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Non-Markovian Corrections to Quantum Regression Theorem for the Strong Coupling Spin-Boson Model

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Abstract

We report the results of an investigation of the effects of non-Markovian corrections to the dynamics of two-time correlation functions of the strong interaction spin-boson model. Beyond quantum regression theorem corrections are taken into account at the low environmental temperatures for a two-level system (TLS) which is in contact with a structured bath. The results indicate that the corrections lead to appreciable (small) quantitative (qualitative) differences for both biased and non-biased TLS settings.

Keywords: Quantum regression theorem, open quantum systems, spin-boson model, quantum dynamics, two-time correlation functions

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1. INTRODUCTION

Two-time correlation functions (TCFs) for an open quantum system are important tools that provide essential information which might not be available within the single-time averages of the physical system quantities. For instance, TCFs of an atom interacting with electromagnetic field allows to evaluate the fluorescence spectrum [1, 2]. Also, in case of current flowing through nanostructure instruments, TCFs of the current play an important role in explaining the transition properties of current deviations and noise spectrum. Quantum regression theorem (QRT) serves as a bridge between the dynamics of the single-time expectation values and that of their corresponding TCFs and is a useful method to evaluate these two-time (multi-time) correlation functions for open quantum systems when the system-environment interaction is Markovian. With the help of QRT, the knowledge of time rate of change of the reduced density matrix of the system lets one to obtain not only the single-time average values, but also two-time correlation functions in the Markovian case. However, the QRT does not work generally for non-Markovian open quantum systems. To handle this problem, there have been many studies that are concerned with finding dynamical equations for the multi-time correlation functions for non-Markovian quantum systems [3–12], such that two-time functions for non-Markovian systems can be computed using a method similar to the QRT. Alonso and de Vega [7–10] have developed a theory for non-Markovian multi-time correlation functions based on stochastic Schrodinger equation and used it to study a weakly interacting dissipative system with non-diagonal interaction and have shown that validity of quantum regression theorem depends strongly on the form of the system environment correlations. The developed approach was applied to compute the emission spectrum of a two-level system in contact with a structured non- Markovian environment, for example the radiation fields in a a photonic crystal [10]. Goan et. al. [4, 11] have derived an useful two-time correlation functions for non-Markovian dissipative quantum systems in finite temperature environments for any separable system-environment initial states (pure or mixed) by using perturbative quantum master equation, and found that calculating the two-time correlation functions of system quantities of non-Markovian dissipative open systems such as a pure-dephasing spin-boson model is not sufficient to use its single-time average, even in the second order approximation. Also, this exact model allows making the non-Markovian environment temperature arbitrary or finite, which is another important result of these studies.

Some groups have tried to develop a non-Markovianity measure based on the violation of QRT [3, 12]. Manirul Ali and coworkers [12] have investigated non-Markovianity measures based on two-time correlation functions for open quantum system by using an exact master equation based on the non-equilibrium Green's functions. They have found that the non-Markovian dynamics for the Fano-Anderson Hamiltonian significantly depends on the strength of the system-bath coupling and various physical parameters, and tend to show two different behaviors depending on time for different spectral densities. Besides, it is observed that non-Markovian memory effect on the system is always reduced by the thermal bath disturbances.

McCutcheon [5] derived a non-Markovian extension to the quantum regression theorem that gives us the facility about the calculation of two-time correlation functions and emission spectrum of weakly driven dissipative two-level system by using projection method. He has found that sideband are related to information flow from the phonon environment to the quantum dot system, which justifies true non-Markovianity and indivisibility of the dynamical map [13]. Cosacchi et. al. [6] present a practical method to calculate the multi-time correlation functions by using path integral method in the presence of the memory of the environment. Cosacchi et. al. [6] observes that the approach of McCutcheon [5] may lead to unphysical results, which presents phonon side-band appears on the wrong side of the Mollow triplet. They study it on the pure-dephasing type coupling to bosonic harmonic environments. They have shown that the resulting method that is used to characterize the emission spectrum of a quantum dot interacting with longitudinal acoustic phonons allows to the phonon sideband appears on the correct side of the zero phonon line, which is not an agreement with the result obtained from the naive application of the quantum regression theorem.

