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On Some Neutrosophic Algebraic Equations

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Abstract — This paper is devoted to studying linear equations, and quadratic equations over a neutrosophic field $F(I)$ and refined neutrosophic field $F(I_1, I_2)$. This work introduces a full description of the solution's algorithm in $F(I)$ and $F(I_1, I_2)$, and discusses the solution's algorithm for a linear system of neutrosophic equations over $F(I)$ and $F(I_1, I_2)$ for the first time.

Keywords – *Neutrosophic field, refined neutrosophic field, neutrosophic linear system, neutrosophic quadratic equation*

1. Introduction

Neutrosophy is a new kind of logic founded by F. Smarandache to deal with indeterminacy in nature and reality. According to Smarandache's work, every idea can be represented by three corresponding values (degree of truth, falsity, and indeterminacy). Recently, neutrosophy has found its way into algebraic studies. Many neutrosophic algebraic structures were defined and handled, such as neutrosophic group, neutrosophic ring, and neutrosophic field. See [1-5].

Refined neutrosophic structures such as refined neutrosophic groups and refined neutrosophic rings were firstly presented in the works of Agboola et al. [6,7] by using Smarandache's idea in splitting the indeterminacy I into many different logical degrees. Laterally, refined neutrosophic algebraic structures were studied widely in [2,8-13].

Through this paper, we try to establish the basic theory of neutrosophic algebraic equations. We introduce a full description of basic algorithms which solve the linear neutrosophic equation, neutrosophic quadratic equation, and neutrosophic linear system in a neutrosophic field $F(I)$ and refined neutrosophic field $F(I_1, I_2)$. Also, we construct some examples to clarify the validity of this work.

Our work's main idea is to transform the neutrosophic equation into an easy equivalent system of classical equations, and then we can build the desired algorithms.

2. Preliminaries

Definition 2.1. [3] Let $(R, +, \times)$ be a ring. Then, $R(I) = \{a + bI : a, b \in R\}$ is called the neutrosophic ring where I is a neutrosophic element with the condition $I^2 = I$.

If R is a field, then $R(I)$ is called a neutrosophic field.
A neutrosophic field is not a field by classical meaning, since I is not invertible.

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Definition 2.2. [1] Let R be a ring and $R(I)$ be the related neutrosophic ring and $P = P_0 + P_1 I = \{a_0 + a_1 I \ : \ a_0 \in P_0 \, , a_1 \in P_1\}$; P_0, P_1 are two subsets of R.

(a) We say that P is an AH-ideal if P_0, P_1 are ideals in the ring R.

(b) We say that P is an AHS-ideal if $P_0 = P_1$.

Remark 2.3. [6] The element I can be split into two indeterminacies I_1 , I_2 with conditions:

$$I_1{}^2 = I_1 \, , I_2{}^2 = I_2 \, , I_1 I_2 = I_2 I_1 = I_1$$

Definition 2.4. [6] If X is a set, then $X(I_1, I_2) = \{(a, bI_1, cI_2) : a \, , b \, , c \in X\}$ is called the refined neutrosophic set generated by X, I_1, I_2.

Definition 2.5. [6] Let $(R, +, \times)$ be a ring, $(R(I_1, I_2) \, , + \, , \times)$ is called a refined neutrosophic ring generated by R, I_1, I_2.

Theorem 2.6. [6] Let $(R(I_1, I_2) \, , + \, , \times)$ be a refined neutrosophic ring, then it is a ring. It is called a neutrosophic field if R is a classical field**.**

3. Main Discussion

Definition 3.1. Let $F(I)$ be a neutrosophic field. Then, a neutrosophic algebraic linear equation is defined as follows:

$$AX + B = 0; A = a_0 + a_1 I, B = b_0 + b_1 I, X = x_0 + x_1 I; x_i, a_i, b_i \in F$$

A neutrosophic quadratic equation is defined as follows:

$$AX^2 + BX + C = 0; A = a_0 + a_1 I, B = b_0 + b_1 I, C = c_0 + c_1 I, X = x_0 + x_1 I; x_i, a_i, b_i, c_i \in F$$

Theorem 3.2. Let $F(I)$ be a neutrosophic field, $AX + B = 0$ be a linear neutrosophic equation. Then, it is equivalent to the following two classical linear equations:

(a) $a_0 x_0 + b_0 = 0$.

