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Theoretical and Experimental Investigations of a Jerk Circuit with Two Parallel Diodes

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ABSTRACT A Jerk circuit with two diodes mounted in parallel is formulated and analyzed in this paper. The system describing the proposed jerk circuit exhibits two or no equilibrium points as function of the system parameters. Studies on equilibrium points stability show the appearance of Hopf bifurcation. The proposed jerk circuit exhibits one scroll chaotic attractor and periodic attractors. An experimental study is presented to support theoretical investigations. The experimental results are shown consistency with numerical simulation results.

KEYWORDS
Jerk circuit,
Two parallel diodes,
Hopf bifurcation,
One scroll chaotic attractor,
Periodic attractor.

INTRODUCTION

The scientific community was surprised when Lorenz Lorenz (1963) discovered in 1963 a chaotic behavior in a simple system based on third order differential equations, with two quadratic non linearities. Lorenz had just discovered the phenomenon of sensitivity to initial conditions and systems possessing this behavior have been characterized by chaotic. Since then, the researchers have been devoting much attention to the famous systems exhibiting chaos as Chen's oscillator Chen and Ueta (1999) the Chua oscillator Chua *et al.* (1993), Rossler oscillator Rössler (1976), the Arneode oscillator Arneodo *et al.* (1981), the Lu oscillator Lü *et al.* (2002) and the Jerk oscillators Sprott (1997).

A huge amount of papers have been hallowed to research on simple systems that can generate chaotic behavior. Jerk Munmuangsaen *et al.* (2011) systems are among these simple systems that are easy to implement. It is a system of three dimensional equations described as:$\dddot{x} = J(\ddot{x}, \dot{x}, x)$ Where "J" refers to the term "Jerk" which is the derivative with respect to the time of the scalar variable x. From standpoint of the dynamic of solids, if the scalar variable is set to be the position of a mobile, then the third order derivative of the position represents the Jerk Schot (1978). Many researchers have mobilized their thoughts on Jerk systems. Sprott and his collaborators presented a Jerk system with exponential nonlinearity Sprott (2010). In 2012, Chunxia at al. Chunxia *et al.* (2012) generate multi-scroll attractors through a Jerk model. In 2013, Omur et al. Umut and Yasar (2013), study a jerk with quadratic nonlinearity initiated in Genesio and Tesi (1992) that present strange attractors.

Sambas and collaborators Sambas *et al.* (2016) brought some modifications on the system study by Omur et al. Umut and Yasar (2013) and they found some striking phenomena. Here, we linger on the Sambas and collaborators system with hyperbolic cosine as nonlinearity term. The reasons of this substitution are: Firstly, the system is simple and easy to implement (replacement of the multiplier by two diodes semiconductor) and secondly, to make it more complex (look for new and enriching phenomena). In our various studies, we have found that our new system can exhibit enriching phenomena such as the existence of multiple attractors.

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These chaotic phenomena are found in several areas of sciences, particularly in biology Djati (2011); in chemistry Nakajima and Sawada (1980); in economics Bouali *et al.* (2012) and even in the medical engineering Shinbrot *et al.* (1992) just to name a few. However, despite their unstable characters, these systems are today to an importance because they can be used in telecommunication for secure information's Sambas *et al.* (2013); in video monitoring Lian *et al.* (2008); in robotics Zang *et al.* (2016) and many other fields in engineering.

The next coming steps are organized as follows. Section 2 presents the description, analytical and numerical investigations of the proposed jerk circuit. Section 3 is devoted to the experimental investigation. Finally, a general conclusion and some remarks are given in section 4.