In the literature, the studies relating to the time evolution of TCFs in the non-Markovian Dynamics considered only the weak system-bath coupling regime. One can naturally ask whether findings of those studies would hold also in the strong coupling regime? Towards an aim to answer that question, in the current work, we will analyze the contribution of non-Markovian corrections to quantum regression theorem of the two-time correlation function of the σ_z operator for the strong coupling spin-boson model with an environment that is described by a structured spectral density.

The article is organized as follows. In Sec. II, we present the strongly driven spin-boson mode in the polaron frame and give basic information about the expressions of time evolution of one and twotime correlation functions for any system operators. The important results of study are the presented in Sec. III. The paper concludes with a brief summary of the findings in Sec. IV.

2. MODEL

We consider a two-level system (TLS) that is interacting with an harmonic environment, of which the energy splitting is ω_A and the tunneling matrix element is V. The Hamiltonian of the closed system composed of the TLS and its environment can be expressed as:

$$H = H_S + H_B + H_I, \tag{1}$$

where H_S is the Hamiltonian of the TLS, H_B is the Hamiltonian of the reservoir which is a collection of independent harmonic oscillators with mode frequencies ω_k, and H_I is the interaction between the TLS and its environment (with $\hbar = 1$):

$$\begin{aligned} H = {} & \frac{\omega_A}{2}\sigma_z + \frac{V}{2}\sigma_x + \sum_k \omega_k a_k^\dagger a_k \\ & + \sigma_z \sum_k g_k \left(a_k^\dagger + a_k\right). \end{aligned} \tag{2}$$

Here $a^\dagger$ (a) is a creation (annihilation) operators of the bath oscillator and g_k is the magnitude of the interaction between the TLS and the k^{th} bath oscillator. Here, we assume that the interaction strength is in the strong coupling regime, so transforming Hamiltonian in Eq. (2) into polaron frame is very natural, which can be obtained by

$$\begin{aligned} H^{'} &= H_0^{'} + H_I^{'}, \\ &= \frac{\omega_A}{2}\sigma_z + V_r\sigma_x + \sum_k \omega_k a_k^\dagger a_k \\ &\quad + \sigma_+ B_- + \sigma_- B_+. \end{aligned} \tag{3}$$

Here the superscript " $'$ " implies that O' is in the polaron frame. In the following we will drop the superscript for simplicity. $\sigma_\pm$ are, then, the spin flip operators of the TLS, V_r is the reduced tunneling matrix element and $B_\pm$ refer the bath correlation operators, and given as a function of system-bath coupling strength g_k with the k^{th} oscillator mode:

$$B_\pm = \langle e^{\mp\sum_k \frac{2g_k}{\omega_k}\left(a_k^\dagger - a_k\right)} \rangle_R\,, \tag{4}$$

where $\langle \ldots \rangle_R$ denotes averaging over the environmental degrees of freedom. It is important to note that the reduced tunneling rate V_r is equal to zero for bath spectral densities whose frequency exponent is less than 2 which is the case in the present study.

The evolution equation of the reduced density matrix of the TLS in the interaction picture can be written as

$$\frac{d}{dt}\rho_S(t) = -\int_0^t dt_1 Tr_R\left[H_I(t), \left[H_I(t_1), \rho_S \otimes \rho_R\right]\right], \tag{5}$$

where Tr_R indicates partial trace over the environmental modes. Second-order evolution equations of any system operator A in the Schrödinger Picture can be evaluated by using

$$\frac{d}{dt_1}\langle A(t_1)\rangle = iTr_{S\otimes R}(\{[H_S, A]\}(t_1)\rho_T(0)) + \int_0^{t_1} d\tau Tr_{S\otimes R}(\{\tilde{H}_I(\tau - t_1)[A, H_I]\}(t_1)\rho_T(0) + \{[H_I, A]\tilde{H}_I(\tau - t_1)\}(t_1)\rho_T(0)). \quad (6)$$

