraction operation. Subject B mentions if the elevator or floor rises, then use the sum operation. Meanwhile, if the elevator or floor goes down, then use a reduction operation. The answer of subject B indicated that in this study, subject B was in the fourth layer, PN. Next, subject B folds back initially from the PN layer to the IM layer. Subject B folded back to the second layer (IM), can be seen from the results of the interview subject B knowing that the top floor is a positive number consisting of numbers 1 to 6. On the ground floor consists of 0 floors, while the underground floor is negative numbers consisting of numbers -1 to -2. Positive numbers, negative numbers, and zeros are identified by pointing and observing the image in the TPS 2. So it can be seen that subject B understands that the top floor is a positive number, the ground floor is a zero, and the underground floor is a negative number through the image. Subject B developed the understanding gained through folding back to the fourth layer, PN again. Subject B remembers that if the elevator or floor rises using the addition operation, whereas if the elevator or floor goes down using subtraction. Subject B also remembers then wrote in the answer column that the top floor is a positive number, the ground floor is a zero number, and an underground floor is a negative number. Furthermore, subject B who was previously in the fourth layer is PN to the sixth layer, namely Observing (Ob). Subject B coordinates the problem with the concepts of addition and subtraction and is able to determine the right algorithm or systematic procedure. The following is a figure of the trajectory of the development of subject B according to the explanation above.

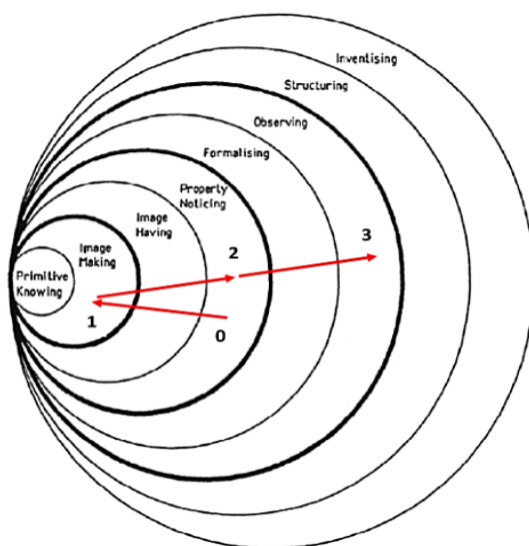


Figure 11.

The Trajectory of Subject B 'Growth in Mathematical Understanding for TPS 2

Subject B folds back from the PN layer to the IM layer. This folding back source comes from self intervention. This self-intervention is done intentionally and focused. The folding back shape performed by subject B is working on deeper layers. The result of folding back is effective.

Discussion and Conclusion

Based on the results of research that has been done, it can be seen that there are differences between folding back students with relational understanding and folding back students with instrumental understanding in the growth in the mathematical understanding of the Pirie-Kieren model. Students with relational understanding are when able to understand about how mathematical rules and the reasons why mathematical rules are used (Skemp, 2013). The characteristics of students with relational understanding are students do not depend on memorization, students do not depend on steps or instructions, students have alternative solutions or procedures, and students have a link between many ideas (Rahmad et al. 2016; Skemp, 2006)