(b) $(a_0 + a_1)(x_0 + x_1) + (b_0 + b_1) = 0$.

Proof. By computing $AX + B = 0$, we find $a_0 x_0 + b_0 + (a_0 x_1 + a_1 x_0 + a_1 x_1 + b_1) I = 0$. Thus,

$$a_0 x_0 + b_0 = 0 \text{ (equation (a))}$$

$$a_0 x_1 + a_1 x_0 + a_1 x_1 + b_1 = 0 \text{ (*)}$$

By adding (a) to (*), we get $(a_0 + a_1)(x_0 + x_1) + (b_0 + b_1) = 0$ (equation (b)).

Remark 3.3. It is easy to get an algorithm to solve neutrosophic linear equation $AX + B = 0$ in a neutrosophic field $F(I)$. We should solve the equivalent system, and then we get the desired solution.

Example 3.4. Consider the following neutrosophic linear equation $(1 + 2I)X + (2 - 3I) = 0$ (*) over the neutrosophic field of real numbers $R(I)$. The equivalent system is:

(a) $x_0 + 2 = 0$. (Its solution is $x_0 = -2$.)

(b) $3(x_0 + x_1) + (-1) = 0$. Its solution is $x_0 + x_1 = \frac{1}{3}$, thus $x_1 = \frac{7}{3}$.

The solution of (*) is $X = -2 + \frac{7}{3} I$.

Example 3.5. Consider the neutrosophic linear equation $(1 + 2I)X + (2 - 3I) = 0$ (*) over $Z_3(I)$ the neutrosophic field of integers modulo 3. The equivalent system is:

(a) $x_0 + 2 = 0$. (Its solution is $x_0 = -2 \equiv 1 (\text{mod } 3)$)

(b) $3(x_0 + x_1) + (-1) = 0$. It is a non solvable in Z_3. Hence (*) is not solvable in $Z_3(I)$.

Theorem 3.6. Let $F(I)$ be a neutrosophic field, $AX^2 + BX + C = 0$ be a quadratic neutrosophic equation. Then, it is equivalent to the following two classical linear equations:

(a) $a_0 {x_0}^2 + b_0 x_0 + c_0 = 0$.

(b) $(a_0 + a_1)(x_0 + x_1)^2 + (b_0 + b_1)(x_0 + x_1) + c_0 + c_1 = 0$.

Proof. By computing $AX^2 + BX + C = 0$, we get

$$(a_0 {x_0}^2 + b_0 x_0 + c_0) + (2a_0 x_0 x_1 + a_0 {x_1}^2 + a_1 {x_0}^2 + 2a_1 x_0 x_1 + a_1 {x_1}^2 + b_0 x_1 + b_1 x_0 + b_1 x_1 + c_1)I = 0$$

Thus, $a_0 {x_0}^2 + b_0 x_0 + c_0 = 0$ (equation (a)) and $2a_0 x_0 x_1 + a_0 {x_1}^2 + a_1 {x_0}^2 + 2a_1 x_0 x_1 + a_1 {x_1}^2 + b_0 x_1 + b_1 x_0 + b_1 x_1 + c_1$(*), by adding (a) to (*) we get $(a_0 + a_1)(x_0 + x_1)^2 + (b_0 + b_1)(x_0 + x_1) + c_0 + c_1 = 0$ (equation (b)).

Remark 3.7. To solve a quadratic neutrosophic equation $AX^2 + BX + C = 0$ in a neutrosophic field $F(I)$. It is sufficient to solve the equivalent system presented in Theorem 3.6.