Here, $\tilde{H}_I\ (t)$ is the time-dependent interaction Hamiltonian:

$$\tilde{H}_I(t) = V\ (\sigma_{-}(t)B_+(t) + \sigma_+(t)B_-(t)), \quad (7)$$

which describes the time evolution in the interaction picture in the polaron frame. Curly brackets in Eqs. (6) and (8) indicate that the expression should be evaluated in the Heisenberg picture and its time should be taken as given in the post bracket. In order to establish the two time correlation functions for the non-Markovian evolution which has the optical results of some physical systems [1, 2], one can use the result of Refs. [4, 5], and its formulation is given as:

$$\frac{d}{dt_1}\langle A(t_1)B(t_2)\rangle = iTr_{S\otimes R}(\{[H_S, A]\}(t_1)B(t_2)\rho_T(0)) + \int_0^{t_1} d\tau Tr_{S\otimes R}(\{\tilde{H}_I(\tau - t_1)[A, H_I]\}(t_1)B(t_2)\rho_T(0) + \{[H_I, A]\tilde{H}_I(\tau - t_1)\}(t_1)B(t_2)\rho_T(0)) + \int_0^{t_2} d\tau Tr_{S\otimes R}\left(\{[H_I, A]\}(t_1)\{[B, \tilde{H}_I(\tau - t_2)]\}(t_2)\rho_T(0)\right). \quad (8)$$

Here, the first two terms on the right hand side of Eq. (8) are the QRT terms while the last term accounts for the corrections for the non-Markovian effects. The last integral term in Eq. (8) is the source of violation of the quantum regression theorem.

3. RESULTS

In the present study, we will investigate the dynamics of the quantity $\langle\sigma_z(t_1)\sigma_z(t_2)\rangle$ for the spinboson model in strong coupling regime within polaron frame for an arbitrary bath spectral density. Choosing $A = \sigma_z$ and $B = \sigma_z$ in Eq. (8), we obtain a set of six coupled differential equations for various combinations of TLS system operator as:

$$\frac{d}{dt_1}\langle\sigma_z(t_1)\sigma_z(t_2)\rangle = -4\Gamma_1(t_1)\langle\sigma_z(t_1)\sigma_z(t_2)\rangle - 4\Gamma_2(t_1)\langle\sigma_z(t_2)\rangle + 2(\Gamma_5(t_1,t_2)\langle\sigma_-(t_1)\sigma_+(t_2)\rangle - \Gamma_6(t_1,t_2)\langle\sigma_+(t_1)\sigma_-t_2)\rangle - \Gamma_7(t_1,t_2)\langle\sigma_+(t_1)\sigma_+(t_2)\rangle + \Gamma_8(t_1,t_2)\langle\sigma_-(t_1)\sigma_-(t_2)\rangle), \quad (9)$$

$$\frac{d}{dt_1}\langle\sigma_+(t_1)\sigma_-(t_2)\rangle = [i\omega_A - 2(\Gamma_1(t_1) + i\Gamma_3(t_1))]\langle\sigma_+(t_1)\sigma_-(t_2)\rangle + 2\Gamma_4(t_1)^*\langle\sigma_-(t_1)\sigma_-(t_2)\rangle + \Gamma_5(t_1,t_2)\langle\sigma_z(t_1)\sigma_z(t_2)\rangle, \quad (10)$$

$$\frac{d}{dt_1}\langle\sigma_-(t_1)\sigma_+(t_2)\rangle = -[i\omega_A + 2(\Gamma_1(t_1) - i\Gamma_3(t))]\langle\sigma_-(t_1)\sigma_+(t_2)\rangle + 2\Gamma_4(t_1)\langle\sigma_+(t_1)\sigma_+(t_2)\rangle + \Gamma_6(t_1,t_2)\langle\sigma_z(t_1)\sigma_z(t_2)\rangle, \quad (11)$$