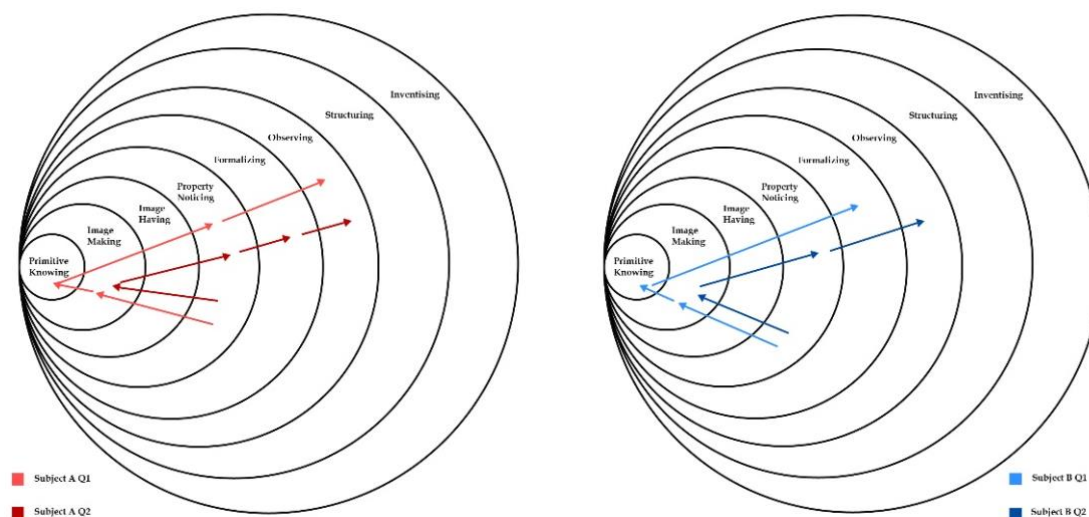


Figure 12.

A Comparison of Folding Back Students who have a Relational and Instrumental Understanding

Students with relational understanding tend to fold back more than once that happens to subject A when solving TPS 1. Subject A does double back, namely at the property noticing to image making layer and at the image making layer to primitive knowing. In addition, subject a has a longer folding back trajectory in the growth in mathematical understanding the Pirie-Kieren model, where subject A to the sixth layer is Observing because it has a link between many ideas to be able to develop ideas more broadly (Skemp, 2006, 2013). But in the case of TPS 2, subject A folds back only once, namely in the property noticing layer to image making.

This folding back source for students with relational understanding comes from self-intervention without any outside intervention such as friends, books, or teachers, where self-intervention is done intentionally and focused. The folding back shape of students with relational understanding tends to lead to work in the deeper layers. The aim is to work at the deeper layers using existing understanding and not off-topic. This form of folding back works at the deeper layers because students have limited understanding of the deeper layers, thereby expanding existing understanding by arranging new understandings at the deeper layers (Martin & Towers, 2016b). But in TPS 1, subject A has a form of folding back collecting in deeper layers. Students try to gain prior knowledge, where students know what is needed, but cannot remember directly about knowledge that was previously possessed. That is, this understanding cannot be easily accessed or recalled. The results of folding back on students with relational understanding are effective, because students successfully apply understanding to the deeper layers and return to work on the outer layers (Martin, 2008). Students also find and apply systematic methods or procedures that are right, and the results of their calculations are also right.

Students with instrumental understanding are only able to understand how mathematical rules are used but do not understand the reasons why mathematical rules are used. The characteristics of students with instrumental and understanding that students depend on memorization, students depend on steps or instructions, students do not have alternative solutions or procedures, and students do not have a connection between many ideas (Bofferding & Enzinger, 2017; Skemp, 2006). Students with instrumental understanding tend to fold back only once, but in TPS 1, subject B folds back twice, namely in the property noticing layer to image making and in the image making layer to primitive knowing.

The source of folding back to students with instrumental and understanding comes from self-intervention without any outside intervention such as friends, books, or teachers, where self-intervention is done intentionally and focused. The folding back shape of students with instrumental understanding tends to lead to gathering in deeper layers. Students try to gain prior knowledge, where students know what is needed, but cannot remember directly about knowledge that was previously possessed (Martin, 2008). That is, this understanding cannot be easily accessed or recalled. So students read back the notebook containing the material needed. But in TPS 1 and TPS 2, subject B had a folding back shape that is working on a deeper layer. The aim is to work at the deeper layers using existing understanding and not off-topic. This form of folding back works at the deeper layers because students have limited understanding of the deeper layers, thereby expanding existing understanding by arranging new understandings at the deeper layers. The results of folding back on students with instrumental understanding are effective, because students successfully apply understanding to the deeper layers and return to work on the outer layers (Skemp, 2013; Weber, 2002). However, students have not been able to find and apply the right method or systematic procedure, and the results of their calculations are also not right.