Example 3.8. Consider the following quadratic neutrosophic equation $(1 + I)X^2 + (2 - I)X + 3I = 0$ (*) over the neutrosophic field of real numbers $R(I)$. The equivalent system is:

(a) ${x_0}^2 + 2x_0 = 0$. (It has two solutions $x_0 = 0$, or $x_0 = -2$.

(b) $(2)(x_0 + x_1)^2 + (1)(x_0 + x_1) + 3 = 0$. (It has no solutions in R, thus (*) is not solvable in $R(I)$.)

Example 3.9. Consider the following quadratic neutrosophic equation $(1 + I)X^2 + (2 - I)X + 3I = 0$ (*) over the neutrosophic field of complex numbers $C(I)$. The equivalent system is:

(a) ${x_0}^2 + 2x_0 = 0$. (It has two solutions $x_0 = 0$, or $x_0 = -2$.)

(b) $(2)(x_0 + x_1)^2 + (1)(x_0 + x_1) + 3 = 0$. It has two solutions $x_0 + x_1 = \frac{-1+i\sqrt{23}}{4}$ or $\frac{-1-i\sqrt{23}}{4}$, thus $x_1 \in \{\frac{-1+i\sqrt{23}}{4}, \frac{-1-i\sqrt{23}}{4}\}$ if $x_0 = 0$. If $x_0 = -2$, then $x_1 \in \{2 + \frac{-1+i\sqrt{23}}{4}, 2 + \frac{-1-i\sqrt{23}}{4}\}$. The solutions of equation (*) are $X = \frac{-1+i\sqrt{23}}{4} I$, or $X = \frac{-1-i\sqrt{23}}{4} I$, or $X = -2 + \left(2 + \frac{-1+i\sqrt{23}}{4}\right) I$, or $X = -2 + \left(2 + \frac{-1-i\sqrt{23}}{4}\right) I$.

Theorem 3.10. Let $A_1 X_1 + A_2 X_2 + \cdots + A_n X_n = C$ (*) be a neutrosophic linear equation with n-variables over a neutrosophic field $F(I)$. Suppose that $C = c_0 + c_1 I$, $A_i = a_0^{(i)} + a_1^{(i)} I, X = x_0^{(i)} + x_1^{(i)} I$; $c_i, x_j^{(i)}, a_j^{(i)} \in F$. Then, (*) has the following equivalent system of classical linear equations:

(a) $a_0^{(1)} x_0^{(1)} + a_0^{(2)} x_0^{(2)} + \cdots + a_0^{(n)} x_0^{(n)} = c_0$.

(b) $(a_0^{(1)} + a_1^{(1)})\left(x_0^{(1)} + x_1^{(1)}\right) + \left(a_0^{(2)} + a_1^{(2)}\right)\left(x_0^{(2)} + x_1^{(2)}\right) + \cdots + \left(a_0^{(n)} + a_1^{(n)}\right)\left(x_0^{(n)} + x_1^{(n)}\right) = c_0 + c_1$.

Proof.

First of all, we should compute $A_i X_i$. We have $A_i X_i = a_0^{(i)} x_0^{(i)} + \left(a_0^{(i)} x_1^{(i)} + a_1^{(i)} x_0^{(i)} + a_1^{(i)} x_1^{(i)}\right) I$, we remark that $a_0^{(i)} x_0^{(i)} + \left(a_0^{(i)} x_1^{(i)} + a_1^{(i)} x_0^{(i)} + a_1^{(i)} x_1^{(i)}\right) = \left(a_0^{(i)} + a_1^{(i)}\right)(x_0^{(i)} + x_1^{(i)})$. Hence,

$$A_i X_i = a_0^{(i)} x_0^{(i)} + \left(a_0^{(i)} x_1^{(i)} + a_1^{(i)} x_0^{(i)} + a_1^{(i)} x_1^{(i)}\right) I = a_0^{(i)} x_0^{(i)} + [\left(a_0^{(i)} + a_1^{(i)}\right)\left(x_0^{(i)} + x_1^{(i)}\right) - a_0^{(i)} x_0^{(i)}] I$$