$$\frac{d}{dt_1}\langle\sigma_+(t_1)\sigma_+(t_2)\rangle = [i\omega_A - 2(\Gamma_1(t_1) + i\Gamma_3(t_1))]\langle\sigma_+(t_1)\sigma_+(t_2)\rangle + 2\Gamma_4(t_1)^*\langle\sigma_-(t_1)\sigma_+(t_2)\rangle - \Gamma_8(t_1,t_2)\langle\sigma_z(t_1)\sigma_z(t_2)\rangle, \quad (12)$$

$$\frac{d}{dt_1}\langle\sigma_-(t_1)\sigma_-(t_2)\rangle = -[i\omega_A + 2(\Gamma_1(t_1) - i\Gamma_3(t_1))]\langle\sigma_-(t_1)\sigma_-(t_2)\rangle$$
$$+2\Gamma_4(t_1)\langle\sigma_+(t_1)\sigma_-(t_2)\rangle - \Gamma_7(t_1,t_2)\langle\sigma_z(t_1)\sigma_z(t_2)\rangle, \quad (13)$$
$$\frac{d}{dt_1}\langle\sigma_z(t_1)\rangle = -4\Gamma_1(t_1)\langle\sigma_z(t_1)\rangle - 4\Gamma_2(t_1). \quad (14)$$

Here, the first two terms in Eq.(9) (originating from the first two terms of Eq.(8)) account for the quantum regression theorem with non-Markovian effects while the rest of Eq.(9) describe the corrections to QRT for the non-Markovian dynamics. These correction terms for QRT allow, then, us to get the last terms in Eq.(10)-Eq.(13), while the other terms in these equations come from the QRT only. Also, the time-dependent coefficients of the differential equations are:

$$\begin{aligned}
\Gamma_1(t_1) &= V^2\int_0^{t_1} d\tau e^{-Q_2(\tau-t_1)}cos(Q_1(\tau-t_1))cos(\omega_A(\tau-t_1)),\\
\Gamma_2(t_1) &= V^2\int_0^{t_1} d\tau e^{-Q_2(\tau-t_1)}sin(Q_1(\tau-t_1))sin(\omega_A(\tau-t_1)),\\
\Gamma_3(t_1) &= V^2\int_0^{t_1} d\tau e^{-Q_2(\tau-t_1)}cos(Q_1(\tau-t_1))sin(\omega_A(\tau-t_1)),\\
\Gamma_4(t_1) &= V^2\int_0^{t_1} d\tau e^{-Q_3(\tau-t_1)}cos(Q_1(\tau-t_1))e^{i\omega_A(\tau-t_1)},\\
\Gamma_5(t_1,t_2) &= V^2\int_0^{t_2} d\tau e^{-Q_2(\tau-t_2-t_1)+iQ_1(\tau-t_2-t_1)}e^{i\omega_A(\tau-t_2)},\\
\Gamma_6(t_1,t_2) &= V^2\int_0^{t_2} d\tau e^{-Q_2(\tau-t_2-t_1)+iQ_1(\tau-t_2-t_1)}e^{-i\omega_A(\tau-t_2)},\\
\Gamma_7(t_1,t_2) &= V^2\int_0^{t_2} d\tau e^{-Q_3(\tau-t_2-t_1)-iQ_1(\tau-t_2-t_1)}e^{i\omega_A(\tau-t_2)},\\
\Gamma_8(t_1,t_2) &= V^2\int_0^{t_2} d\tau e^{-Q_3(\tau-t_2-t_1)-iQ_1(\tau-t_2-t_1)}e^{-i\omega_A(\tau-t_2)},
\end{aligned} \quad (15)$$

