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On Some Special Elements in Neutrosophic Rings and Refined Neutrosophic Rings

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Abstract — Idempotent elements in a ring R are the elements with the condition $\boldsymbol{a^2 = a}$. This paper introduces the criterion of any element in a refined neutrosophic ring to be idempotent. Also, the concept of symmetric and supersymmetric elements in a neutrosophic ring $R(I)$, and a refined neutrosophic ring $R(I_1, I_2)$ are defined. Also, the invertibility of these elements is discussed.

Keywords – *Neutrosophic ring, refined neutrosophic ring, idempotent element, symmetric element, supersymmetric element*

1. Introduction

Neutrosophic algebra is a new trend in pure mathematics; it is considered a combination between the neutrosophic set introduced by Smarandache and classical algebra.

Many neutrosophic algebraic structures were defined and studied in a wide range such as neutrosophic groups, neutrosophic rings, neutrosophic vector spaces, and neutrosophic modules. See [1-6].

Recently, many generalized concepts came to light, such as refined neutrosophic rings, Boolean rings, and n-refined neutrosophic rings [7-14]. These generalizations were built over the idea of splitting the indeterminacy I into many logical degrees. In the case of refined structures, I is splitting into two sub-indeterminacies I_1, I_2 with the following property $I_1 I_2 = I_1, {I_1}^2 = I_1, {I_2}^2 = I_2$ [9]. Also, I is splitting into n sub-indeterminacies $I_1, \ldots, I_n$ in the case of n-refined structures. See [12,13].

Idempotents in a ring R are the elements with the property $a^2 = a$. They were handled and classified in neutrosophic rings with semi idempotents in [15,16]. Through this paper, we introduce the condition of any element in a refined neutrosophic ring $R(I_1, I_2)$ to be idempotent. Two new kinds of special elements (symmetric and supersymmetric elements) in neutrosophic rings and refined neutrosophic rings are presented and classified. These elements have many interesting properties, especially in neutrosophic fields and refined neutrosophic fields. Also, their algebraic structure will be discussed in previous cases.

2. Preliminaries

Definition 2.1. [13] Let $(R, +, \times)$ be a ring. Then, $R(I) = \{a + bI : a, b \in R\}$ is called the neutrosophic ring where I is a neutrosophic element with the condition $I^2 = I$.

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If R is a field, then $R(I)$ is called a neutrosophic field.

A neutrosophic field is not a field by classical meaning, since I is not invertible.

Definition 2.2. [1] Let R be a ring and $R(I)$ be the related neutrosophic ring and $P = P_0 + P_1 I = \{a_0 + a_1 I \, : \, a_0 \in P_0 \, , a_1 \in P_1\}$; P_0, P_1 are two subsets of R.

(a) We say that P is an AH-ideal if P_0, P_1 are ideals in the ring R.

(b) We say that P is an AHS-ideal if $P_0 = P_1$.

Remark 2.3. [11] The element I can be split into two indeterminacies I_1 , I_2 with conditions:

$$\mathrm{I}_1{}^2 = \mathrm{I}_1 \, , I_2{}^2 = I_2 \, , I_1 I_2 = I_2 I_1 = I_1$$

Definition 2.4. [11] If X is a set, then $X(I_1, I_2) = \{(a, bI_1, cI_2) : a \, , b \, , c \in X\}$ is called the refined neutrosophic set generated by X , I_1, I_2.

Definition 2.5. [11] Let $(R, +, \times)$ be a ring, $(R(I_1, I_2) \, , + \, , \times)$ is called a refined neutrosophic ring generated by R ,I_1, I_2.

Theorem 2.6. [11] Let $(R(I_1, I_2) \, , + \, , \times)$ be a refined neutrosophic ring, then it is a ring. It is called a neutrosophic field if R is a classical field**.**

Definition 2.7. [17] Let R be a ring, a be any element in R. Then, it is called idempotent if and only if $a^2 = a$.

3. Idempotents in $R(I_1, I_2)$

Theorem 3.1. Let R be any ring (noncommutative ring in general), $R(I_1, I_2)$ be its corresponding refined neutrosophic ring. Assume that $x = (a, bI_1, cI_2)$ is an arbitrary element in $R(I_1, I_2)$. Then, x is idempotent in $R(I_1, I_2)$ if and only if $a, a + c, a + b + c$ are idempotents in R.

