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ON FRONTIER AND EXTERIOR IN INTUITIONISTIC SUPRA $\alpha-$ CLOSED SET

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Abstract. The main aim of the study of this paper is to work with the properties of frontier and exterior in intuitionistic supra topological spaces. Considering this we have introduced intuitionistic supra α-frontier and intuitionistic α-exterior in intuitionistic supra topological space. We have also deliberated the properties of intuitionistic suppra α-frontier and intuitionistic supra α-exterior in intuitionistic supra topological space. The comparative study has been done with the use of intuitionistic supra α-open set between Intuitionistic supra frontier, Intuitionistic supra exterior and intuitionistic supra α-frontier, intuitionistic α-exterior in intuitionistic supra topological space.

1. Introduction

In 1970, Levine[4] introduced the concept of generalized closed sets in topological spaces. Njastad.O[12] and Maki.H et al[6] introduced α-closed sets and gα-closed sets in topological spaces. In 1965 ,O.Njastad[12] introduced α-open sets. The concept of intuitionistic set and intuitionistic topological spaces was introduced by Coker[1][2]. Supra topology was introduced by A.S.Mashhour et.al[6] Intuitionistic supra α-open set was introduced by the Author[8] on intuitionistic supra topological spaces and discussed the properties of Intuitionistic supra α-open sets in supra topological spaces.

The purpose of this paper is to study the properties of α-frontier and α-exterior in intuitionistic supra topological spaces. Also to study the comparison between Intuitionistic supra frontier, Intuitionistic supra exterior and intuitionistic supra α-frontier, α-exterior in intuitionistic supra topological space.

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2. Preliminaries

Definition 2.1 [1] Let X be a non-empty set,an intuitionistic set (IS in short) A is an object having the form $A = \langle X, A_1, A_2 \rangle$, where A_1 and A_2 are subsets of X satisfying $A_1 \cap A_2 = \phi$. The set A_1 is called the set members of A, while A_2 is called the set of non-members of A.

Definition 2.2 [1] Let X be a non-empty set, $A = \langle X, A_1, A_2 \rangle$ and $B = \langle X, B_1, B_2 \rangle$ be IS's on X and let $\{A_i : i \in J\}$ be an arbitrary family of IS's in X, where $A_i = \left\langle X, A_i^{(1)}, A_i^{(2)} \right\rangle$. Then

(i) $A \subseteq B$ iff $A_1 \subseteq B_1$ and $A_2 \supseteq B_2$.
(ii) $A = B$ iff $A \subseteq B$ and $B \subseteq A$.
(iii) $\bar{A} = \langle X, A_2, A_1 \rangle$.
(iv) $A \cup B = \langle X, A_1 \cup B_1, A_2 \cap B_2 \rangle$.
(v) $A \cap B = \langle X, A_1 \cap B_1, A_2 \cup B_2 \rangle$.
(vi) $\bigcup A_i = \langle X, \bigcup A_i^1, \bigcap A_i^2 \rangle$.
(vii) $\bigcap A_i = \langle X, \bigcap A_i^1, \bigcup A_i^2 \rangle$.
(viii) $A\text{-}B = A \cap \bar{B}$.
(ix) $[]\, A = \langle X, A_1, (A_1)^c \rangle$.
(x) $\langle\rangle\, A = \langle X, (A_2)^c, A_2 \rangle$.
(xi) $\underset{\sim}{X} = \langle X, X, \phi \rangle$.
(xii) $\underset{\sim}{\phi} = \langle X, \phi, X \rangle$.

Definition 2.3 [6] An intuitionistic topology on a non-empty set X is a family τ of IS's in X satifying the following axioms:

(i) $\underset{\sim}{X}, \underset{\sim}{\phi} \in \tau$.
(ii) $A_1 \cap A_2 \in \tau$ for any $A_1, A_2 \in \tau$.
(iii) $\cup A_i \in \tau$ for any arbitrary family $\{A_i : i \in J\} \subseteq \tau$.

The pair (X, τ) is called an intuitionistic topological space (ITS in short) and IS in τ is known as an intuitionitic open set (IOS in short) in X, the complement of IOS is called an intuitionistic closed set (ICS in short).

Definition 2.4 [6] The supra closure of a set A is denoted by $cl^{\mu}(A)$, and is defined as,
supra $cl(A) = \bigcap \{B : B \quad is \;\; supra \;\; closed \;\; and \;\; A \subseteq B\}$.
The supra interier of a set A is denoted by $int^{\mu}(A)$, and is defined as
supra $int(A) = \bigcup \{B : B \quad is \;\; supra \;\; open \;\; and \;\; A \supseteq B\}$.