Now, we can write $\sum_{i=1}^{n} A_i X_i = \left(\sum_{i=1}^{n} a_0^{(i)} x_0^{(i)}\right) + I(\sum_{i=1}^{n} \left(a_0^{(i)} + a_1^{(i)}\right)\left(x_0^{(i)} + x_1^{(i)}\right) - \sum_{i=1}^{n} a_0^{(i)} x_0^{(i)}) = C = c_0 + c_1 I$. Thus, $\left(\sum_{i=1}^{n} a_0^{(i)} x_0^{(i)}\right) = c_0$ (equation (a)). And $\sum_{i=1}^{n} \left(a_0^{(i)} + a_1^{(i)}\right)\left(x_0^{(i)} + x_1^{(i)}\right) - \sum_{i=1}^{n} a_0^{(i)} x_0^{(i)} = c_1$ (*), by adding (a) to (*) we get $\sum_{i=1}^{n} \left(a_0^{(i)} + a_1^{(i)}\right)\left(x_0^{(i)} + x_1^{(i)}\right) = c_0 + c_1$. (equation (b)).

Remark 3.11. We can solve a linear system of neutrosophic equations in a neutrosophic field $F(I)$ by solving its equivalent system in the classical field F.

Example 3.12. Consider the following neutrosophic linear system over the neutrosophic field of real numbers:

(1) $(1+I)X+(2-I)Y=1+3I$.

(2) $(2+I)X+5IY=-1+I$.

The equivalent system of (1) is

$$x_0+2y_0=1 \text{ (I)}$$
$$2(x_0+x_1)+(y_0+y_1)=4 \text{ (II)}$$

The equivalent system of (2) is

$$2x_0+0.y_0=-1 \text{ (III)}$$
$$3(x_0+x_1)+5(y_0+y_1)=0 \text{ (IV)}$$

From (I), (III), we get $x_0=-\frac{1}{2}, y_0=\frac{3}{4}$. From (II), (IV), we get $x_0+x_1=\frac{20}{7}, y_0+y_1=-\frac{12}{7}$.

Definition 3.13. Let $F(I_1,I_2)$ be a refined neutrosophic field. We define

(a) $AX+B=(0,0,0); A=(a_0,a_1I_1,a_2I_2), B=(b_0,b_1I_1,b_2I_2), X=(x_0,x_1I_1,x_2I_2); b_i,a_i,x_i \in F$. (Refined neutrosophic linear equation with one variable).

(b) $AX^2+BX+C=(0,0,0); C=(c_0,c_1I_1,c_2I_2)$. (Refined quadratic neutrosophic equation).

Theorem 3.14. Let $F(I_1,I_2)$ be a refined neutrosophic field, $AX+B=(0,0,0)$ be a refined linear neutrosophic equation. Then, it is equivalent to the following system of classical linear equations:

(a) $a_0x_0+b_0=0$.

(b) $(a_0+a_2)(x_0+x_2)+(b_0+b_2)=0$.

(c) $(a_0+a_1+a_2)(x_0+x_1+x_2)+(b_0+b_1+b_2)=0$.

PROOF.

We compute

$$AX+B=(a_0x_0+b_0,[a_0x_1+a_1x_0+a_1x_1+a_1x_2+a_2x_1+b_1]I_1,[a_0x_2+a_2x_0+a_2x_2+b_2]I_2)$$

So, we get

$$a_0x_0+b_0=0 \text{ (equation (a))}$$
$$a_0x_2+a_2x_0+a_2x_2+b_2=0 \text{ (*)}, a_0x_1+a_1x_0+a_1x_1+a_1x_2+a_2x_1+b_1 \text{ (**)}$$

By adding (a) to (*), we find $(a_0+a_2)(x_0+x_2)+(b_0+b_2)=0$ (equation (b)).