Proof. Let $x = (a, bI_1, cI_2)$ be an idempotent in $R(I_1, I_2)$, then

$$x^2 = (a^2, [a.b + b.a + b^2 + b.c + c.b]I_1, [a.c + c.a + c^2]I_2) = x$$

Thus, $[a^2 = a]$,(*)$[\, a.b + b.a + b^2 + b.c + c.b = b]$, (**)$[a.c + c.a + c^2 = c]$, hence a is an idempotent in R.

Now, we compute $(a + c)^2 = a^2 + a.c + c.a + c^2$, we can find from (**) that $(a + c)^2 = a^2 + c = a + c$, thus $a + c$ is idempotent in R.

Also, we have $(a + b + c)^2 = a^2 + b^2 + c^2 + a.b + b.a + a.c + c.a + b.c + c.b$, by (*) we get

$$(a + b + c)^2 = (a^2 + c^2 + a.c + c.a) + (a.b + b.a + b^2 + b.c + c.b) = (a + c) + b = a + b + c$$

Thus, $a + b + c$ is idempotent in R.

For the converse, we suppose that $a, a + c, a + b + c$ are idempotents in R, then we get

(1) $[a^2 = a]$.

(2) $(a + c)^2 = a^2 + a.c + c.a + c^2 = a + c$. By using equation (1), we get $a.c + c.a + c^2 = c$.

(3) $(a + b + c)^2 = a^2 + b^2 + c^2 + a.b + b.a + a.c + c.a + b.c + c.b = a + b + c$. By using (1) and (2), we get $(a + b + c)^2 = a + b^2 + a.b + b.a + (a.c + c.a + c^2) + b.c + c.b = a + b + c$. Thus,

$$a + b^2 + a.b + b.a + (c) + b.c + c.b = a + b + c$$

Hence, $b^2 + a.b + b.a + b.c + c.b = b$.

Now, we compute $x^2 = (a^2, [a.b + b.a + b^2 + b.c + c.b]I_1, [a.c + c.a + c^2]I_2) = (a, bI_1, cI_2) = x$. So, it is idempotent in the refined neutrosophic ring $R(I_1, I_2)$.

Example 3.2. Let $R = Z_3$ be the ring of integers modulo 3, $R(I_1, I_2) = \{(a, bI_1, cI_2) : a, b, c \in Z_3\}$ be its corresponding refined neutrosophic ring. The set of idempotents in R is

$M = \{0,1\}$, the set of idempotents in $R(I_1, I_2)$ according to Theorem 3.1 is:

$$N = \{(1, I_1, 2I_2), (1,0,2I_2), (1,2I_1, 0), (1,0,0), (0,0,0), (0, I_1, 0), (0,0, I_2), (0,2I_1, I_2)\}$$

The following theorem determines the number of idempotents in $R(I_1, I_2)$.

Theorem 3.3. If the ring R has m idempotents, then the corresponding refined neutrosophic ring $R(I_1, I_2)$ has m^3 idempotents.

Proof. According to Theorem 3.1, for each idempotent $a \in R$, we have $x = (a, bI_1, cI_2)$ is idempotent in $R(I_1, I_2)$, if and only if $a + c, a + b + c$ are idempotents in R, thus c can be taken by m ways, and b is the same. By this argument, we get the fact that $R(I_1, I_2)$ has $m \times m \times m = m^3$ idempotents.

4. Symmetric Elements

This section is devoted to studying a new kind of special elements in a neutrosophic ring and a refined neutrosophic ring with its algebraic structures.

Definition 4.1. Let R be a ring, $R(I)$ be the corresponding neutrosophic ring. An arbitrary element $x = a + bI \in R(I)$ is called symmetric if and only if $a = b$. The set of all symmetric elements in a neutrosophic ring is denoted by $S(I)$.

Definition 4.2. Let R be a ring, $R(I_1, I_2)$ be the corresponding refined neutrosophic ring. An arbitrary element $x = (a, bI_1, cI_2) \in R(I_1, I_2)$ is called symmetric if and only if $a = b = c$. The set of all symmetric elements in a refined neutrosophic ring is denoted by $S(I_1, I_2)$.

Theorem 4.3. Let $R(I)$ be a neutrosophic ring, $S(I)$ be the set of all symmetric elements. Then, $(S(I), +)$ is a subgroup of $(R(I), +)$ and $(S(I), +) \cong (R, +)$.