Definition 2.5 [1] An Intuitionistic supra topology on a non-empty set X is a family τ of IS's in X satisfying the following axioms:

(i) $\underset{\sim}{X}, \underset{\sim}{\phi} \in \tau$.
(ii) $\cup A_i \in \tau$ for any arbitrary family $\{A_i : i \in J\} \subseteq \tau$.

The pair(X,τ) is called intuitionistic supra topological space (ISTS in short) and IS in τ is known as an intuitionistic supra open set (ISOS in short) in X, the complement of ISOS is called intuitionistic supra closed set(ISCS in short).

Definition 2.6 [1] Let (X,τ) be an ISTS and let $A = \langle X, A_1, A_2 \rangle$ be an IS in X, then the supra closure and supra interior of A are defined by:
$cl^{\mu}(A) = \bigcap \{K : K \text{ is an } ISCS \text{ in } X \text{ and } A \subseteq K\}$.
$int^{\mu}(A) = \bigcup \{K : K \text{ is an } ISOS \text{ in } X \text{ and } A \supseteq K\}$.

Definition 2.7 [8] Let (X,τ) be an ISTS and let $A = \langle X, A_1, A_2 \rangle$ be an IS in X , then the supra α closure and supra α interior of A are defined by:
$I\alpha cl^{\mu}(A) = \bigcap \{k : k \text{ is an } IS\alpha CS \text{ in } X \text{ and } A \subseteq k\}$
$I\alpha int^{\mu}(A) = \bigcup \{k : k \text{ is an } IS\alpha OS \text{ in } X \text{ and } A \supseteq k\}$

Definition 2.8 [1] Let (X,τ) be an intuitionistic supra topological space. An intuitionistic set A is called intuitionistic supra α-closed set (ISαCS in short) if $cl^{\mu}(int^{\mu}(cl^{\mu}(A))) \subseteq$ U, whenever A $\subseteq$U, U is intuitionistic supra α-open set (ICαOS).
The complement of intuitionistic supra α-closed set is intuitionistic supra α-open set (ISαOS in short).

3. Intuitionistic supra Frontier

Definition 3.1 Let X be an ISTS and for a subset A of a ISTS X, $IFr^{\mu}(A) = Icl^{\mu}(A) - Iint^{\mu}(A)$ is said to be Intuitionistic supra Frontier of A.

Theorem 3.2 Let X be an ISTS then and for any a subset A of IS in ISTS X , the following statements hold:

(i) $IFr^{\mu}(A) = Icl^{\mu}(A) \cap Icl^{\mu}(X - A)$.
(ii) $IFr^{\mu}(A) = IFr^{\mu}(X - A)$.
(iii) $IFr^{\mu}(IFr^{\mu}(A)) \subseteq IFr^{\mu}(A)$.
(iv) $Icl^{\mu}(A) = Iint^{\mu}(A) \cup IFr^{\mu}(A)$.
(v) $Iint^{\mu}(A) \cap IFr^{\mu}(A)$=$\phi$.
(vi) $IFr^{\mu}(\underset{\sim}{X}) = \underset{\sim}{\phi}$,$IFr^{\mu}(\underset{\sim}{\phi}) = \underset{\sim}{X}$.
(vii) $IFr^{\mu}(Icl^{\mu}(A)) \subseteq IFr^{\mu}(A)$.

Proof. Let A be a IS in ISTS X.

(i) $IFr^{\mu}(A) = Icl^{\mu}(A) - Iint^{\mu}(A)$=$Icl^{\mu}(A) \cap Icl^{\mu}(X - A)$.
(ii) $IFr^{\mu}(A) = Icl^{\mu}(A) - Iint^{\mu}(A)$=$(X - Iint^{\mu}(A)) - (X - Icl^{\mu}(A))$=$Icl^{\mu}(X - A) - Iint^{\mu}(X - A)$=$IFr^{\mu}(X - A)$.