Characteristics of folding back students in the development of the understanding of the Pirie-Kieren model on relational understanding and instrumental understanding that students with relational understanding tend to double fold back, namely the property noticing layer to image making and the image making layer to primitive knowing. Students with relational and understanding have a longer folding back trajectory in the growth in mathematical understanding the Pirie-Kieren model, where the trajectory reaches the sixth layer, observing. This folding back source for students with relational understanding comes from intentional and focused self-intervention. The folding back shape of students with relational understanding tends to lead to working in deeper layers (Greer, 2012; Martin & Towers, 2016a). The results of folding back on students with relational understanding are effective. Students also find and apply systematic methods or procedures that are right, and the results of their calculations are also right.

The characteristics of students in folding back are influenced by three mathematical abilities in completing mathematical tasks, namely problem solving, conceptual understanding, and reasoning (Fatimah & Prabawanto, 2020; Syazali, Listiani, & Farid, 2019). Deep conceptual understanding helps students in completing mathematical assignments. Building this understanding can be helped by presenting problems in everyday life (Diana, Suryadi & Dahlan, 2020; Orhun, 2013).

Characteristics of students with instrumental understanding tend to fold back only once, namely the property noticing layer to the image making. Students with instrumental and understanding have shorter folding back trajectories in the development of the understanding of the Pirie-Kieren model, where the trajectory only reaches the fifth layer, which is formalizing. Folding back source for students with instrumental and understanding comes from self-intervention that is done intentionally and focused. The folding back shape of students with instrumental understanding tends to lead to gathering in deeper layers. The results of folding back on students with instrumental understanding are effective, but students have not succeeded in finding and applying an appropriate systematic method or procedure, and the results of their calculations are also not right.

Recommendations

In general, the pupils applied reflective thinking with different strengths. Based on the conclusion, it is good for teacher in stimulating pupils' reflective thinking. Giving the treatments can be ICT learning tool usage, fresh learning model, and giving new experience for pupils. The teachers need to pay more attention to develop instrument in term of increasing pupils' achievement.

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Appendix

Test of Understanding Type

Do the operations below include subtraction operation? Give your reasons.

1. $5 - 2$
2. $7 - (-3)$
3. $-4 - 1$
4. $6 + (-2)$
5. $-8 - (-6)$

Task of Problem Solving 1

Dimas, Nina, and Adit will shop at a mall.

If Dimas is on the ground floor go up to the 3rd floor, go down to the 3rd floor underground, go up to the 6th floor, and go down to the 4th floor.

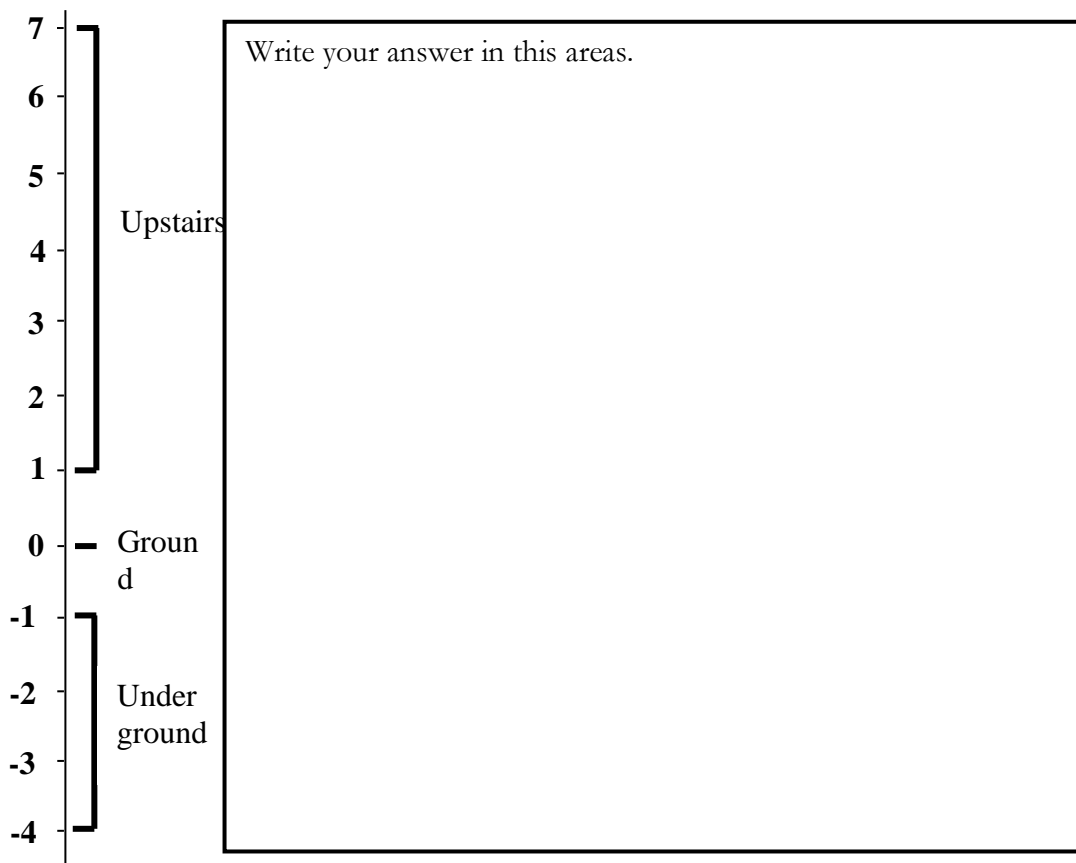
If Nina is on the ground floor go down to the 2nd floor underground, go up to the 1st floor, go up to the 3rd floor, and go down to the 3rd floor underground.

