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A note on nearly hyperbolic cosymplectic manifolds

Nearly hiperbolik kosimplektik manifoldlar üzerine bir not

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A Note on Nearly Hyperbolic Cosymplectic Manifolds

Highlights

- *Sub manifods of nearly hyperbolic cosymplectic manifolds were investigated.*
- *Distrubutions of Sub manifods were defined.*
- *Integrability conditions were investigated.*
- *Integrability conditions were determined for the distributions of these sub manifolds.*
- *A pseudo submanifold M of a nearly $\widetilde{M}$ hyperbolic cosymplectic manifold which is totally geodesic, D_θ,D_μ distributions with $D_\mu \oplus \langle\xi\rangle$ and $D_\theta \oplus D_\mu$ are proved integrable but $D_\theta \oplus \langle\xi\rangle$ or not.*

Graphical Abstract

$\varphi^2 X = X + \eta(X)\xi$

$g(X,\xi) = \eta(X),\ \eta(\xi) = -1$

$\varphi(\xi) = 0,\ \eta \circ \varphi = 0$

$g(\varphi X, \varphi Y) = g(X,Y) - \eta(X)\eta(Y)$

i-) $TM = D_\theta \oplus D_\mu \oplus \langle\xi\rangle$,

ii-) D_θ is slant distribution with slant angle $\theta \neq \frac{\pi}{2}$,

iii-) D_μ is anti-invariant, that is; $\varphi D_\mu \subseteq T^\perp M$.

It is shown that $[X,Y] \in D_\theta \oplus D_\mu$ for any vector fields X,Y in $D_\theta \oplus D_\mu$. On the other hand,
$(\overline{\nabla}_X g)(Y,\xi) = \overline{\nabla}_X g(Y,\xi) - g(\overline{\nabla}_X Y,\xi) - g(Y,\overline{\nabla}_X \xi) = 0$
and from the last equation;
$g(\overline{\nabla}_X Y,\xi) = 0$
for any vector fields X,Y in $D_\theta \oplus D_\mu$. In the same way, it can be written,
$g(\overline{\nabla}_Y X,\xi) = 0$
for any vector fields X and Y in $D_\theta \oplus D_\mu$. If (3.14) and (3.15) use, it can be defined by
$g([X,Y],\xi) = 0$
for any vector fields X,Y in $D_\theta \oplus D_\mu$. In this case, the proof is complete.

Aim

Determine the integrability requirements of the distribution for the sub manifolds of the Nearly Hyperbolic Cosimplectic Manifolds.

Design & Methodology

Definition, theorem and proof method are used.

Originality

Setting the distribution integrability requirements for the sub manifolds of Nearly Hyperbolic Cosimplectic Manifolds.

Findings

A pseudo submanifold M of a nearly hyperbolic cosymplectic manifold $\widetilde{M}$ which is totally geodesic, D_θ,D_μ distributions with $D_\mu \oplus \langle\xi\rangle$ and $D_\theta \oplus D_\mu$ are proved integrable but $D_\theta \oplus \langle\xi\rangle$ or not.

Conclusion

*It is think that, the results which are obtained in this study are important for differential geometers who are dealing with nearly hyperbolic cosymplectic metric manifolds, the results which are stated in this study can be handled in different form***.**

Declaration of Ethical Standards

The author(s) of this article declare that the materials and methods used in this study do not require ethical committee permission and/or legal-special permission.

Nearly Hiperbolik Kosimplektik Manifoldlar Üzerine Bir Not

Araştırma Makalesi / Research Article

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ÖZ

Son yıllarda hiperbolik kosimplektik manifoldlara olan ilgi artmaktadır. Bu çalışmada bir nearly hiperbolik kosimplektik manifoldun pseudo-slant altmanifoldları derinlemesine araştırılıp incelenmiştir. Özellikle bu tip manifoldların distribusyonlarının integrallenebilirlik koşulları araştırılmıştır. Böylece total geodezik bir nearly hiperbolik kosimplektik $\widetilde{M}$ manifoldunun pseudo slant M altmanifoldu için D_θ ve D_μ distribusyonları ile $D_\mu \oplus \langle\xi\rangle$ ve $D_\theta \oplus D_\mu$ nın integrallenebilir olduğu, $D_\theta \oplus \langle\xi\rangle$ nın integrallenebilir olmadığı ispatlanmıştır.

Anahtar Kelimeler: Pseudo slant altmanifold, nearly hiperbolik kosimplektik manifold, integrallenebilirlik şartları.

A Note on Nearly Hyperbolic Cosymplectic Manifolds

ABSTRACT

Interest in hyperbolic cosymplectic manifolds has increased in recent years. In this paper, pseudo slant submanifolds of a nearly hyperbolic cosymplectic manifold have been explored deeply. In particular, the integrability conditions of distributions on such manifolds have been investigated. So, for a pseudo slant submanifold M of a nearly hyperbolic cosymplectic manifold $\widetilde{M}$ which is totally geodesic, D_θ,D_μ distributions with $D_\mu \oplus \langle\xi\rangle$ and $D_\theta \oplus D_\mu$ are proved integrable but $D_\theta \oplus \langle\xi\rangle$ or not.