where

$$\begin{aligned}
Q_1(t) &= \frac{1}{2\pi}\int_0^\infty d\omega\frac{J(\omega)}{\omega^2}sin(\omega t),\\
Q_2(t) &= \frac{1}{2\pi}\int_0^\infty d\omega\frac{J(\omega)}{\omega^2}coth\left(\frac{\beta\omega}{2}\right)(1-cos(\omega t)),\\
Q_3(t) &= \frac{1}{2\pi}\int_0^\infty d\omega\frac{J(\omega)}{\omega^2}coth\left(\frac{\beta\omega}{2}\right)(1+cos(\omega t)),
\end{aligned}$$

where $Q_{1,2,3}(t)$ are the real and the imaginary parts of the bath correlation functions as a function of time, inverse temperature $\beta = 1/k_BT$. $J(\omega)$ is the bath spectral function which quantifies the frequency distribution of the bath oscillators as well as the interaction between the two level system and the bath. The reorganization energy which is defined as $E_r = \int_0^\infty d\omega\, J(\omega)/\omega$ is a rough measure of interaction strength between the TLS and its environment. In the current study, we choose $J(\omega)$ as a structured spectral density function that defines the environment as a single harmonic oscillator whose levels are broadened by its interaction with an Ohmic environment of non-interacting harmonic oscillators [14]:

$$J(\omega) = 8\kappa^2\frac{\gamma\omega_0\omega}{(\omega^2-\omega_0^2)^2+4\gamma^2\omega^2}\ , \quad (16)$$

where ω_0 is the frequency of the central harmonic oscillator, κ is the strength of TLS-bosonic bath coupling, γ is the broadening term of the oscillator levels due to its interaction with the Ohmic bath. It should be noted that the reorganization energy for the chosen $J(\omega)$ is equal to κ^2/ω_0. For the structured spectral density, the bath correlation function $\exp(-Q_3(t))$ is zero which leads to vanishing of the kernels Γ_4, Γ_7, and Γ_8 in Eqs. (9)-(14). Those equations in simplified form become:

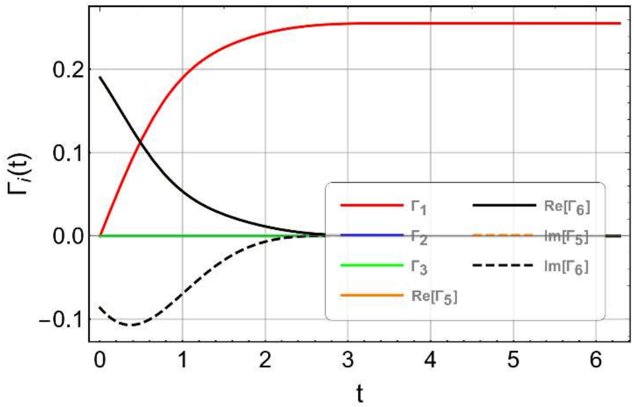
$$\frac{d}{dt_1}\langle\sigma_z(t_1)\sigma_z(t_2)\rangle = -4\Gamma_1(t_1)\langle\sigma_z(t_1)\sigma_z(t_2)\rangle - 4\Gamma_2(t_1)\langle\sigma_z(t_2)\rangle$$
$$+2(\Gamma_5(t_1,t_2)\langle\sigma_-(t_1)\sigma_+(t_2)\rangle - \Gamma_6(t_1,t_2)\langle\sigma_+(t_1)\sigma_-t_2)\rangle), \quad (17)$$
$$\frac{d}{dt_1}\langle\sigma_+(t_1)\sigma_-(t_2)\rangle = [i\omega_A - 2(\Gamma_1(t_1) + i\Gamma_3(t_1))]\langle\sigma_+(t_1)\sigma_-(t_2)\rangle$$

$$+4\Gamma_5(t_1,t_2)\langle\sigma_z(t_1)\sigma_z(t_2)\rangle, \tag{18}$$
$$\frac{d}{dt_1}\langle\sigma_-(t_1)\sigma_+(t_2)\rangle = -[i\omega_A + 2(\Gamma_1(t_1) - i\Gamma_3(t))]\langle\sigma_-(t_1)\sigma_+(t_2)\rangle$$
$$+4\Gamma_6(t_1,t_2)\langle\sigma_z(t_1)\sigma_z(t_2)\rangle, \tag{19}$$
$$\frac{d}{dt_1}\langle\sigma_z(t_1)\rangle = -4\Gamma_1(t_1)\langle\sigma_z(t_1)\rangle - 4\Gamma_2(t_1). \tag{20}$$