By adding (b) to (**), we find $(a_0+a_1+a_2)(x_0+x_1+x_2)+(b_0+b_1+b_2)=0$ (equation (c)).

Example 3.15. Consider the following refined neutrosophic linear equation (*) $(2,I_1,3I_2)X+(4,7I_1,-5I_2)=(0,0,0)$ over the refined neutrosophic field $Q(I_1,I_2)$.

The equivalent system is:

(a) $2x_0+4=0$. It has a solution $x_0=-2$.

(b) $5(x_0+x_2)+(-1)=0$. It has a solution $x_0+x_2=\frac{1}{5}$, hence $x_2=\frac{11}{5}$.

(c) $6(x_0+x_1+x_2)+(6)=0$. It has a solution $x_0+x_1+x_2=-1$, hence $x_1=\frac{-6}{5}$.

The solution of equation (*) is $X=(-2,\frac{-6}{5}I_1,\frac{11}{5}I_2)$.

Theorem 3.16. Let $F(I_1, I_2)$ be a refined neutrosophic field, $AX^2 + BX + C = (0,0,0)$ be a refined quadratic neutrosophic equation over $F(I_1, I_2)$. Then, it is equivalent to the following system of classical quadratic equations:

(a) $a_0{x_0}^2 + b_0x_0 + c_0 = 0$.

(b) $(a_0 + a_2)(x_0 + x_2)^2 + (b_0 + b_2)(x_0 + x_2) + c_0 + c_2 = 0$.

(c) $(a_0 + a_1 + a_2)(x_0 + x_1 + x_2)^2 + (b_0 + b_1 + b_2)(x_0 + x_1 + x_2) + c_0 + c_1 + c_2 = 0$.

Proof. The proof is similar to the previous theorem.

Example 3.17. Consider the following refined neutrosophic quadratic equation over the refined neutrosophic field of complex numbers $C(I_1, I_2)$,

$(*)$ $(1,0,I_2)X^2 + (1,I_1,0)X + (-2,I_1,I_2) = (0,0,0)$, the equivalent system is

(a) ${x_0}^2 + x_0 - 2 = 0$. It has two solutions $x_0 = 1$ or $x_0 = -2$.

(b) $2(x_0 + x_2)^2 + (x_0 + x_2) - 1 = 0$. It has two possible solutions $x_0 + x_2 = -1$ or $x_0 + x_2 = \frac{1}{2}$. Thus if $x_0 = -2$, then $x_2 = 1$ or $x_2 = \frac{5}{2}$, and if $x_0 = 1$, then $x_2 = -2$ or $x_2 = \frac{-1}{2}$.

(c) $2(x_0 + x_1 + x_2)^2 + 2(x_0 + x_1 + x_2) = 0$. It has two possible solutions $x_0 + x_1 + x_2 = 0$ or $x_0 + x_1 + x_2 = -1$.

If $x_0 + x_2 = -1$, then $x_1 = 1$ or $x_1 = 0$, and if $x_0 + x_2 = \frac{1}{2}$, then $x_1 = \frac{-1}{2}$ or $x_1 = -\frac{3}{2}$.

The set of solutions of equation (*) are

$$\left\{(1,I_1,-2I_2),\left(1,\frac{-1}{2}I_1,-\frac{1}{2}I_2\right),(-2,I_1,I_2),\left(-2,-\frac{1}{2}I_1,\frac{5}{2}I_2\right),(1,0,-2I_2),\left(1,-\frac{3}{2}I_1,-\frac{1}{2}I_2\right),(-2,0,I_2),(-2,-\frac{3}{2}I_1,\frac{5}{2}I_2)\right\}$$

Theorem 3.18. Let $A_1X_1 + \cdots + A_nX_n = C$; $C = (c_0, c_1I_1, c_2I_2)$, $X_i = \left(x_0^{(i)}, x_1^{(i)}I_1, x_2^{(i)}I_2\right)$, $A_i = \left(a_0^{(i)}, a_1^{(i)}I_1, a_2^{(i)}I_2\right)$ be a linear equation with n-variables over a refined neutrosophic field $F(I_1, I_2)$. Then, it is equivalent to the following system of classical linear equations over the classical field F:

(a) $\sum_{i=1}^n a_0^{(i)}x_0^{(i)} = c_0$.