(iii) $IFr^{\mu}(IFr^{\mu}(A))=Icl^{\mu}(IFr^{\mu}(A))\cap Icl^{\mu}(X-IFr^{\mu}(A))\subseteq Icl^{\mu}(IFr^{\mu}(A))=IFr^{\mu}(X-A)$. Hence $IFr^{\mu}(IFr^{\mu}(A))\subseteq IFr^{\mu}(A)$.
(iv) $Iint^{\mu}(A)\cup IFr^{\mu}(A)=Iint^{\mu}(A)\cup(Icl^{\mu}(A)-Iint^{\mu}(A))=(Iint^{\mu}(A)\cup Icl^{\mu}(A))-(Iint^{\mu}(A)\cup Iint^{\mu}(A))=(Iint^{\mu}(A)\cup Icl^{\mu}(A))-Iint^{\mu}(A)=Icl^{\mu}(A)$.
(v) $Iint^{\mu}(A)\cap IFr^{\mu}(A)=Iint^{\mu}(A)\cap(Icl^{\mu}(A)-Iint^{\mu}(A))=\underset{\sim}{\phi}$.
(vi) $IFr^{\mu}(\underset{\sim}{X})=\underset{\sim}{\phi}, IFr^{\mu}(\underset{\sim}{\phi})=\underset{\sim}{X}$.
(vii) $IFr^{\mu}(Icl^{\mu}(A))=Icl^{\mu}(Icl^{\mu}(A))-Iint^{\mu}(Icl^{\mu}(A))\subseteq Icl^{\mu}(A)-Iint^{\mu}(A)=IFr^{\mu}(A)$. Hence $IFr^{\mu}(Icl^{\mu}(A))\subseteq IFr^{\mu}(A)$.

□

The proof of the above theorem is shown in the following example:

Example 3.3 Let $X=\{a,b,c\}$. $\tau=\left\{\underset{\sim}{X},\underset{\sim}{\phi},A_1,A_2,A_3\right\}$, where $A_1=\langle X,\{a\},\{b,c\}\rangle$, $A_2=\langle X,\{b\},\{c\}\rangle$ and $A_3=\langle X,\{a,b\},\{c\}\rangle$.
Let $A=\langle X,\{a\},\{c\}\rangle$. $X-A=\langle X,\{c\},\{a\}\rangle$. $Iint^{\mu}(A)=\langle X,\{a\},\{b,c\}\rangle$. $Icl^{\mu}(A)=\underset{\sim}{X}$. $IFr^{\mu}(A)=\langle X,\{b,c\},\{a\}\rangle$.
$Iint^{\mu}(X-A)=\underset{\sim}{\phi}$. $Icl^{\mu}(X-A)=\langle X,\{b,c\},\{a\}\rangle.IFr^{\mu}(X-A)=\langle X,\{b,c\},\{a\}\rangle$.

(i) $IFr^{\mu}(A)=Icl^{\mu}(A)-Iint^{\mu}(A)=\langle X,\{b,c\},\{a\}\rangle$ and $Icl^{\mu}(A)\cap Icl^{\mu}(X-A)=\langle X,\{b,c\},\{a\}\rangle$.
(ii) $IFr^{\mu}(A)=\langle X,\{b,c\},\{a\}\rangle$ and $IFr^{\mu}(X-A)=\langle X,\{b,c\},\{a\}\rangle$.
(iii) $IFr^{\mu}(A)=\langle X,\{b,c\},\{a\}\rangle$. $IFr^{\mu}(IFr^{\mu}(A))=\langle X,\{b,c\},\{a\}\rangle$.Hence $IFr^{\mu}(IFr^{\mu}(A))\subseteq IFr^{\mu}(A)$.
(iv) $Iint^{\mu}(A)\cup IFr^{\mu}(A)=\underset{\sim}{X}.cl^{\mu}(A)=\underset{\sim}{X}$.Hence $Icl^{\mu}(A)=Iint^{\mu}(A)\cup IFr^{\mu}(A)$.
(v) $Iint^{\mu}(A)\cap IFr^{\mu}(A)=\underset{\sim}{\phi}$.
(vi) $IFr^{\mu}(\underset{\sim}{X})=\underset{\sim}{\phi}, IFr^{\mu}(\underset{\sim}{\phi})=\underset{\sim}{X}$.
(vii) $IFr^{\mu}(Icl^{\mu}(A))=\langle X,\{b,c\},\{a\}\rangle.IFr^{\mu}(A)=\langle X,\{b,c\},\{a\}\rangle$.Hence $IFr^{\mu}(Icl^{\mu}(A))\subseteq IFr^{\mu}(A)$.

Definition 3.4 Let X be an ISTS and for a subset A of a ISTS, $I\alpha Fr^{\mu}(A)=I\alpha cl^{\mu}(A)-I\alpha int^{\mu}(A)$ said to be Intuitionistic supra α-Frontier of A.

Theorem 3.5 For a subset A of ISTS, I$\alpha Fr^{\mu}(A)\subseteq IFr^{\mu}(A)$.

proof Let x $\in I\alpha Fr^{\mu}(A)$ then x $\in I\alpha cl^{\mu}(A)-I\alpha int^{\mu}(A)$, implies x $\in Icl^{\mu}(A)-Iint^{\mu}(A)$, since every intuitionistic supra closed set is intuitionistic supra α-closed set. Hence x $\in IFr^{\mu}(A)$. Therefore $I\alpha Fr^{\mu}(A)\subseteq IFr^{\mu}(A)$.
Converse of the above theorem need not be true. It is shown in the following example.

