Pupils’ reflective thinking in solving linear equation system problem

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Abstract

Reflective thinking begins with perplexity. Efforts to overcome perplexity by involving the experience and knowledge. In the process of reflective thinking, problem solvers need to use mathematical abilities. The research reveals pupils’ reflective thinking in solving Linear Equation System (LES) problem in term of their mathematical abilities. The qualitative research employed 46 Indonesian reflective thinkers. A reflective thinker is a pupil employees experience and knowledge in solving LES problem. The instruments have been validated by mathematicians and education experts. Data collected by some methods among others: test, observation, and in-depth interview. Triangulation conducted as data validation process. We successfully concluded that different mathematical abilities provide differences in pupils’ reflective thinking for solving LES problems. It is necessary to develop reflective thinking skills in pupils with low mathematical abilities.

Introduction

Mathematics is a subject with systematic concepts from simple concepts to more complicated concepts (Radovic et al. 2018). Pupils who have not mastered the basic concepts will certainly face a trouble in mastering more complicated concepts. This has led to speculation that mathematics as a difficult subject to understand, complicated, and even unpleasant compared to others (Dubinsky, 2002). This resulted in pupils’ learning outcomes tend to be weak (Sammons et al. 2011). Mathematical learning outcomes are changes in pupil behavior after mastering the lesson (Sandt, 2007). The outcomes measured by pupils’ thinking processes for solving mathematical problems (Hanley et al. 2015) for instance reflective thinking (Dewey, 1933; Rodgers, 2002).

Reflective thinking begins with the appearance of perplexity overcome by conducting re-investigation for problem solving (Rodgers, 2002). Moreover, a problem solver employees knowledge and experience to deal with perplexity. Reflective thinking supports pupils in making meaning out of experiences at the highest critical level (Howlett et al. 2015). By employing reflective thinking, pupils able to control themselves in their learning by actively accessing what they already know, what they need to know, and how to solve the problems (Stark & Krause, 2009). Reflective thinking encourages problem solvers to explore effective, efficient, and appropriate strategies for solving problems. Therefore reflective thinking contains cognitive and affective aspects (Afshar & Farahani, 2018). Unfortunately, not all teachers understand the importance of exploring and developing pupils’ reflective thinking (Sezer, 2008). By employing reflective thinking for problem solving, pupils can reach better achievement both in affective and cognitive (Ghanizadeh, 2017; Hsieh & Chen, 2012; Kaune, 2006).
In fact, reflective thinking receives less attention from the teacher (Sezer, 2008). In problem solving, they tend to see whether pupils’ answers are correct or not without knowing how pupils get the solution. This affects pupils to only focus on answering questions correctly without going through the process of analysis, evaluation, and monitoring (Susandi & Widyawati, 2017). That’s why research focuses on reflective thinking needs to be conducted continuously. Hong & Choi (2011) explores the level of reflective thinking as single loop, double loop, and triple loop. The difference lies in the extent to which the problem solver’s ability to control the experience and knowledge they have.

Reflective thinking category is concluded by Suharina (2018) as productive, connective, and clarificative reflective thinking. Pupils with productive reflective thinking categories overcome the perplexity by solving problem more than one solution. It is conducted to provide confidence that their answers are correct. Pupils with connective reflective thinking categories overcome the perplexity by connecting relevant concepts, theorems, and operations in mathematics. Meanwhile, pupils with clarificative reflective thinking categories overcome the perplexity by clarifying and re-monitoring their solution.

Aspects of reflective thinking on mathematical problem solving conveyed by Zehavi & Mann (2005) among others are techniques, monitoring, insight, and conceptualization. Researchers have conducted preliminary research to develop reflective thinking indicators based on those aspects. The results of the preliminary study described as follows.

Techniques aspect means selection of strategies to facilitate problem solving. The preliminary research concluded the aspect contains three indicators among others: understanding the information, understanding the question, and understanding the most effective and efficient way. Monitoring is an activity of analyzing and evaluating steps and answers for problem solving. The aspect contains three indicators as follows: monitoring the steps of solution, and monitoring whether the solutions are correct or not. Insight is individual ingenuity in managing experience and knowledge for problem solving. The aspects contains two indicators among others: being ready to overcome perplexity and understanding how to prevent any difficulty. Conceptualization aspect means relating relevant concepts to solve the problems. Pupils must be able to relate some concepts in mathematics for solving a problem.