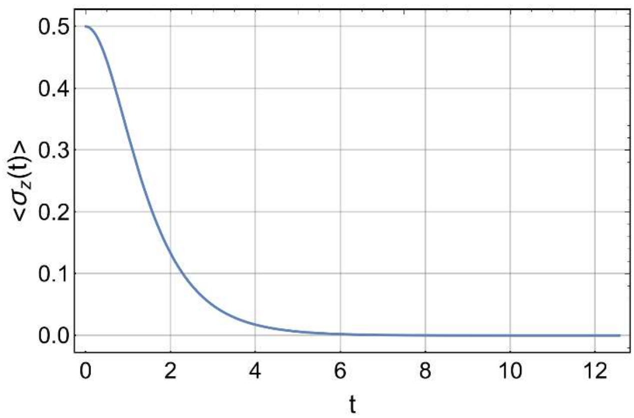
It is significant to note that the time rate of change of $\langle\sigma_z(t_1)\sigma_z(t_2)\rangle$ does not depend on $\langle\sigma_+(t_1)\sigma_+(t_2)\rangle$ and $\langle\sigma_-(t_1)\sigma_-(t_2)\rangle$ for the environment with the structured spectral density. As a result, six coupled differential equations of Eqs. (9)-(14) are reduced to four differential equations (Eqs. 17-20). In these equations, the terms that contain the kernel function ($\Gamma_{1,2,3}$) with a single time argument are due to non-Markovian quantum regression theorem while ($\Gamma_{5,6}$) with two time arguments account for the corrections to QRT for the non-Markovian dynamics which will be referred to as QRT+ for the rest of the paper. Non-Markovian QRT predictions reported below are obtained by neglecting $\Gamma_{5,6}$ terms in Eqs. (17)-(20).

As the numerical solutions of the dynamical equations of the two time correlation functions of the spin boson model in the current study, we will consider a low temperature environment with relatively strong system-bath coupling and investigate the difference between the non-Markovian QRT and QRT+ dynamics for two different values of level splitting of the two-level system. Initial state of the TLS+ bath closed system is taken as $\rho_0 = \rho_S(0) \otimes \rho_E(0)$ where $\rho_E(0)$ is the thermal equilibrium of the environment while $\rho_S(0)$ is the density matrix of the pure TLS state $|\psi\rangle = \frac{\sqrt{3}}{2}|e\rangle + \frac{1}{2}|g\rangle$.

First, we consider the zero-bias case for the TLS and display the kernel functions, magnetization and the two-time correlation functions obtained by numerically solving Eq. (17)-(20) in Fig. 1, respectively. The other system and the environment parameters which indicate a relatively strong system-bath interaction ($E_r = 1$) and low environmental temperature ($\beta = 100$) are provided in the caption of the figure. As can be seen from Fig. 1(a), for the non-biased TLS, only Γ_1 and Γ_6 kernels are nonzero. In the $t \to \infty$ limit, both real and imaginary parts of Γ_6 vanishes. Behaviour of both $\langle\sigma_z(t)\rangle$ and the two time correlation functions can be understood easily based on the time dependence of these correlation functions: All the considered quantities should exponentially decay to zero at long time limit with a decay rate which is a function of the kernel Γ_1. The dynamics of two-time correlation functions obtained with non-Markovian QRT and QRT+ approaches (dashed and straight lines in Fig. 1(c) indicate that the qualitative difference between the results obtained by employing those two approaches is very small while the quantitative difference for $\langle\sigma_-(t_1)\sigma_+(t_2)\rangle$ and $\langle\sigma_+(t_1)\sigma_-(t_2)\rangle$ is larger than that for $\langle\sigma_z(t_1)\sigma_z(t_2)\rangle$.