Keywords: Pseudo slant submanifold, nearly hyperbolic cosymplectic manifold, integrability condition.

1. INTRODUCTION

Upadhyay and Dube [1] have investigated and described almost hyperbolic metric (ϕ,ξ,η,g) structure. Joshi and Dube [2] have explored semi invariant submanifolds of an almost r-contact hyperbolic submanifold. Dogan and Karadag [3] have researched on slant submanifolds of an almost hyperbolic contact metric manifolds. Uddin, Wong and Mustafa [4] have surveyed on warped product pseudo slant submanifolds of a nearly cosymplectic manifold. Ahmad and Ali [5] have introduced the notion of semi invariant submanifolds of a nearly hyperbolic cosymplectic manifold. In addition ; Ahmad and Ali [6] have studied CR-submanifolds of a nearly hyperbolic cosymplectic manifold. Pseudo-slant submanifolds of a Sasakian manifold is studied by Khan and Khan [7]. De and Sarkar [8] have studied pseudo-slant submanifolds of trans-sasakian manifold. Blair [9] has been working on geometry of contact and symplectic manifolds for a long time. Fujumoto and Olszak [10,11] have many studies on cosymplectic manifolds. In [12] and [13] have studied contact metric manifolds. Endo, Yano and Goldberg have studied on almost cosymplectic manifolds [14,15]. Pseudo slant submanifolds of a nearly hyperbolic cosymplectic manifold hava been explored and then a solution has been tried to find on a pseudo submanifold of a nearly hyperbolic cosymplectic manifold has been evaluated and determined. In this study, pseudo slant submanifolds of nearly hyperbolic cosymplectic manifold have been examined and some results have been obtained on this manifolds.In particular, the integrability conditions of the distributions of these manifolds have been investigated. Firstly, n-dimensional almost hyperbolic contact metric manifold $\widetilde{M}$ which is given with almost hyperbolic contact metric structure (ϕ,ξ,η,g) has been defined. Also, it is obtained some interesting results for integrability of a nearly hyperbolic cosymplectic manifold.

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2. MATERIAL AND METHOD

2. 1. Preliminaries

The aim of this study was redefined and studied for nearly cosymplectic metric manifolds based on [1], [2] and [4]. Now, it can give a brief description and some results on distribution of nearly hyperbolic cosymplectic manifolds. Let $\widetilde{M}$ be an $(2n+1)$-dimensional almost hyperbolic contact metric manifold with almost hyperbolic contact metric structure (ϕ,ξ,η,g). Here φ is a tensor of type $(1,1)$, ξ is a vector field called structure vector field and η is the 1-form which are satisfying the followings

$$\varphi^2X = X + \eta(X)\xi \tag{1}$$

$$g(X,\xi) = \eta(X),\ \eta(\xi) = -1 \tag{2}$$

$$\varphi(\xi) = 0,\ \eta\circ\varphi = 0 \tag{3}$$

$$g(\varphi X,\varphi Y) = g(X,Y) - \eta(X)\eta(Y) \tag{4}$$

for any vector fields X and Y in TM. Also, it is showed that

$$g(\varphi X,Y)=-g(X,\varphi Y) \tag{5}$$

An almost hyperbolic contact metric manifold with almost hyperbolic contact metric structure (φ,ξ,η,g) is said to be nearly hyperbolic cosymplectic manifold [4] if

$$(\overline{\nabla}_X\varphi)Y+\varphi(\overline{\nabla}_Y X)=0 \tag{6}$$

$$\overline{\nabla}_X\xi=0 \tag{7}$$

for all $X,Y\in TM$.

Let M be submanifold of a nearly hyperbolic cosymplectic manifold $\widetilde{M}$ with induced metric g and if ∇ and $\nabla^{\perp}$ are the induced connections on the tangent bundle TM and the normal bundle $T^{\perp}M$ of M, respectively, then Gauss and Weingarten formulae are given by

$$\overline{\nabla}_X Y=\nabla_X Y+h(X,Y) \tag{8}$$

and

$$\overline{\nabla}_X N=-A_N X+\nabla_X^{\perp}N \tag{9}$$

For all X,Y in TM and $N\in T^{\perp}M$, where h and A_N are the second fundamental form and the shape operator, respectively. Also, it is showed that:

$$g(h(X,Y),N)=g(A_N X,Y) \tag{10}$$

where g denotes the Riemannian metric on M as well as induced on $\widetilde{M}$. For any vector field X in TM, let

$$\varphi X=PX+FX, \tag{11}$$

where PX is the tangential component and FX is the normal component of φX, respectively. Similarly, for any $N\in T^{\perp}M$, let

$$\varphi N=tN+fN\ , \tag{12}$$

where tN is the tangential component and fN is the normal component of φN, respectively.

A submanifold M of a nearly hyperbolic cosymplectic manifold $\widetilde{M}$ is said to be a pseudo slant submanifold if there exist two orthogonal complementary distributions D_θ and D_μ satisfy such that [4] ;

i-) $TM=D_\theta\oplus D_\mu\oplus\langle\xi\rangle$,

ii-) D_θ is slant distribution with slant angle $\theta\neq\frac{\pi}{2}$,

iii-) D_μ is anti-invariant, that is; $\varphi D_\mu\subseteq T^{\perp}M$.