Reflective thinking begins from a problem solvers’ perplexity (Rodgers, 2002). Therefore, a problem solving type that employed for understanding reflective thinking is a question that can cause a problem solver’s perplexity. It may be a non-routine question (Hong & Choi, 2011). It is an unfamiliar question for pupils so it will cause them to have reflective thinking (Hidajat et al. 2019). The study employs a non-routine question in Linear Equation System (LES) content. LES content is easy developed into a non-routine question. Thus non-routine problems in LES content can be employed to reveal pupils’ reflective thinking.

**Problem of Research**

There are still opportunities to explore reflective thinking seen from various reviews. The research focuses on problem of how are pupils’ reflective thinking in solving LES problem in term of their mathematical abilities? Pupils categorized in three mathematical abilities namely expert, moderate, and novice. The similarities and differences of pupils’ reflective thinking at all three levels will be addressed qualitatively.

**Methods**

**Research Design**

The research is a descriptive qualitative study, because it depicts all facts without manipulation (Sagala et al. 2019). Qualitative research based on natural object conditions in order to obtain data in depth and meaningfully. The study reveals pupils’ reflective thinking in solving LES problem in term of their mathematical abilities.

**Participants**

The participants are 46 reflective thinkers enrolling study in Middle School in Central Java – Indonesia. The subjects are reflective thinkers who are able to deal with perplexity by employing knowledge and experience to solve problems. By considering the standard deviation and mean of final semester tests, their mathematical abilities categorized into 11 pupils in expert level, 21 pupils in moderate level, and 14 pupils in novice level.

**Instruments**

The instruments are test, observation sheet, and in-depth interview guideline. All instruments have been validated by two validators from mathematicians and education experts. Revision conducted to improve the quality of instrument until declared valid. The test focuses on LES content to delve pupils’ reflective thinking process. It contains of one item non-routine question. The validator suggests that the test instrument demands a higher level of analysis so that it can cause confusion to stimulate reflective thinking. Observation sheet employed to record whether subjects
conduct reflective thinking indicators or not. The researchers as observers affix a checklist to the column “YES” if the subjects show the reflective thinking indicator, while, in the column “NOT” if the subjects do not conduct reflective thinking indicator. If the observers experience doubts whether the subject performs reflective thinking indicators or not, then the observers put a checklist in the “DOUBT” column. In term of ensuring this result, the researchers conducted in-depth interviews based on the guidelines. The in-depth interviews may be conducted more than once until the researchers obtain complete data. Both validators stated that the observation sheet instrument and in-depth interview guidelines did not need to be revised.

**Data Analysis**

Data analyzed through three stages. First, the reduction phase conducted to determine important data to achieve the research objectives. Then, the reduced data presented to describe pupils’ reflective thinking data based on aspects of techniques, monitoring, insight, and conceptualization under expert, moderate, and novice pupils. The last one, researchers drew a conclusion of the data to answer the research question. The research procedure presented in Figure 1.

![Figure 1](image1.png)

**The Research Procedure**

**Prodecure**

Data obtained by tests, observation sheets, in-depth interviews, video recordings, and documentation methods. Documentation data employed to categorize pupils’ mathematical abilities. Researchers observe the subject solving the test with think aloud techniques. Interesting findings noted on observation sheets. At the same time, audio-visual recording employed to record the problem solving process. In exploring pupils’ reflective thinking more deeply researchers conducted interviews. The LES problem to understand pupils’ reflective thinking presented in Figure 2.

**The Question to Understand Pupils’ Reflective Thinking**

Reza will buy some donuts and brownies. The price of one box of brownies is twice the price of one box of donuts. If the price of three boxes of brownies and two boxes of donuts is 480,000 IDR, then what is the price of two boxes of brownies and four boxes of donuts? How much change does Reza receive if he pays 500,000 IDR to the seller?

![Figure 2](image2.png)

**The LES Problem**
Results

In this article, we present findings of one subject for each mathematical ability category. One subject represent pupils’ reflective thinking in each category. S1 is a pupil categorized in expert, S2 in moderate, while S3 in novice mathematical ability.

Expert pupil’s reflective thinking (S1)

S1’s answer sheet presented in Figure 3.