Example 3.6 Let $X=\{a,b,c\}$. $\tau=\left\{\underset{\sim}{X},\underset{\sim}{\phi},A_1,A_2,A_3\right\}$, where$A_1=\langle X,\{a\},\{b,c\}\rangle$. $A_2=\langle X,\{b\},\{c\}\rangle$ and $A_3=\langle X,\{a,b\},\{c\}\rangle$.

Let $A = \langle X, \{\phi\}, \{a\}\rangle$, $IFr^{\mu}(A) = \langle X, \{b, c\}, \{a\}\rangle$, and $I\alpha Fr^{\mu}(A) = \langle X, \phi\ \{a\}\rangle$. Here $I\alpha Fr^{\mu}(A) \subseteq IFr^{\mu}(A)$ is true but converse is not true.

Theorem 3.7 Let X be an ISTS then and for any a subset A of IS in ISTS X, the following statements holds:

(i) $I\alpha Fr^{\mu}(A) = I\alpha cl^{\mu}(A) \cap I\alpha cl^{\mu}(X - A)$.
(ii) $I\alpha Fr^{\mu}(A) = I\alpha Fr^{\mu}(X - A)$.
(iii) $I\alpha Fr^{\mu}(I\alpha Fr^{\mu}(A)) \subseteq I\alpha Fr^{\mu}(A)$.
(iv) $I\alpha cl^{\mu}(A) = I\alpha int^{\mu}(A) \cup I\alpha Fr^{\mu}(A)$.
(v) $I\alpha int^{\mu}(A) \cap I\alpha Fr^{\mu}(A) = \underset{\sim}{\phi}$.
(vi) $I\alpha Fr^{\mu}(\underset{\sim}{X}) = \underset{\sim}{\phi}$, $I\alpha Fr^{\mu}(\underset{\sim}{\phi}) = \underset{\sim}{X}$.
(vii) $I\alpha Fr^{\mu}(I\alpha cl^{\mu}(A)) \subseteq I\alpha Fr^{\mu}(A)$.

Proof

(i) $I\alpha Fr^{\mu}(A) = I\alpha cl^{\mu}(A) - I\alpha int^{\mu}(A) = I\alpha cl^{\mu}(A) \cap I\alpha cl^{\mu}(X - A)$.
(ii) $I\alpha Fr^{\mu}(A) = I\alpha cl^{\mu}(A) - I\alpha int^{\mu}(A) = (X - I\alpha int^{\mu}(A)) - (X - I\alpha cl^{\mu}(A)) = I\alpha cl^{\mu}(X - A) - I\alpha int^{\mu}(X - A) = I\alpha Fr^{\mu}(X - A)$.
(iii) $I\alpha Fr^{\mu}(I\alpha Fr^{\mu}(A)) = I\alpha cl^{\mu}(I\alpha Fr^{\mu}(A)) \cap I\alpha cl^{\mu}(X - I\alpha Fr^{\mu}(A)) \subseteq I\alpha cl^{\mu} I\alpha Fr^{\mu}(A)) = I\alpha Fr^{\mu}(X - A)$. Hence $I\alpha Fr^{\mu}(I\alpha Fr^{\mu}(A)) \subseteq I\alpha Fr^{\mu}(A)$.
(iv) $I\alpha int^{\mu}(A) \cup I\alpha Fr^{\mu}(A) = I\alpha int^{\mu}(A) \cup (I\alpha cl^{\mu}(A) - I\alpha int^{\mu}(A)) = (I\alpha int^{\mu}(A) \cup I\alpha cl^{\mu}(A)) - (I\alpha int^{\mu}(A) \cup I\alpha int^{\mu}(A)) = (I\alpha int^{\mu}(A) \cup I\alpha cl^{\mu}(A)) - I\alpha int^{\mu}(A) = I\alpha cl^{\mu}(A)$.
(v) $I\alpha int^{\mu}(A) \cap I\alpha Fr^{\mu}(A) = I\alpha int^{\mu}(A) \cap (I\alpha cl^{\mu}(A) - I\alpha int^{\mu}(A)) = \underset{\sim}{\phi}$.
(vi) $I\alpha Fr^{\mu}(\underset{\sim}{X}) = \underset{\sim}{\phi}, I\alpha Fr^{\mu}(\underset{\sim}{\phi}) = \underset{\sim}{X}$.
(vii) $I\alpha Fr^{\mu}(I\alpha cl^{\mu}(A)) = I\alpha cl^{\mu}(I\alpha cl^{\mu}(A)) - I\alpha int^{\mu}(I\alpha cl^{\mu}(A)) \subseteq I\alpha cl^{\mu}(A) - I\alpha int^{\mu}(A) = I\alpha Fr^{\mu}(A)$. Hence $I\alpha Fr^{\mu}(I\alpha cl^{\mu}(A)) \subseteq I\alpha Fr^{\mu}(A)$.