Proof. Let $x = a + aI, y = b + bI$ be two arbitrary elements in $S(I)$, $x - y = (a - b) + (a - b)I \in S(I)$, thus $S(I)$ is a subgroup of $(R(I), +)$. (It is known that $(R(I), +)$ is an abelian group by the definition of the ring).

We define $f: R \to S(I); f(a) = a + aI$, suppose that $a, b \in R$, then $f(a + b) = (a + b) + (a + b)I = f(a) + f(b)$.

f is a well-defined map since if $a = b$, then $a + aI = b + bI$, i.e. $f(a) = f(b)$. Clearly, f is bijective; thus, it is an isomorphism.

Theorem 4.4: Let $R(I_1, I_2)$ be a refined neutrosophic ring, $S(I_1, I_2)$ be the set of all symmetric elements. Then, $(S(I_1, I_2), +)$ is a subgroup of $(R(I_1, I_2), +)$ and $(S(I_1, I_2), +) \cong (R, +)$.

Proof. The proof is similar to that of Theorem 4.3.

Theorem 4.5. Let $K(I)$ be a neutrosophic field, $S(I)$ be the set of all symmetric elements. If $Char(K) = 2$, then $S(I)$ is a field and $S(I) \cong K$.

Proof. We must prove that $(S(I)/\{0\}, .)$ is a group. Let $x = a + aI, y = b + bI$ be two arbitrary elements in $S(I)/\{0\}$, we have $x.y = (a.b) + (a.b + a.b + a.b)I = (a.b) + (a.b)I \in S(I)$, since $a.b + a.b = 2a.b = 0$ (under the assumption $Char(K) = 2$). The inverse of x is $x^{-1} = a^{-1} + a^{-1}I$ because $x.x^{-1} = (aa^{-1}) + (aa^{-1} + aa^{-1} + aa^{-1})I = 1 + I$. $1 + I$ is an identity concerning multiplication, that is because $(a + aI).(1 + I) = a + (a + a + a)I = a + aI$.

We define $f: S(I) \to K; f(a + aI) = a$, f is a well-defined bijective map.

Let $x = a + aI, y = b + bI$ be two arbitrary elements in $S(I)$, $f(x + y) = (a + b) = f(x) + f(y), f(x.y) = f(a.b + a.bI) = a.b = f(x).f(y)$. Hence f is an isomorphism.

Example 4.6. Let $K = Z_2$ be a field with $Char(K) = 2$, $K(I) = \{0,1, I, 1 + I\}$, $S(I) = \{0,1 + I\}$.

We can see that $S(I)$ is a field, the identity concerning multiplication is $1 + I$, and $S(I) \cong Z_2 = K$.

Theorem 4.7. Let $K(I_1, I_2)$ be a refined neutrosophic field, $S(I_1, I_2)$ be the set of all symmetric elements.

If $Char(K) = 2$, then $S(I_1, I_2)$ is a field and $S(I_1, I_2) \cong K$.

Proof. We must prove that $(S(I_1, I_2)/\{0\}, .)$ is a group. Let $x = (a, aI_1, aI_2), y = (b, bI_1, bI_2)$ be two arbitrary elements in $S(I_1, I_2)/\{0\}$, we have $x.y = (a.b, [5a.b]I_1, [3a.b]I_2) = (a.b, a.bI_1, a.bI_2) \in S(I_1, I_2)$, since $5a.b = 3a.b = a.b$ (under the assumption $Char(K) = 2$).

The inverse of x is $x^{-1} = (a^{-1}, a^{-1}I_1, a^{-1}I_2)$ because $x.x^{-1} = (a.a^{-1}, [5a.a^{-1}]I_1, [3a.a^{-1}]I_2) = (1,1.I_1, 1.I_2)$. ($5aa^{-1} = 5 = 1 + 4 = 0, 3aa^{-1} = 3 = 1 + 2 = 1$, under the assumption $Char(K) = 2$).

$(1,1.I_1, 1.I_2)$ is an identity concerning multiplication, that is because $(a, aI_1, aI_2).(1,1.I_1, 1.I_2) = (a, aI_1, aI_2)$.

We define $f: S(I_1, I_2) \to K; f(a, aI_1, aI_2) = a$, f is an isomorphism (It can be proved by a similar way to the previous theorem.).