If Adit is also on the ground floor go down to the 3rd floor underground, go up to the 1st floor underground, go up to the 6th floor, go down to the 2nd floor underground.

Who passes the most mall floor levels?

Who passes the mall floor level in the fewest?

Who will get to the ground floor first?



Task of Problem Solving 2

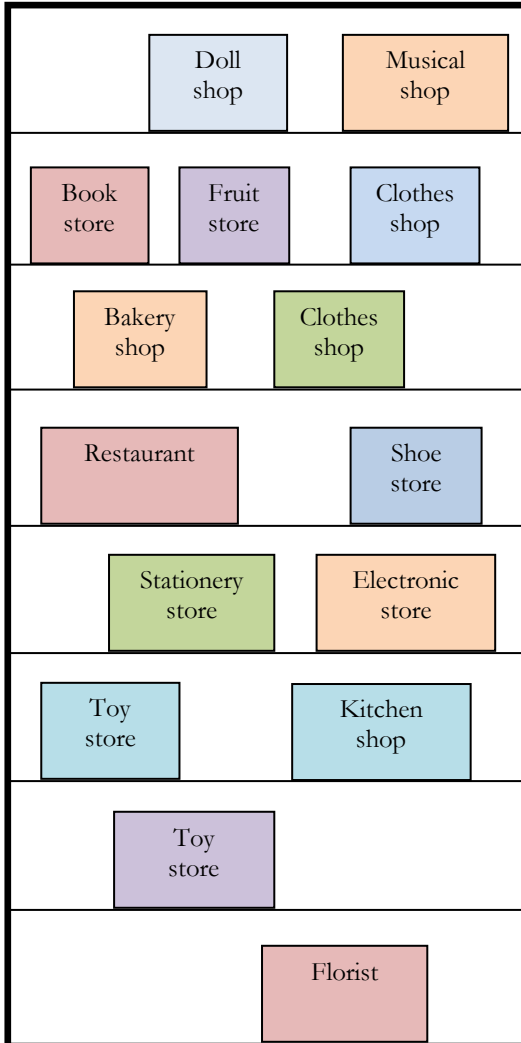
Father and mother are on the 5th floor of a mall.

Mother headed for the kitchen shop on the ground floor

Daddy went down 4 floors, then went down 1 floor, and went up 3 floors again.

Uncle is also at the same mall wanting to buy lunch for father and mother. Uncle is on the 3rd floor, down 2 floors, then up 1 floor, and up again 2 floors.

How do you get fathers, mothers, and uncles to meet on the same floor?



Write your answer in this areas.