CONCEPTION AND ANALYSIS OF THE PROPOSED JERK CIRCUIT WITH TWO PARALLEL DIODES

Sambas and other researchers Sambas *et al.* (2016) recently proposed the jerk model with quadratic nonlinearity defined by:

$$\frac{dx_1}{dt} = x_2, \tag{1a}$$
$$\frac{dx_2}{dt} = kx_3, \tag{1b}$$
$$\frac{dx_3}{dt} = -cx_1 - bx_2 - ax_3 + \mu x_1^2, \tag{1c}$$

where t is the time, x_1, x_2 and x_3 are the state variables of system (1) and the parameters a, b, c, k, μ are the positive system parameters. It is important to note that deep studies conducted on this system have resulted in interesting phenomena. In this paper, the quadratic term in system (1) is substituted by a cosine hyperbolic nonlinearity. Therefore, system (1) becomes:

$$\frac{dx_1}{dt} = x_2, \tag{2a}$$
$$\frac{dx_2}{dt} = kx_3, \tag{2b}$$
$$\frac{dx_3}{dt} = -cx_1 - bx_2 - ax_3 + \mu cosh(x_1), \tag{2c}$$

The cosine hyperbolic nonlinearity of system (2) is easily realized with two semiconductor diodes mounted in parallel as indicated in Fig.1.

The schematic circuit of system (2) of consists of six operational amplifiers, twelve resistors, three capacitors and two semiconductor diodes Fig. 1 mounted in parallel. The jerk circuit of Fig. 1, is easy to implement and less expensive (substitution of the multiplier by two diodes).

System (2) is invariant under any transformation $(x_1, x_2, x_3) \leftrightarrow (-x_1, -x_2, -x_3)$ It is also disspative because $\frac{\partial \dot{x}_1}{\partial x_1} + \frac{\partial \dot{x}_2}{\partial x_2} + \frac{\partial \dot{x}_3}{\partial x_3} = -a < 0$ The equilibrium points of system (2) can be derived by setting $\frac{dx_1}{dt} = \frac{dx_2}{dt} = \frac{dx_3}{dt} = 0$. Resolution lead us to:

$$x_2^* = x_3^* = 0, \tag{3a}$$
$$-cx_1^* + \mu \cosh(x_1^*) = 0 \tag{3b}$$

Analytical resolution of Eq. (3b) is impossible but by using the Newton-Raphson method we can get the value of x_1^*. Equation (3b) has no roots or two or four roots as function of the value of parameters $(\mu,\ c)$ as shown in Fig. 2 (a). At equilibrium point $E = \left(\ x_1^*,\ 0,\ 0\right)$ evaluation of characteristic equation of system (2) give:

$$\lambda^3 + a\lambda^2 + kb\lambda - k\left[-c + \mu \sinh\left(x_1^*\right)\right] = 0. \tag{4}$$

It can be obviously verified via Routh-Hurwitz stability criterion, that the real parts of eigenvalues are less than zero if:

$$c - \mu \sinh\left(x_1^*\right) > 0, \tag{5a}$$
$$ab - c + \mu \sinh\left(x_1^*\right) > 0. \tag{5b}$$

because $a > 0$ and $k > 0$. The stability analysis of equilibrium points $E = \left(x_1^*, 0, 0\right)$ versus the parameter c is shown in Fig. 2 (b).

Figure 2 (a) reveals that system (2) has two or no equilibrium points regions as function of the parameters μ and m. One of the equilibrium points E_1 is always unstable (see Fig. 2 (b)). For $c \leq 3.352$, the other equilibrium point E_2 is stable while for $c > 3.352$, it is unstable as shown in Fig. 2 (b). So system (1) can exhibit a Hopf or transcritical bifurcation at the equilibrium point E_2 by varying the parameter c.