Example 4.8. Let $K = Z_2$ be a field with $Char(K) = 2$,

$$K(I_1, I_2) = \{(0,0,0), (1,0,0), (0,1.I_1, 0), (0,0,1.I_2), (1,1.I_1, 1I_2), (1,1I_1, 0), (0,1.I_1, 1.I_2), (1,0,1.I_2\}$$

$S(I_1, I_2) = \{(0,0,0), (1,1.I_1, 1I_2)\}$, which is a field isomorphic to $K = Z_2$.

The following theorem determines which elements in a neutrosophic field $K(I)$ are invertible.

Theorem 4.9. Let K be a field, $K(I)$ be the corresponding neutrosophic field. An arbitrary element $z = a + bI \in K(I)$ is invertible if and only if $a \neq 0$ and $a \neq -b$.

Proof. Let $z = a + bI \in K(I)$ be an invertible element in $K(I)$. There is $m = x + yI \in K(I); z.m = 1$. Thus, $(a.x) + (a.y + b.x + b.y)I = 1$, this means $x = a^{-1}, a.y + b(a^{-1}) + by = 0$. Hence,

$$y = \frac{-b.a^{-1}}{a+b}, \text{ this implies } a \neq 0 \text{ and } a \neq -b$$

Conversely, suppose that $a \neq 0$ and $a \neq -b$, then there is $m = x + yI \in K(I)$, where $x = a^{-1}$, $y = \frac{-b.a^{-1}}{a+b}$ with $z.m = 1$.

Result 4.10. If $K(I)$ is a neutrosophic field with $Char(K) \neq 2$, then all symmetric elements different from zero are invertible.

Proof. Let K be a field with $Char(K) \neq 2$, $x = a + aI$ be a symmetric element different from zero. It is clear that $a \neq -a$, thus x is invertible according to Theorem 4.10.

Example 4.11. Let $K = Z_5$ be the field of integers modulo 5. We have $x = 3 + 3I$ a symmetric element. The inverse of x is $x^{-1} = 2 + 4I$.

The inverse of a symmetric element is not supposed to be symmetric in general.

The following theorem determines which elements are invertible in a refined neutrosophic field $K(I_1, I_2)$.

Theorem 4.12. Let $K(I_1, I_2)$ be a refined neutrosophic field. An arbitrary element $t = (a, bI_1, cI_2)$ is invertible if and only if $a \neq 0, a + c \neq 0, a + b + c \neq 0$.

Proof. Suppose that $t = (a, bI_1, cI_2)$ is invertible. Then, there is $m = (x, yI_1, zI_2); m.t = 1_{K(I_1, I_2)}$.

$m.t = (a.x, [a.y + b.x + b.z + b.y + c.y]I_1, [a.z + c.x + c.z]I_2) = (1,0,0)$, this means $x = a^{-1}$, $z = \frac{-ca^{-1}}{a+c}$, $y = (a + b + c)^{-1}.(-b.a^{-1} + \frac{bc}{a(a+c)})$, which implies that $a \neq 0, a + c \neq 0, a + b + c \neq 0$.

Conversely, if $a \neq 0, a + c \neq 0, a + b + c \neq 0$, then there is $m = (x, yI_1, zI_2); m.t = 1_{K(I_1,I_2)}$, where $x = a^{-1}, z = \frac{-ca^{-1}}{a+c}, y = (a + b + c)^{-1}.(-b.a^{-1} + \frac{bc}{a(a+c)})$.

Result 4.13. Let K be a field with $Char(K) \neq 2$ and $Char(K) \neq 3$, then all symmetric elements different from zero in the corresponding refined neutrosophic field $K(I_1, I_2)$ are invertible since the conditions of Theorem 4.12 are true in this case.

Example 4.14. Let $K = Z_5$ be the field of integers modulo 5 and $Char(K) = 5$, let $x = (3, 3I_1, 3I_2)$ is an element in the refined neutrosophic field $K(I_1, I_2)$. According to Theorem 12.3, the inverse of x is $x^{-1} = (2,3I_1, 4I_2)$.

The inverse of a symmetric element in a refined neutrosophic field is not supposed to be a symmetric element in general.

5. Super Symmetric Elements

The following section is discussing a generalized kind of symmetric elements.