(b) $\sum_{i=1}^n \left(a_0^{(i)} + a_2^{(i)}\right)\left(x_0^{(i)} + x_2^{(i)}\right) = c_0 + c_2$.

(c) $\sum_{i=1}^n \left(a_0^{(i)} + a_1^{(i)} + a_2^{(i)}\right)\left(x_0^{(i)} + x_1^{(i)} + x_2^{(i)}\right) = c_0 + c_1 + c_2$.

Proof.

We shall prove that $\sum_{i=1}^n A_iX_i = (\sum_{i=1}^n a_0^{(i)}x_0^{(i)}, \left[\sum_{i=1}^n \left(a_0^{(i)} + a_1^{(i)} + a_2^{(i)}\right)\left(x_0^{(i)} + x_1^{(i)} + x_2^{(i)}\right) - \sum_{i=1}^n \left(a_0^{(i)} + a_2^{(i)}\right)\left(x_0^{(i)} + x_2^{(i)}\right)\right] I_1, \left[\sum_{i=1}^n \left(a_0^{(i)} + a_2^{(i)}\right)\left(x_0^{(i)} + x_2^{(i)}\right) - \sum_{i=1}^n a_0^{(i)}x_0^{(i)}\right] I_2)$.

We will use induction on n. For $n = 1$, the theorem is true easily. Suppose that it is true for k. We must prove it for $k + 1$.

$$\sum_{i=1}^{k+1} A_iX_i = \sum_{i=1}^{k} A_iX_i + A_{k+1}X_{k+1}$$

$$= \left(\sum_{i=1}^{k} a_0^{(i)}x_0^{(i)}, \left[\sum_{i=1}^{k}(a_0^{(i)} + a_1^{(i)} + a_2^{(i)})(x_0^{(i)} + x_1^{(i)} + x_2^{(i)}) - \sum_{i=1}^{k}(a_0^{(i)} + a_2^{(i)})(x_0^{(i)} + x_2^{(i)})\right] I_1,\right.$$

$$\left[\sum_{i=1}^{k}(a_0^{(i)}+a_2^{(i)})(x_0^{(i)}+x_2^{(i)})-\sum_{i=1}^{k}a_0^{(i)}x_0^{(i)}\right]I_2\Bigg)+(a_0^{(k+1)},a_1^{(k+1)}I_1,a_2^{(k+1)}I_2)(x_0^{(k+1)},x_1^{(k+1)}I_1,x_2^{(k+1)}I_2)$$
$$=(m,nI_1,tI_2)$$

We have $m=(\sum_{i=1}^{k}a_0^{(i)}x_0^{(i)})+a_0^{(k+1)}x_0^{(k+1)}=\sum_{i=1}^{k+1}a_0^{(i)}x_0^{(i)}$.

$$t=\sum_{i=1}^{k}(a_0^{(i)}+a_2^{(i)})(x_0^{(i)}+x_2^{(i)})-\sum_{i=1}^{k}a_0^{(i)}x_0^{(i)}+[a_0^{(k+1)}x_2^{(k+1)}+a_2^{(k+1)}x_0^{(k+1)}+a_2^{(k+1)}x_2^{(k+1)}]$$
$$=\sum_{i=1}^{k}(a_0^{(i)}+a_2^{(i)})(x_0^{(i)}+x_2^{(i)})-\sum_{i=1}^{k}a_0^{(i)}x_0^{(i)}+[(a_0^{(k+1)}+a_2^{(k+1)})(x_0^{(k+1)}+x_2^{(k+1)})-a_0^{(k+1)}x_0^{(k+1)}]$$
$$=\sum_{i=1}^{k+1}\left(a_0^{(i)}+a_2^{(i)}\right)\left(x_0^{(i)}+x_2^{(i)}\right)-\sum_{i=1}^{k+1}a_0^{(i)}x_0^{(i)}$$