Example 3.8 Let $X = \{a, b, c\}$. $\tau = \left\{\underset{\sim}{X}, \underset{\sim}{\phi}, A_1, A_2, A_3\right\}$, where $A_1 = \langle X, \{a\}, \{b, c\}\rangle$, $A_2 = \langle X, \{c\}, \{a, b\}\rangle$, and $A_3 = \langle X, \{a, c\}, \{b\}\rangle$.
Let $A = \langle X, \{b\}, \{a, c\}\rangle$. $X - A = \langle X, \{a, c\}, \{b\}\rangle$. $I\alpha int^{\mu}(A) = \underset{\sim}{\phi}$, $I\alpha cl^{\mu}(A) = \langle X, \{b\}, \{a, c\}\rangle$. $I\alpha Fr^{\mu}(A) = \langle X, \{b\}, \{a, c\}\rangle$.
$I\alpha int^{\mu}(X - A) = \langle X, \{a, c\}, \{b\}\rangle$. $I\alpha cl^{\mu}(X - A) = \langle X, \{a, c\}, \{b\}\rangle$. $I\alpha Fr^{\mu}(X - A) = \langle X, \{a, c\}, \{b\}\rangle$.

(i) $I\alpha Fr^{\mu}(A) = I\alpha cl^{\mu}(A) - I\alpha int^{\mu}(A) = \langle X, \{b\}, \{a, c\}\rangle$, , and $I\alpha cl^{\mu}(A) \cap I\alpha cl^{\mu}(X - A) = \langle X, \{b\}, \{a, c\}\rangle$.
(ii) $I\alpha Fr^{\mu}(A) = \langle X, \{b\}, \{a, c\}\rangle$ and $I\alpha Fr^{\mu}(X - A) = \langle X, \{b\}, \{a, c\}\rangle$.
(iii) $I\alpha Fr^{\mu}(A) = \langle X, \{b\}, \{a, c\}\rangle$. $I\alpha Fr^{\mu}(I\alpha Fr^{\mu}(A)) = \langle X, \{b\}, \{a, c\}\rangle$. Hence $I\alpha Fr^{\mu}(I\alpha Fr^{\mu}(A)) \subseteq I\alpha Fr^{\mu}(A)$.
(iv) $I\alpha int^{\mu}(A) \cup I\alpha Fr^{\mu}(A) = \langle X, \{b\}, \{a, c\}\rangle . I\alpha cl^{\mu}(A) = \langle X, \{b\}, \{a, c\}\rangle$. Hence $Icl^{\mu}(A) = I\alpha int^{\mu}(A) \cup I\alpha Fr^{\mu}(A)$.

(v) $I\alpha int^{\mu}(A) \cap I\alpha Fr^{\mu}(A) = \underset{\sim}{\phi}$.

(vi) $I\alpha Fr^{\mu}(\underset{\sim}{X}) = \underset{\sim}{\phi}, I\alpha Fr^{\mu}(\underset{\sim}{\phi}) = \underset{\sim}{X}$.

(vii) $I\alpha Fr^{\mu}(I\alpha cl^{\mu}(A)) = \langle X, \{b\}, \{a,c\}\rangle$. Hence $I\alpha Fr^{\mu}(I\alpha cl^{\mu}(A)) \subseteq I\alpha Fr^{\mu}(A)$.

4. Intuitionistic supra Exterior

Definition 4.1 Let X be an ISTS and for a subset A of a ISTS X, $IExt^{\mu}(A) = Iint^{\mu}(\underset{\sim}{X} - A)$ s said to be Intuitionistic supra Exterior of A.

Theorem 4.2 Let X be an ISTS then and for any a subset A of IS in ISTS X, the following statements hold:

(i) $IExt^{\mu}(A) = \underset{\sim}{X} - Icl^{\mu}(A)$.

(ii) $IExt^{\mu}(IExt^{\mu}(A)) = Iint^{\mu}(Icl^{\mu}(A)) \supseteq Iint^{\mu}(A)$.

(iii) $A \subseteq B = IExt^{\mu}(B) \subseteq IExt^{\mu}(A)$.

(iv) $IExt^{\mu}(A \cup B) = IExt^{\mu}(A) \cap IExt^{\mu}(B)$.