In solving LES problems, S1 begins with perplexity. S1 understands the problem by reading the problem carefully. The first step, S1 identifies and writes down given information and question clearly and precisely. Next, S1 transform the LES problem into a mathematical model as $b = \text{brownies} = 2d$, $d = \text{donut}$, $3b + 2d = 480,000$, and $2b + 4d =$ ?. S1 develops a plan to solve LES problem. S1 substitute $b = 2d$ into $3b + 2d = 480,000$ to conclude the price a box of donut. S1 seems silent, this shows symptoms of perplexity. In overcoming the perplexity, S1 re-read the question, then S1 substitute the price of a box of donut into $3b + 2d = 480,000$ to conclude the price of a box of brownies. S1 experienced another perplexity marked by scribbles on the answer sheet. Perplexity appears when S1 will determine the change Reza receives. S1 thinks of a way to determine the change. The way taken by calculating the bill and then deduct it with Reza’s cash. S1 succesfully conclude that the change should be 20,000 IDR. To bring up the belief that the answer is correct, S1 re-monitors the steps and conclusions with an optimistic attitude and believes that the results he got are correct. S1 avoids difficulties by trying to remember and reread questions.

Moderate pupil’s reflective thinking (S2)

S2’s answer sheet presented in Figure 4.
Figure 4.

**Answer Sheet of S2**

First, S2 reads and understands questions. S2 understands the problem by analysing the problem and determining how to reach a solution. Plans arranged to prevent difficulties. S2 experienced doubts when writing mathematical models. To overcome doubts, S2 reread information and questions until S2 obtained a mathematical model. S2 determines the price of a box donut correctly, but is wrong in determining the price of a box brownies. If the price of a box donut 60,000 IDR, so the price of a box brownies must be 120,000 IDR. Because of price of a box brownies is twice the price a box of donut. In addition, S2 overcomes difficulties and doubts by monitoring steps and answers whether they are correct or not. S2 made a mistake but he didn't realize. Mistakes in determining the price a box of brownies cause inaccurate conclusions. He sums up that Reza does not get change if paying 500,000 IDR.

**Novice pupil’s reflective thinking (S3)**

S3’s answer sheet presented in Figure 5.

<table>
<thead>
<tr>
<th>Original version:</th>
<th>Translate version:</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Information:</strong></td>
<td><strong>Solution:</strong></td>
</tr>
<tr>
<td>Price 1 box brownies = 2x price a box of donut</td>
<td><strong>Brownies = x</strong></td>
</tr>
<tr>
<td>Price 3 boxes brownies and 2 boxes donut = 480,000 IDR</td>
<td><strong>Donut = y</strong></td>
</tr>
<tr>
<td>Question: What is the price of two boxes of brownies and four boxes of donuts? How much change does Reza receive if he pays 500,000 IDR to the seller?</td>
<td>x = 2y</td>
</tr>
<tr>
<td><strong>Solution:</strong></td>
<td>**3x + 2y = 480,000 **→ 4x + 4y = 960,000</td>
</tr>
<tr>
<td></td>
<td>**2x + 4y = 500,000 **→ 6x + 12y = 1,500,000</td>
</tr>
<tr>
<td></td>
<td><strong>4y = 540,000</strong></td>
</tr>
<tr>
<td></td>
<td><strong>y = 70,000</strong></td>
</tr>
<tr>
<td></td>
<td><strong>Change = 500,000 – 466,000</strong> = 34,000</td>
</tr>
<tr>
<td></td>
<td><strong>x = 113,000</strong></td>
</tr>
</tbody>
</table>

Figure 5.

**Answer Sheet of S3**

In solving the LES problem, S5 understands the problem by reading the problem carefully and converting the information in the LES problem into variables. Brownies as x, while, donut as y. Furthermore, S3 is able to change problems into mathematical models as 3x + 2y = 480,000 and 2x + 4y = 500,000. S3 tries to find effective and efficient ways to solve the problem. S3 was silent for a long time due to perplexity when understanding the LES problem. By using the elimination method, S3 gets price a box of donut is 70,000 IDR. S3 substituted value y into equation 3x + 2y = 480,000 so he concluded price a box of brownies is 113,000 IDR. S5 experiences perplexity because he is not
sure about his answer. Re-monitoring conducted but does not bring the change. By substituting value $x$ and $y$, he concluded that the change should be 34,000 IDR.