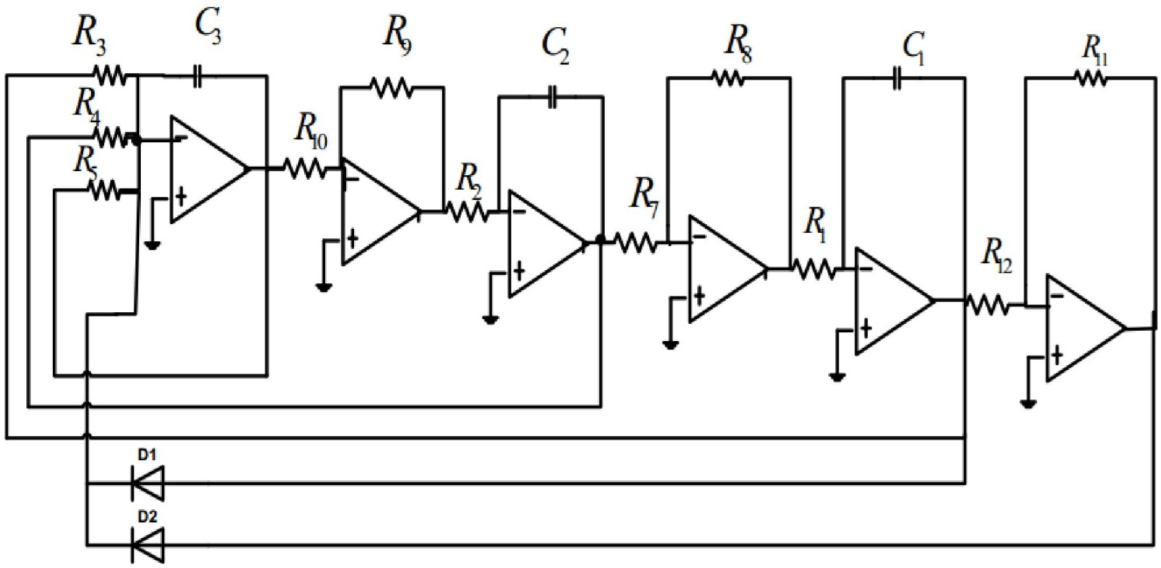

Figure 1 Electronic circuit of system (2).

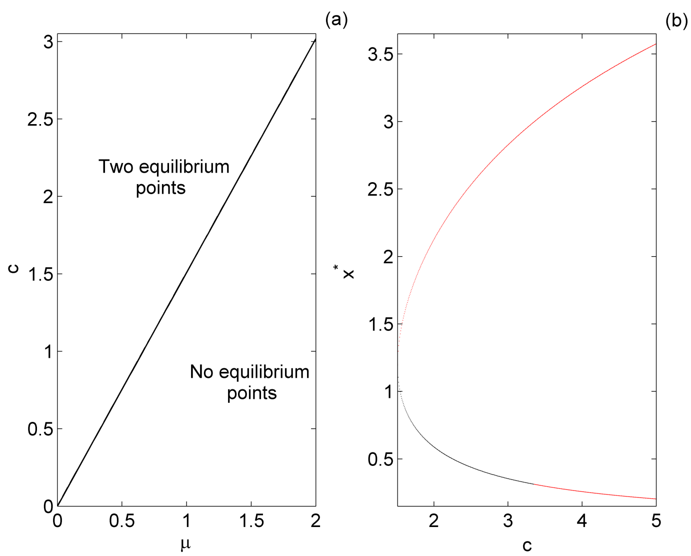

Figure 2 (Color online) (a) Distribution of equilibrium points of system (2) versus the parameters μ and c and (b) stability diagram of equilibrium points $E = (x_1*, 0, 0)$ as function of the parameter c for $\mu = 1.0$. In panels (b), red lines indicate unstable branches and solid black lines stable branches. The remaining parameters are $\alpha = 1.0$and $b = 3.03$.

Theorem: System (2) exhibits a Hopf bifurcation at the equilibrium point located E_2 for $bk > 0$ and the parameter α passes through the value $c_H = -ab + \mu sinh(x_1*)$.

Proof: By substituting $\lambda = i\omega \,(\omega > 0)$ into Eq. (4), we obtain

$$\omega = \omega_0 = \sqrt{bk}, \tag{6a}$$
$$c = c_H = -ab + \mu \sinh\left(x_1^*\right). \tag{6b}$$

By differentiating both sides of Eq. (4) with respect to c, it is obtained

$$3\lambda^2 \frac{d\lambda}{dc} + 2a\lambda \frac{d\lambda}{dc} + bk \frac{d\lambda}{dc} + k = 0 \tag{7a}$$
$$\frac{d\lambda}{dc} = \frac{-k}{3\lambda^2 + 2a\lambda + bk}. \tag{7b}$$

then

$$\mathrm{Re}\left(\left. \frac{d\lambda}{dc} \right|_{c=c_H, \lambda = i\omega_0} \right) = \frac{k}{2\left(bk + a^2\right)} \neq 0. \tag{8}$$

Therefore the conditions for Hopf bifurcation to happen are fulfilled. System (2) exhibits a Hopf bifurcation at E_2 when $c_H = -ab + \mu \sinh\left(x_1^*\right)$ and periodic solutions exit in the vicinity of the point c_H (for $bk > 0$). If $\alpha = 1.0$, $b = 3.03$ and $k = 2.0$, the critical value is $c = c_H \approx 3.352$ and the time traces and the phase portraits of system (2) for two values of c around $c_H \approx 3.352$.