(v) $IExt^{\mu}(A \cap B) = IExt^{\mu}(A) \cup IExt^{\mu}(B)$.

(vi) $IExt^{\mu}(\underset{\sim}{X}) = \underset{\sim}{\phi}, IExt^{\mu}(\underset{\sim}{\phi}) = \underset{\sim}{X}$.

(vii) $IExt^{\mu}(A) = IExt^{\mu}(\underset{\sim}{X} - IExt^{\mu}(A))$.

Proof

(i) $IExt^{\mu}(A) = Iint^{\mu}(\underset{\sim}{X} - A) = \underset{\sim}{X} - Icl^{\mu}(A)$.

(ii) $IExt^{\mu}(IExt^{\mu}(A)) = Iint^{\mu}(\underset{\sim}{X} - (IExt^{\mu}(A)) = Iint^{\mu}(Icl^{\mu}(A)) \supseteq Iint^{\mu}(A)$.

(iii) $A \subseteq B \Longrightarrow Iint^{\mu}(A) \subseteq Iint^{\mu}(B)$.
$IExt^{\mu}(B) = Iint^{\mu}(\underset{\sim}{X} - B) \subseteq Iint^{\mu}(\underset{\sim}{X} - A) = IExt^{\mu}(A) \Longrightarrow IExt^{\mu}(B) \subseteq IExt^{\mu}(A)$.

(iv) $IExt^{\mu}(A \cup B) = Iint^{\mu}(\underset{\sim}{X} - (A \cup B)) = Iint^{\mu}((\underset{\sim}{X} - A) \cap (\underset{\sim}{X} - B)) \subseteq Iint^{\mu}(\underset{\sim}{X} - A) \cap Iint^{\mu}(\underset{\sim}{X} - B) = IExt^{\mu}(A) \cap IExt^{\mu}(B)$.
Hence $IExt^{\mu}(A \cup B) = IExt^{\mu}(A) \cap IExt^{\mu}(B)$.

(v) $IExt^{\mu}(A \cap B) = Iint^{\mu}(\underset{\sim}{X} - (A \cap B)) = Iint^{\mu}((\underset{\sim}{X} - A) \cup (\underset{\sim}{X} - B)) \supseteq Iint^{\mu}(\underset{\sim}{X} - A) \cup Iint^{\mu}(\underset{\sim}{X} - B) = IExt^{\mu}(A) \cup IExt^{\mu}(B)$.
Hence $IExt^{\mu}(A \cap B) = IExt^{\mu}(A) \cup IExt^{\mu}(B)$.

(vi) $IExt^{\mu}(\underset{\sim}{X}) = \underset{\sim}{\phi}, IExt^{\mu}(\underset{\sim}{\phi}) = \underset{\sim}{X}$.

(vii) $IExt^{\mu}(\underset{\sim}{X} - IExt^{\mu}(A)) = IExt^{\mu}(\underset{\sim}{X} - Iint^{\mu}(\underset{\sim}{X} - A)) = Iint^{\mu}(\underset{\sim}{X} - A) = IExt^{\mu}(A)$.

The proof of the above theorem is shown in the following example:

Example 4.3 Let $X = \{a, b, c\}$. $\tau = \left\{\underset{\sim}{X}, \underset{\sim}{\phi}, A_1, A_2, A_3\right\}$, where $A_1 = \langle X, \{a\}, \{b, c\}\rangle$, $A_2 = \langle X, \{b\}, \{c\}\rangle$, and $A_3 = \langle X, \{a, b\}, \{c\}\rangle$.

Let $A = \langle X, \{a,b\}, \{c\}\rangle$. $B = \langle X, \{a,b\}, \phi\rangle$. $X - A = \langle X, \{c\}, \{a,b\}\rangle$. $Iint^{\mu}(A) = \langle X, \{a,b\}, \{c\}\rangle$. $Icl^{\mu}(A) = \underset{\sim}{X}$.
$Iint^{\mu}(X - A) = \langle X, \{c\}, \{a,b\}\rangle$. $Icl^{\mu}(X - A) = \underset{\sim}{\phi}$. $Iext^{\mu}(A) = \underset{\sim}{\phi}$, $Iext^{\mu}(B) = \underset{\sim}{\phi}$