(a)



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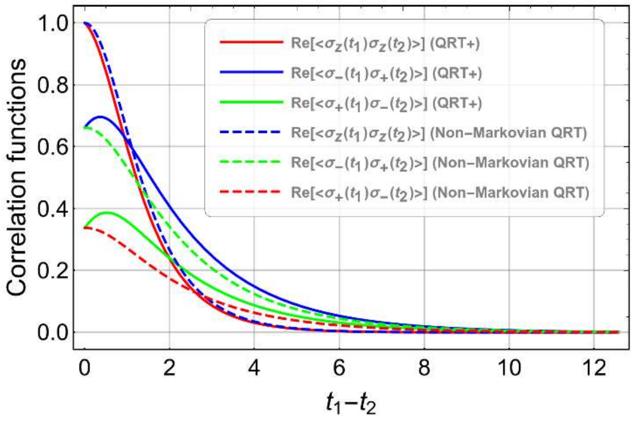
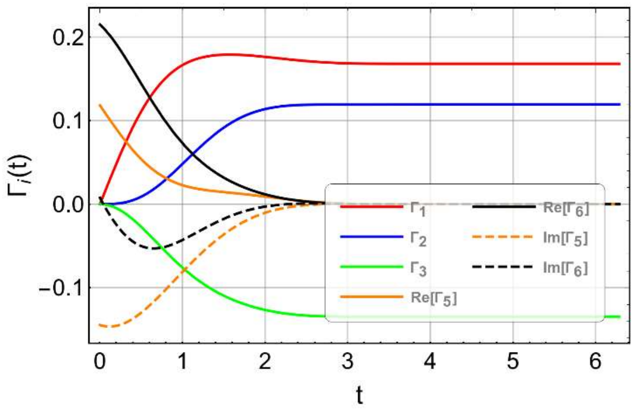


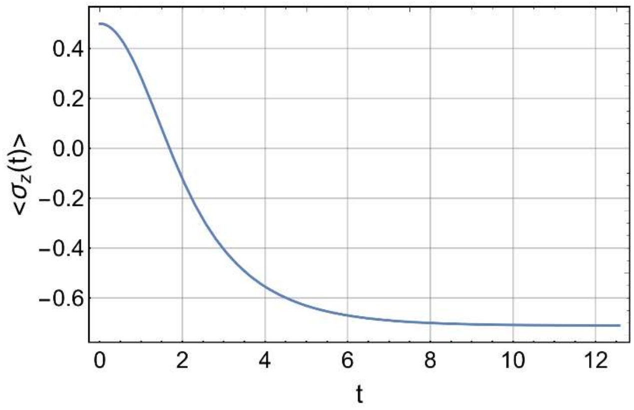
(c)

Figure 1. (a) Time-dependence of the kernels $\Gamma_{1,2,3}(t)$, and the real and the imaginary parts of $\Gamma_{5,6}(t_1, t_2)$ for the set of Eqs.(17)-(20), (b) dynamics of $\langle\sigma_z(t)\rangle$, and (c) time evolutions of the real parts of system operators for two cases: QRT (solid line) and non-Markovian QRT (dashed line). Other parameters used are $t_2 = 1$, $V = 0.5$, $\omega_0 = 1$, $\gamma = 0.1$, $\kappa = 1$, $\beta = 100$, $E_r = \kappa^2/\omega = 1$ and the TLS is considered to be non-biased ($\omega_0 = 0$).