Definition 5.1. Let R be a ring, $R(I)$ be the corresponding neutrosophic ring. An arbitrary element $x = a + bI \in R(I)$ is called supersymmetric if and only if $a = m.c, b = n.c\ ; c \in R$ and $m, n \in Z$. The set of all supersymmetric elements in a neutrosophic ring is denoted by $SS(I)$.

Definition 5.2. Let R be a ring, $R(I_1, I_2)$ be the corresponding refined neutrosophic ring. An arbitrary element $x = (a, bI_1, cI_2) \in R(I_1, I_2)$ is called supersymmetric if and only if $a = m.d, b = n.d, c = s.d; d \in R$ and $m, n, s \in Z$. The set of all supersymmetric elements in a refined neutrosophic ring is denoted by $SS(I_1, I_2)$.

Theorem 5.3. Let $R(I)$ be a neutrosophic ring, $SS(I)$ be the set of all supersymmetric elements. Then, $SS(I)$ is closed under the multiplication of $R(I)$.

Proof. Let $x = m.a + n.aI, y = s.b + t.bI; a, b \in R$ and $m, n, s, t \in Z$, we have $x.y = (mn)(a.b) + [(mt + ns + nt)(a.b)]I \in SS(I)$. Since $mn, mt + ns + nt \in Z$.

Remark 5.4. $SS(I)$ is not an additive subgroup of $(R(I), +)$ in general. We clarify it by the following example:

Let R be the ring of real numbers, $x = \sqrt{2} + 3\sqrt{2I}, y = \sqrt{3} - 4\sqrt{3}I$ be two elements in $SS(I)$, $x + y = (\sqrt{2} + \sqrt{3}) + (3\sqrt{2} - 4\sqrt{3})I$, we can see that $x + y$ is not in $SS(I)$, since $3\sqrt{2} - 4\sqrt{3}$ cannot be written as $m.(\sqrt{2} + \sqrt{3})$, where m is an integer.

Theorem 5.5. Let $R(I_1, I_2)$ be a refined neutrosophic ring, $SS(I_1, I_2)$ be the set of all supersymmetric elements. Then, $SS(I_1, I_2)$ is closed under the multiplication of $R(I_1, I_2)$.

Proof. The proof is similar to that of Theorem 5.3.

Remark 5.6. $SS(I_1, I_2)$ is not an additive subgroup of $(R(I_1, I_2), +)$ in general. We illustrate an example.

Let R be the ring of real numbers, $x = (\sqrt{2}, \sqrt{2}\,I_1, 3\sqrt{2}\,I_2), y = (\sqrt{3}, 2\sqrt{3}I_1, 5\sqrt{3}I_2)$ be two elements in $SS(I_1, I_2)$, $x + y = (\sqrt{2} + \sqrt{3}, [\sqrt{2} + 2\sqrt{3}]I_1, [3\sqrt{2} + 5\sqrt{3}]I_2)$, we can see that $\sqrt{2} + 2\sqrt{3}$ can not be written as $m.(\sqrt{2} + \sqrt{3})$ where m is an integer.

The following theorem introduces a special case, which $SS(I)$ and $SS(I_1, I_2)$, will be two additive subgroups of $(R(I), +)$ and $(R(I_1, I_2), +)$, respectively.

Theorem 5.7. Let $R = Z$ be the ring of integers. Then,

(a) $(SS(I), +)$ is a subgroup of $(R(I), +)$.

(b) $(SS(I), +.)$ is a subring of $(R(I), +, .)$.

(c) $(SS(I_1, I_2), +)$ is a subgroup of $(R(I_1, I_2), +)$.

(d) $(SS(I_1, I_2), +, .)$ is a subring of $(R(I_1, I_2), +, .)$.

Proof.

(a) Let $x = m.a + n.aI, y = s.b + t.bI; a, b \in Z$ and $m, n, s, t \in Z$, we have $x - y = (m.a - s.b)(1) + [(n.a - t.b)(1)]I \in SS(I)$, thus $(SS(I), +)$ is an additive subgroup of $(R(I), +)$.

(b) It holds directly from (a) and Theorem 5.3.

(c) It can be proved by a similar argument of (a).

(d) It holds directly from (c) and Theorem 5.5.

We discuss the invertibility properties of supersymmetric elements in a neutrosophic field and a refined neutrosophic field by the following theorem.