(i) $\underset{\sim}{X} - Icl^{\mu}(A) = \underset{\sim}{\phi}$. Hence $IExt^{\mu}(A) = \underset{\sim}{X} - Icl^{\mu}(A)$.
(ii) $IExt^{\mu}(IExt^{\mu}(A)) = \underset{\sim}{X}$ Hence $IExt^{\mu}(IExt^{\mu}(A)) \supseteq Iint^{\mu}(A)$.
(iii) $A = \langle X, \{a,b\}, \{c\}\rangle \subseteq B = \langle X, \{a,b\}, \phi\rangle$ implies $IExt^{\mu}(A) = \underset{\sim}{\phi} and IExt^{\mu}(B) = \underset{\sim}{\phi}$ implies $IExt^{\mu}(B) \subseteq IExt^{\mu}(A)$.
(iv) $IExt^{\mu}(A \cup B) = \underset{\sim}{\phi}, IExt^{\mu}(A) = \underset{\sim}{\phi} \cup IExt^{\mu}(B) = \underset{\sim}{\phi}$.
(v) $IExt^{\mu}(A \cap B) = \underset{\sim}{\phi}, IExt^{\mu}(A) = \underset{\sim}{\phi} \cap IExt^{\mu}(B) = \underset{\sim}{\phi}$.
(vi) $IExt^{\mu}(\underset{\sim}{X}) = \underset{\sim}{\phi}, IExt^{\mu}(\underset{\sim}{\phi}) = \underset{\sim}{X}$.
(vii) $IExt^{\mu}(\underset{\sim}{X} - IExt^{\mu}(A)) = \underset{\sim}{\phi}$.

Definition 4.4 Let X be an ISTS and for a subset A of a ISTS X, $I\alpha Ext^{\mu}(A) = I\alpha int^{\mu}(\underset{\sim}{X} - A)$ said to be Intuitionistic supra α-Exterior of A.

Theorem 4.5 Let X be an ISTS then and for any a subset A of IS in ISTS X, the following statements hold:

(i) $I\alpha Ext^{\mu}(A) = \underset{\sim}{X} - I\alpha cl^{\mu}(A)$.
(ii) $I\alpha Ext^{\mu}(I\alpha Ext^{\mu}(A)) = I\alpha int^{\mu}(I\alpha cl^{\mu}(A)) \supseteq I\alpha int^{\mu}(A)$.
(iii) $A \subseteq B = I\alpha Ext^{\mu}(B) \subseteq I\alpha Ext^{\mu}(A)$.
(iv) $I\alpha Ext^{\mu}(A \cup B) = I\alpha Ext^{\mu}(A) \cap I\alpha Ext^{\mu}(B)$.
(v) $I\alpha Ext^{\mu}(A \cap B) = I\alpha Ext^{\mu}(A) \cup I\alpha Ext^{\mu}(B)$.
(vi) $I\alpha Ext^{\mu}(\underset{\sim}{X}) = \underset{\sim}{\phi}, I\alpha Ext^{\mu}(\underset{\sim}{\phi}) = \underset{\sim}{X}$.
(vii) $I\alpha Ext^{\mu}(A) = I\alpha Ext^{\mu}(\underset{\sim}{X} - I\alpha Ext^{\mu}(A))$.

Proof

(i) $I\alpha Ext^{\mu}(A) = I\alpha int^{\mu}(\underset{\sim}{X} - A) = \underset{\sim}{X} - I\alpha cl^{\mu}(A)$.
(ii) $I\alpha Ext^{\mu}(I\alpha Ext^{\mu}(A)) = I\alpha int^{\mu}(\underset{\sim}{X} - (I\alpha Ext^{\mu}(A)) = I\alpha int^{\mu}(I\alpha cl^{\mu}(A)) \supseteq I\alpha int^{\mu}(A)$.
(iii) $A \subseteq$ B implies $I\alpha int^{\mu}(A) \subseteq I\alpha int^{\mu}(B)$.
$I\alpha Ext^{\mu}(B) = I\alpha int^{\mu}(\underset{\sim}{X} - B) \subseteq I\alpha int^{\mu}(\underset{\sim}{X} - A) = I\alpha Ext^{\mu}(A)$ implies $I\alpha Ext^{\mu}(B) \subseteq I\alpha Ext^{\mu}(A)$.
(iv) $I\alpha Ext^{\mu}(A \cup B) = I\alpha int^{\mu}(\underset{\sim}{X} - (A \cup B)) = I\alpha int^{\mu}((\underset{\sim}{X} - A) \cap (\underset{\sim}{X} - B)) \subseteq I\alpha int^{\mu}(\underset{\sim}{X} - A) \cap I\alpha int^{\mu}(\underset{\sim}{X} - B) = I\alpha Ext^{\mu}(A) \cap I\alpha Ext^{\mu}(B)$.
Hence $I\alpha Ext^{\mu}(A \cup B) = I\alpha Ext^{\mu}(A) \cap I\alpha Ext^{\mu}(B)$.