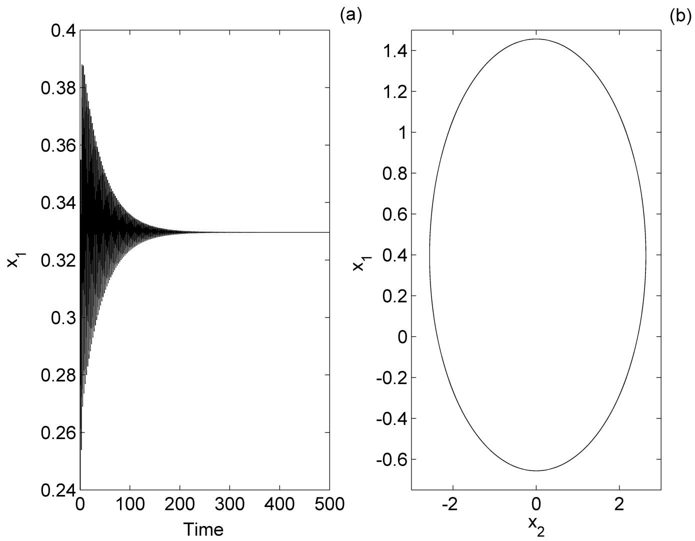

Figure 3 The time traces and the phase portraits of system (2) for two values of c around $c_H \approx 3.352$: (a) $c = 3.2 < c_H$ and $c = 3.2 > c_H$. The initials conditions are $x_1(0) = 0.1$, $x_2(0) = 0.1$ and $x_3(0) = 0.1$. The others parameters are $a = 1.0, b = 3.03, k = 2.0$ and $\mu = 1$.

In Fig. 3 (a) for $c = 3.2 < c_H$, the trajectories of system (2) converge to the equilibrium point E_2 whereas for $c = 3.2 > c_H$, , system (2) displays a perid-1-oscillation as shown in Fig. 3 (b). The appearance of the Hopf bifurcation is independent on the parameter k as shown in Eq. (6b). The bifurcation diagrams of $x_1(t)$ versus the parameter c for $\alpha = 1.0, b = 3.03, \mu = 1$ and three different values of the parameter k is show in Fig. 4 in order to confirm the analytical results presented in Eq. (6b).

In Fig. 4, limit cycle is exhibited up to $c = c_H = -ab + \mu \sinh\left(x_1^*\right) \approx 3.352$ where a Hopf bifurcation occurs followed by the convergence of the trajectories of system (2) to the line equilibrium point E_2. The analysis of the behavior of system (2) can be developed by plotting the bifurcation diagram and largest Lyaponov exponent (LLE) as function of the parameter c as illustrated in Fig. 5.

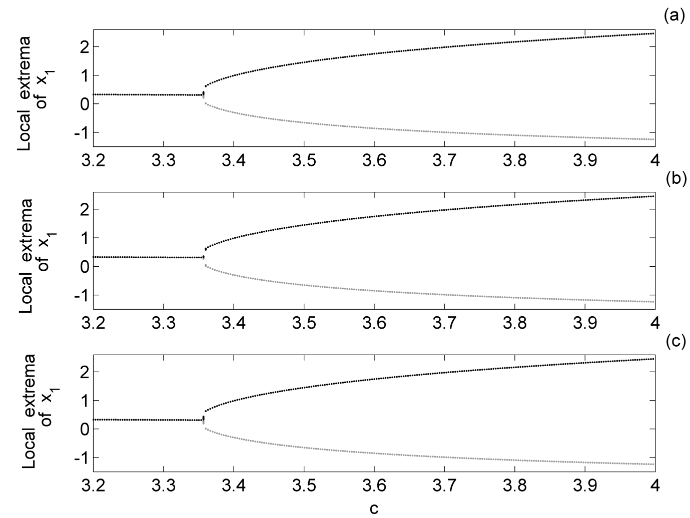

Figure 4 The bifurcation chart showing maxima (black dots) and minima (gray dots) of $x_1(t)$ versus the parameter c for different values of parameter k: (a) $k = 2$,(b) $k = 8$ and (c) $k = 15$. The others parameters are for $\alpha = 1.0, b = 3.03$ and $\mu = 1$.