**Discussion and Conclusion**

Referring the results of the data analysis, it can be argued that the pupils with expert mathematical abilities categories are able to satisfy aspects and indicators of reflective thinking for problem solving. Meanwhile, both moderate and novice pupils do not. Expert pupils successfully employed knowledge, experience, and attitude when overcoming the perplexity in problem solving. It is relevant to research by Huang et al. (2010). The smart and intelligent pupils are able to manage their knowledge and experience in solving problems. This is because smart pupils always try new problems so they can control themselves to solve unfamiliar problems (Bishop, 2012). On the other hands, novice pupils lack motivation to find the correct solution to the problem (Kulkarni, 2017; Stahlberg et al. 2016). They lack awareness of the importance of problem-solving skills as well as they do not understand what the components must be applied for problem solving (Hidajat et al. 2019).

In general, pupils understand the meaning of technical aspects. This aspect satisfied because pupils have an effort to understand the problem and determine the solution. All pupils understand how to transform informations and questions into mathematical models. It is relevant to research by Ramasamy & Puteh (2018). Pupils able to transform information into mathematical problem despite experience difficulties. Suharna (2018) argued the ability of pupils to understand information and question for problem solving is classified at the understanding of the problem stage. Other findings show that not all pupils think of effective and efficient ways. Experience has the big role of problem solvers in selecting the right and accurate strategy for problem solving (Mann et al. 2017).

In monitoring aspect, pupils tend to re-monitor the written step and solution as whether these are correct or not. However, many pupils remonitor them inaccurately so it is found the incorrect solutions. The monitor process is not optimal because the problem solver has no accuracy, is not focused, and does not understand the problem. The role of monitoring in problem solving has been argued by Van Haneghan & Baker (1989). Monitoring has a benefit role in correcting problem solvers’ mistakes (DiDonato, 2013). In minimizing errors, the problem solver needs to be monitored carefully and increase self-awareness (Parmin et al. 2020).

In insight aspect, some pupils are ready to correct the wrong answers. It is relevant to the research by Önder (2016). It is concluded that the pupils are ready to correct the wrong answers. There are pupils who are not ready to correct mistakes due to boredom and lack of passion (Pressley et al. 2003). It can be overcome by implementing cooperative learning model in the class. Cooperative learning models encourage pupils to increase learning enthusiasm (Cavanagh, 2011). Discussions between mates often provide useful new experiences and knowledge (Titikusumawati et al. 2013). The peer discussion provide various alternative ways to prevent difficulties for problem solving (Oliver, 2011). Gaining insight from friends provides valuable experience and information (Pravesti et al. 2020) because individual has his own way in avoiding difficulties to share with mates.

In conceptualization aspect, the pupils can relate mathematical concepts such as transformation of information into mathematical models as well as the concept of elimination or substitution. It is relevant to the researches by Handayani et al. (2020) & Ikram et al. (2020). The pupils are able to relate some concepts for problem solving despite the old concept. In addition, they do not only focused on solving problems, but also understanding the concepts of mathematical solutions (Annisavitri et al. 2020). Actually pupils have networks or connections between concepts (Sa’dijah et al. 2020). Connections or bridges between concepts get stronger if the problem solver makes a recall. Strengthening connections between concepts can be conducted by applying scaffolding (Zayyadi et al. 2020).

We successfully concluded that expert pupils perform the whole indicator of reflective thinking. Perplexity can be overcome by controlling the experience and knowledge possessed. The experience and knowledge possessed by expert pupils are honed so they can overcome perplexity with a relatively short time. Insight and monitoring by expert pupils are more meaningful because they can generate ideas that are solutive, effective, and efficient in unfamiliar problems. Equally important, moderate pupils perform the whole indicator of reflective thinking. Unfortunately, experience and knowledge in problem solving are less meaningful. Insight and monitoring do not provide the right problem solution. They need a lot of knowledge and experience to solve unfamiliar problems. Experience and knowledge cannot be recalled optimally. Novice pupils’ reflective thinking are at the lowest level. They need time to increase experience and knowledge. Moreover, they have no the passion to solve problems as well as do not care about the perplexity that arises. Thinking skills in problem solving is not an important thing for them.
Recommendations

In general, the pupils applied reflective thinking with different strengths. Based on the conclusion, it is good for teacher in stimulating pupils’ reflective thinking. Giving the treatments can be ICT learning tool usage, fresh learning model, and giving new experience for pupils. The teachers need to pay more attention to develop instrument in term of increasing pupils’ achievement.

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