By following the same argument, we find that

$$n=\sum_{i=1}^{k+1}\left(a_0^{(i)}+a_1^{(i)}+a_2^{(i)}\right)\left(x_0^{(i)}+x_1^{(i)}+x_2^{(i)}\right)-\sum_{i=1}^{k+1}\left(a_0^{(i)}+a_2^{(i)}\right)\left(x_0^{(i)}+x_2^{(i)}\right)$$

By putting m,n,t in equation (*) we get:

(a) $\sum_{i=1}^{n}a_0^{(i)}x_0^{(i)}=c_0$.

(I) $\sum_{i=1}^{n}\left(a_0^{(i)}+a_2^{(i)}\right)\left(x_0^{(i)}+x_2^{(i)}\right)-\sum_{i=1}^{n}a_0^{(i)}x_0^{(i)}=c_2$.

(II) $\sum_{i=1}^{n}\left(a_0^{(i)}+a_1^{(i)}+a_2^{(i)}\right)\left(x_0^{(i)}+x_1^{(i)}+x_2^{(i)}\right)-\sum_{i=1}^{n}\left(a_0^{(i)}+a_2^{(i)}\right)\left(x_0^{(i)}+x_2^{(i)}\right)=c_1$.

We add (a) to (I) to get equation (b). Also, we add (b) to (II) to get equation (c). Hence our proof is complete.

Remark 3.19. According to the previous theorem, we can solve any linear system of refined neutrosophic linear equations by transforming it into a classical equivalent system.

Example 3.20. Consider the following system of refined linear neutrosophic equations over the refined neutrosophic field of real numbers $R(I_1,I_2)$.

(1) $(1,I_1,0)X+(0,I_1,I_2)Y=(1,0,I_2)$.

(2) $(2,0,I_2)X+(-1,I_1,-I_2)Y=(3,0,0)$. Where $X=(x_0,x_1I_1,x_2I_2),Y=(y_0,y_1I_1,y_2I_2)$.

The equivalent system of equation (1) is:

$1.x_0+0.y_0=1$ **(I)**,

$1.(x_0+x_2)+1.(y_0+y_2)=2$ **(II)**,

$2.(x_0+x_1+x_2)+2(y_0+y_1+y_2)=2$ (**III**).

The equivalent system of equation (2) is:

$2.x_0-1.y_0=3$ **(a)**,

$3.(x_0+x_2)-2(y_0+y_2)=3$ (**b**),

$3.(x_0+x_1+x_2)-1(y_0+y_1+y_2)=3$ **(c)**.

By solving (I) with (a), we find $x_0=1,y_0=-1$.

By solving (II) with (b), we find $x_0 + x_2 = \frac{7}{5}, y_0 + y_2 = \frac{3}{5}$. Thus, $x_2 = \frac{2}{5}, y_2 = \frac{8}{5}$.

By solving (III) with (c), we find $x_0 + x_1 + x_2 = 1, y_0 + y_1 + y_2 = 0$. Thus, $x_1 = -\frac{2}{5}, y_1 = -\frac{3}{5}$.

The solution of the system (1) and (2) is:

3. $X = (1, -\frac{2}{5}I_1, \frac{2}{5}I_2), Y = (-1, -\frac{3}{5}I_1, \frac{8}{5}I_2)$.

4. Conclusion

In this article, we have introduced an algorithm to solve linear and quadratic equations in a neutrosophic field $F(I)$ and refined neutrosophic field $F(I_1, I_2)$. Also, we have introduced an algorithm to solve a linear system of neutrosophic equations over $F(I)$ and $F(I_1, I_2)$ by turning it into an easy classical equivalent system of linear equations.

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