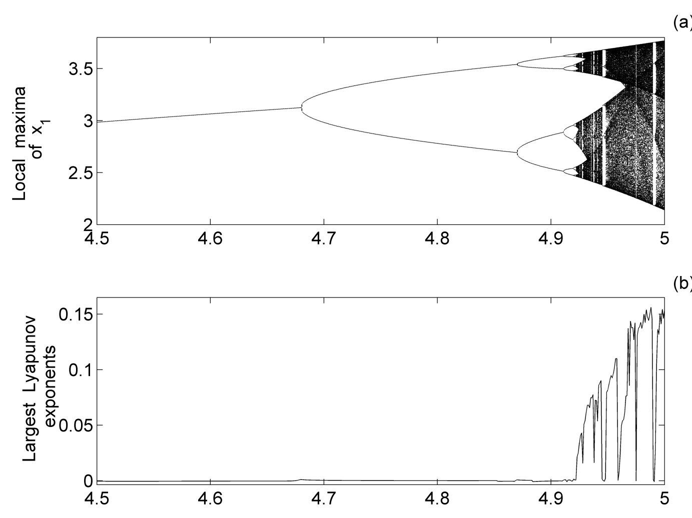

Figure 5 The bifurcation chart showing the local maxima of $x_1(t)$ (a) and the corresponding LLE (b) as function of the parameter c and $a = 1.0, b = 3.03, k = 2.0$ and $\mu = 1$

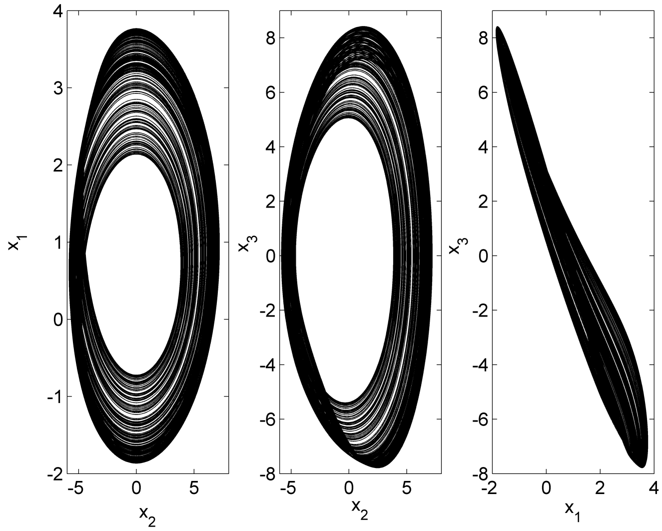

Figure 6 Numerical phase portraits of the chaotic attractors of system (2) for $c = 5$ and using initials conditions $x_1(0) = 0.1$, $x_2(0) = 0.1$ and $x_3(0) = 0.1$. The others parameters are $\alpha = 1.0$, $b = 3.03$, $k = 2.0$ and $\mu = 1$

A period-doubling route to chaos interspersed with periodic regions is observed in Fig. 5 (a). The LLE of Fig. 5 (b) confirms the dynamical behaviors obtained in Fig. 5 (a). The phase portraits in different planes to illustrating the chaotic behavior of system (2) are depicted in Fig. 6 for $c = 5$. One-scroll chaotic attractor is shown in Fig. 6.

ELECTRONIC REALIZATION OF THE OF THE PROPOSED JERK CIRCUIT WITH TWO PARALLEL DIODES

The electronic circuit of Fig.1 describing by system (2) is implemented as shown in Fig. 7.