We display, also, the same set of graphs in Figs.2(a)-(c) for the resonance case for which the splitting frequency of the TLS is equal to the central frequency of the environmental oscillator. Contrary to the non-biased case (Fig.1(a)), one can see that all kernels are non-zero at the short time while only the single time argument ones remain non-zero at the long time limit. This result causes that the average of $\sigma_z(t)$ to exponentially decay to a terminal value determined by the ratio $\Gamma_2(\infty)/\Gamma_1(\infty)$, as can be seen from Fig.2(b). It is also interesting to note that the dynamics of $\langle\sigma_z(t_1)\sigma_z(t_2)\rangle$ for the biased and non-biased TLS are very similar with somewhat different steady-state values as displayed Fig.1(c). The dynamics of $\langle\sigma_\pm(t_1)\sigma_\mp(t_2)\rangle$ for biased TLS differ from the those for the non-biased case in Fig.2(c). This result can readily be explained by the fact that both Γ_3 and ω_A are non-zero, which drives these system operators to coherent decay with decay rate related to the integrals of Γ_1, Γ_2, Γ_5, Γ_6 and oscillation frequency which is related to ω_A and Γ_3. The oscillations of two-time correlation functions are more pronounced in the resonance case. The difference between the results obtained by using QRT+ and non-Markovian QRT approaches is found to be smaller for the $\langle\sigma_\pm(t_1)\sigma_\mp(t_2)\rangle$ than that for $\langle\sigma_z(t_1)\sigma_z(t_2)\rangle$ quantitatively, contrary to the findings in the non-biased case discussed above. In general, the results presented above show that the Dynamics for both QRT+ and the non-Markovian QRT have noticeably similar behavior for all time intervals for the strongly driven spin-boson model. This result is not valid for the weak system-bath coupling for the certain system parameters [4, 11]. Finally, we can conclude that the non-Markovian two-time correlation functions obtained in the polaron frame has somewhat different behavior depending on the TLS splitting term ω_A.



(a)



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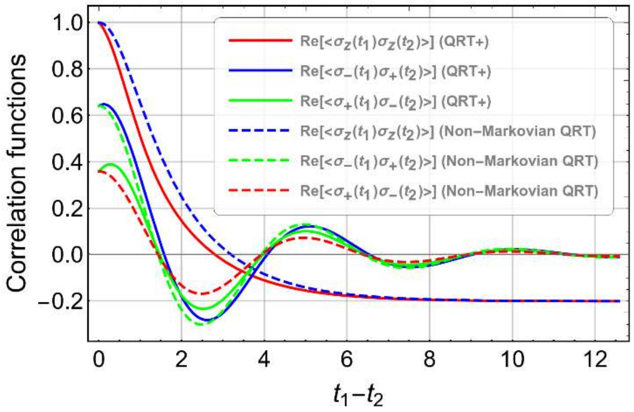

(c)

Figure 2. Same as Fig.(1) for the resonance between the TLS and the environmental oscillator ($\omega_A = \omega_0 = 1$).

4. CONCLUSIONS

In this work, we have investigated that timedependence of non-Markovian two-time correlation functions for a TLS that is in contact with a harmonic oscillator which in turn is interacting with a thermal bath of harmonic oscillators by transforming the system Hamiltonian into polaron frame when the strength of system-bath interaction is strong. We have investigated the effect of including non-Markovian corrections to the dynamics of the twocorrelation functions beyond the quantum regression theorem. In the low temperature, relatively strong system-bath interaction regime we have found that although the QRT corrections lead to small qualitative changes in the dynamics, overall qualitative time-dependence of the two time-correlation functions does not change appreciably by inclusion of the QRT corrections. This result was found to be true for both biased and non-biased system settings. If QRT violations can be considered as an indication of non-Markovianity of the dynamics, one can claim that the dynamics of the model is near Markovian for the parameter regime studied in the current work.

Research and Publication Ethics

This paper has been prepared within the scope of international research and publication ethics.

Ethics Committee Approval

This paper does not require any ethics committee permission or special permission.

Conflict of Interests

The author declared no potential conflicts of interest with respect to the research, authorship, and/or publication of this paper.

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