Theorem 5.8. Let K be a field, $K(I)$ be the corresponding neutrosophic field, $K(I_1, I_2)$ be the corresponding refined neutrosophic field. Then,

(a) An arbitrary supersymmetric element $x = m.a + n.aI \in K(I); m, n \in Z$ and $a \in K$ is invertible if and only if $a \neq 0$ and $(m + n).a \neq 0$.

(b) An arbitrary supersymmetric element $x = (m.a, n.aI_1, s.aI_2) \in K(I_1, I_2); m, n, s \in Z$ and $a \in K$ is invertible if and only if $a \neq 0, (m + s).a \neq 0, (m + n + s).a \neq 0$.

Proof.

(a) According to Theorem 4.9, $x = a + bI$ is invertible if and only if $a \neq 0$ and $a + b \neq 0$, thus $x = m.a + n.aI$ is invertible if and only if $a \neq 0$ and $m.a + n.a = (m + n).a \neq 0$.

(b) It can be proved by a similar argument of section (a), and by using Theorem 4.12.

6. Conclusion

In this article, we have determined the criterion of idempotency in a refined neutrosophic ring. Also, we have introduced two new kinds of special elements in neutrosophic rings and refined neutrosophic rings. We have studied the invertibility conditions of these elements and their algebraic structure. This work should be extended to the case of n-refined neutrosophic rings defined in [12], where the necessary and sufficient condition for nilpotency is still unknown.

References

[1] M. Abobala, *On Some Special Substructures of Neutrosophic Rings and Their Properties*, International Journal of Neutrosophic Science 4 (2020) 72-81.

[2] M. Abobala, *Classical Homomorphisms Between Refined Neutrosophic Rings and Neutrosophic Rings*, International Journal of Neutrosophic Science 5 (2020) 72-75.

[3] M. Abobala, *AH-Subspaces in Neutrosophic Vector Spaces*, International Journal of Neutrosophic Science 6 (2020) 80-86.

[4] M. Abobala, R. Alhamido, *AH-Substructures in Neutrosophic Modules*, International Journal of Neutrosophic Science 7 (2020) 79-86.

[5] M. Abobala, *Classical Homomorphisms Between n-refined Neutrosophic Rings*, International Journal of Neutrosophic Science 7 (2020) 74-78.

[6] A. Hatip, N. Olgun, *On Refined Neutrosophic R-Module*, International Journal of Neutrosophic Science 7 (2020) 87-96.

[7] M. Abobala, *On Some Special Substructures of Refined Neutrosophic Rings,* International Journal of Neutrosophic Science 5 (2020) 59-66.

[8] A. A. A. Agboola, *On Refined Neutrosophic Algebraic Structures*, Neutrosophic Sets and Systems 10 (2015) 99-101.

[9] E. O. Adeleke, A. A. A. Agboola, F. Smarandache, *Refined Neutrosophic Rings I*, International Journal of Neutrosophic Science 2(2) (2020) 77-81.

[10] E. O. Adeleke, A. A. A. Agboola, F. Smarandache, *Refined Neutrosophic Rings II*, International Journal of Neutrosophic Science 2(2) (2020) 89-94.

[11] V. W. B. Kandasamy, F. Smarandache, *Some Neutrosophic Algebraic Structures and Neutrosophic N-Algebraic Structures*, Hexis, Phoenix, Arizona, 2006.

[12] F. Smarandache, M. Abobala, *n-Refined Neutrosophic Rings*, International Journal of Neutrosophic Science 6 (2020) 83-90.

[13] F. Smarandache, M. Abobala, *n-Refined Neutrosophic Vector Spaces*, International Journal of Neutrosophic Science 7 (2020) 47-54.

[14] T. Chalapathi, L. Madhavi, *Neutrosophic Boolean Rings*, Neutrosophic Sets and Systems 33 (2020) 57-66.

[15] Y. Ma, X. Zhang, F. Smarandache, J. Zhang. *The Structure of Idempotents in Neutrosophic Rings and Neutrosophic Quadruple Rings*, Symmetry 11(10) (2019) 1-15.

[16] V. W. B. Kandasamy, K. Ilanthenral, F. Smarandache, *Semi-idempotents in Neutrosophic Rings*, Mathematics 7 (2019) 1-7.

[17] M. A. Ibrahim, A. A. A. Agboola, B. S. Badmus, S. A. Akinleye, *On Refined Neutrosophic Vector Spaces I*, International Journal of Neutrosophic Science 7 (2020) 97-109.