(v) $I\alpha Ext^{\mu}(A\cap B) = I\alpha int^{\mu}(\underset{\sim}{X}-(A\cap B)) = I\alpha int^{\mu}((\underset{\sim}{X}-A)\cup(\underset{\sim}{X}-B)) \supseteq I\alpha int^{\mu}(\underset{\sim}{X}-A)\cup I\alpha int^{\mu}(\underset{\sim}{X}-B) = I\alpha Ext^{\mu}(A)\cup I\alpha Ext^{\mu}(B)$.
Hence $I\alpha Ext^{\mu}(A\cap B) = I\alpha Ext^{\mu}(A)\cup I\alpha Ext^{\mu}(B)$.

(vi) $I\alpha Ext^{\mu}(\underset{\sim}{X}) = \underset{\sim}{\phi}, I\alpha Ext^{\mu}(\underset{\sim}{\phi}) = \underset{\sim}{X}$.

(vii) $I\alpha Ext^{\mu}(\underset{\sim}{X} - I\alpha Ext^{\mu}(A)) = I\alpha Ext^{\mu}(\underset{\sim}{X} - I\alpha int^{\mu}(\underset{\sim}{X}-A)) = I\alpha int^{\mu}(\underset{\sim}{X} - A) = I\alpha Ext^{\mu}(A)$.

The proof of the above theoremis shown in the following example:

Example 4.6 Let $X = \{a, b, c\}$. $\tau = \left\{\underset{\sim}{X}, \underset{\sim}{\phi}, A_1, A_2, A_3\right\}$, where $A_1 = \langle X, \{b\}, \{c\}\rangle$, $A_2 = \langle X, \{a\}, \{c\}\rangle$, and $A_3 = \langle X, \{a, b\}, \{c\}\rangle$.
Let $A = \langle X, \{a, c\}, \{\phi\}\rangle$. $B = \langle X, X, \phi\rangle$. $X - A = \langle X, , \{\phi\}, \{a, c\}\rangle$. $I\alpha int^{\mu}(A) = \langle X, , \{a, c\}, \{b, c\}\rangle$, $I\alpha cl^{\mu}(A) = \underset{\sim}{X}$. $\alpha Fr^{\mu}(A) = \langle X, \{b, c\}, \{\phi\}\rangle$.
$I\alpha int^{\mu}(X{-}A) = \langle X, \{\phi\}, \{a, c\}\rangle$. $I\alpha cl^{\mu}(X{-}A) = \underset{\sim}{\phi}$. $I\alpha Fr^{\mu}(X{-}A) = \langle X, \{\phi\}, \{a, c\}\rangle$.
$I\alpha ext^{\mu}(A) = \underset{\sim}{\phi}$, $I\alpha Ext^{\mu}(B) = \underset{\sim}{\phi}$.

(i) $\underset{\sim}{X} - I\alpha cl^{\mu}(A) = \underset{\sim}{\phi}$. Hence $I\alpha Ext^{\mu}(A) = \underset{\sim}{X} - I\alpha cl^{\mu}(A)$.

(ii) $I\alpha Ext^{\mu}(I\alpha Ext^{\mu}(A)) = \underset{\sim}{X}$. Hence $I\alpha Ext^{\mu}(I\alpha Ext^{\mu}(A)) \supseteq I\alpha int^{\mu}(A)$.

(iii) $A = \langle X, \{a, c\}, \{\phi\}\rangle \subseteq B = \langle X, X, \phi\rangle\, implies I\alpha Ext^{\mu}(A) = \underset{\sim}{\phi}$ and $I\alpha Ext^{\mu}(B) = \underset{\sim}{\phi}$ implies $I\alpha Ext^{\mu}(B) \subseteq I\alpha Ext^{\mu}(A)$.

(iv) $I\alpha Ext^{\mu}(A\cup B) = \underset{\sim}{\phi}$, $I\alpha Ext^{\mu}(A) = \underset{\sim}{\phi} \cup I\alpha Ext^{\mu}(B) = \underset{\sim}{\phi}$.

(v) $I\alpha Ext^{\mu}(A\cap B) = \underset{\sim}{\phi}$, $I\alpha Ext^{\mu}(A) = \underset{\sim}{\phi} \cap I\alpha Ext^{\mu}(B) = \underset{\sim}{\phi}$.

(vi) $I\alpha Ext^{\mu}(\underset{\sim}{X}) = \underset{\sim}{\phi}, I\alpha Ext^{\mu}(\underset{\sim}{\phi}) = \underset{\sim}{X}$.

(vii) $I\alpha Ext^{\mu}(\underset{\sim}{X} - I\alpha Ext^{\mu}(A)) = \underset{\sim}{\phi}$.

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