The circuit of Fig. 7 is made of resistors, capacitors, operational amplifiers (TL084) and two diodes (1N4148) represent the nonlinearity element. The three variables (x_1, x_2, x_3) are respectively represented by the voltages across the capacitors C_1, C_2 and C_3 and the non-linearity is represented by two diodes mounted in parallel. The values of the circuit components are fixed as follow: $R_1 = R_5 = R_6 = R_7 = R_8 = R_9 = R_{10} = R_{11} = 100k\Omega$; $R_2 = 50k\Omega$, $R_3 = 50k\Omega$, $R_4 = 26.5k\Omega$, $C_1 = C_2 = C_2 = 1nF$. The phase portraits obtained from the oscilloscope are illustrated in Fig. 8.

The experimental results of Fig. 8 agree qualitatively with the numerical simulations results of Fig. 6.

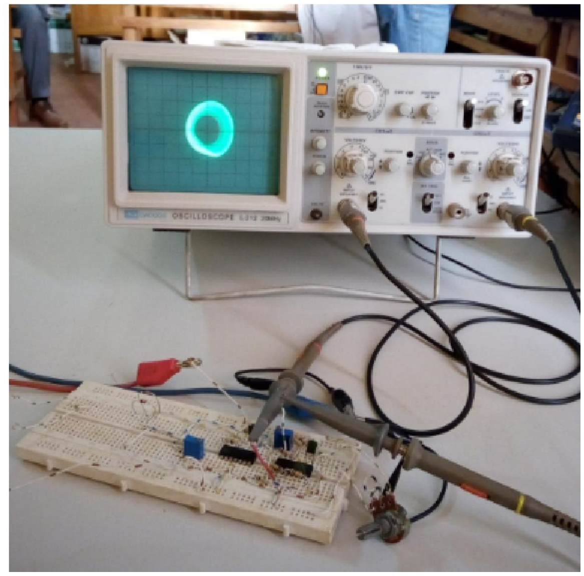

Figure 7 Image representing the experimental device of the jerk oscillator with hyperbolic cosine nonlinearity powered on $\pm$ 12V visualized on the oscilloscope.

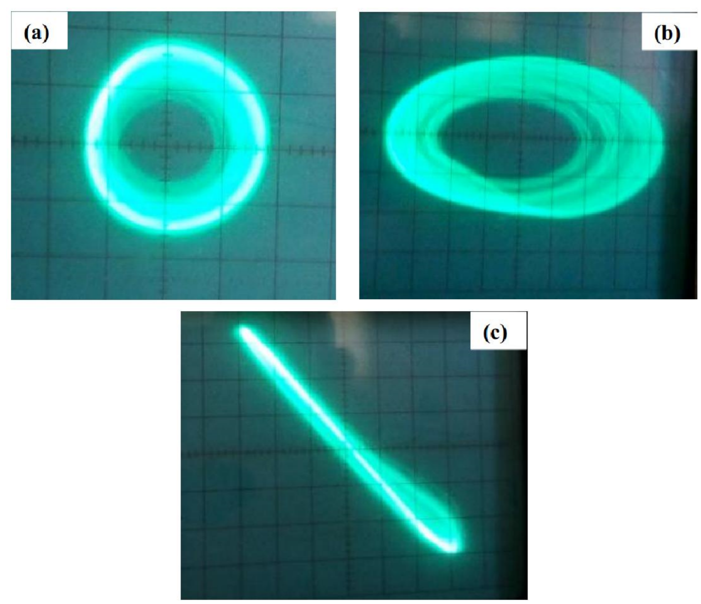

Figure 8 Phase portraits of chaotic behavior of the jerk system observed from oscilloscope for $R_3 = 26.3k\Omega$: (a): $V(x_2) - V(x_1)$ plane; (b): $V(x_2) - V(x_3)$ plane; (c): $V(x_1) - V(x_3)$ plane.

CONCLUSION

This paper reported on analytical, numerical and experimental studies of an introduced jerk with two diodes mounted in parallel. The presence of Hopf Bifurcation in the proposed jerk circuit was established. The numerical analysis of the proposed jerk circuit was revealed that it exhibit periodic attractors and one scroll chaotic attractor. The experimental study was done to verify the accuracy of the numerical results simulations obtained. The experimental results agree well with the numerical simulations.

Conflicts of interest

The corresponding author on behalf the authors declares that there is no conflict of interest in this research article.

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