A pseudo slant submanifold M of a nearly hyperbolic cosymplectic manifold $\widetilde{M}$ is totally geodesic if

$$h(X,Y)=0 \tag{13}$$

for any vector fields X,Y in TM.

3. RESULTS AND DISCUSSION

3.1. Integrability Conditions

In this section, it works out integrability conditions of distributions on these manifolds as [7] and also, obtain a few interesting results of this setting.

Let M be a pseudo slant submanifold of a nearly hyperbolic cosymplectic manifold $\widetilde{M}$ and D_θ, D_μ be distributions and ξ be the vector field. Thus, it can be given following theorems:

Theorem 1 Let M be a pseudo slant submanifold of a nearly hyperbolic cosymplectic manifold $\widetilde{M}$. If M is total geodesic, then D_μ is integrable.

Proof. It is assume that M be a pseudo slant submanifold of a nearly hyperbolic cosymplectic manifold $\widetilde{M}$. From Eq. 6, it can show

$$(\overline{\nabla}_Z\varphi)W+\varphi(\overline{\nabla}_W Z)=0 \tag{14}$$

$$(\overline{\nabla}_Z\varphi)W-\varphi(\overline{\nabla}_Z W)+\varphi(\overline{\nabla}_W Z)=0 \tag{15}$$

$$(\overline{\nabla}_Z\varphi)W=\varphi[Z,W] \tag{16}$$

for any vector fields Z,W in D_μ.

Now, if takes scalar product
$(\overline{\nabla}_Z\varphi)W$ and X in TM, from Eq. 9, it is obtained

$$g(\overline{\nabla}_Z\varphi W,X)=-g(A_{\varphi W}Z,X)+g(\nabla_Z^{\perp}\varphi W,X) \tag{17}$$

$$g(\overline{\nabla}_Z\varphi W,X)=-g(h(Z,X),\varphi W) \tag{18}$$

for any vector fields Z, W in D_μ and $X\in TM$.

Since M is totally geodesic and g is non degenere it is obtained

$$g(\overline{\nabla}_Z\varphi W,X)=0. \tag{19}$$

Thus, equation can be defined as $\overline{\nabla}_Z\varphi W\in T^{\perp}M$. From (16), it is seen that $[Z,W]\in D_\mu$. Hence D_μ is integrable.

Theorem 2 Let M be a pseudo slant submanifold of a nearly hyperbolic cosymplectic manifold $\widetilde{M}$. Then the slant distribution D_θ is integrable.

Proof. It is shown that $[X,Y]\in D_\theta$ for any vector field $X,Y\in D_\theta$. Since M is nearly hyperbolic cosymplectic manifold, from Eq. 6; f or any vector field $X,Y\in D_\theta$

$$(\overline{\nabla}_X\varphi)Y+\varphi(\overline{\nabla}_Y X)=0. \tag{20}$$

In this case, it is obtained for $X,Y\in D_\theta$

$$\overline{\nabla}_X\varphi Y-\varphi\overline{\nabla}_X Y+\varphi\overline{\nabla}_Y X=0 \tag{21}$$

and

$$\overline{\nabla}_X\varphi Y=\varphi[X,Y]. \tag{22}$$

If it uses Eq.11 in Eq. 22, it can get

$$\overline{\nabla}_X PY+\overline{\nabla}_X FY=P[X,Y]+F[X,Y] \tag{23}$$

for any vector fields $X,Y\in D_\theta$. If it uses Eq. 8 and Eq. 9 in Eq. 23 and it can be considered tangent and normal component, it can be obtained

$$\nabla_X PY-A_{FY}X=P[X,Y] \tag{24}$$

and

$$h(X,PY)+\nabla_X^{\perp}FY=F[X,Y] \tag{25}$$

for any vector fields $X,Y\in D_\theta$.

Let θ be an angle between $P[X,Y]$ and $\varphi[X,Y]$. İt is shown that θ is constant. It can be defined,

$$\cos\theta = \frac{g(P[X,Y],\varphi[X,Y])}{\|P[X,Y]\|\|\varphi[X,Y]\|} \tag{26}$$

for any vector fields $X, Y \in D_\theta$. From equations 8, 9, 11, 22 and 24, it is shown that

$$\cos\theta = \frac{g\left(P[X,Y],\overline{\nabla}_X \varphi Y\right)}{\|P[X,Y]\|\|\varphi[X,Y]\|},$$

$$= \frac{g\left(P[X,Y],\overline{\nabla}_X PY\right)}{\|P[X,Y]\|\|\varphi[X,Y]\|} + \frac{g\left(P[X,Y],\overline{\nabla}_X FY\right)}{\|P[X,Y]\|\|\varphi[X,Y]\|},$$

$$= \frac{g\left(P[X,Y],\overline{\nabla}_X PY\right)}{\|P[X,Y]\|\|\varphi[X,Y]\|} + \frac{g(P[X,Y],-A_{FY}X)}{\|P[X,Y]\|\|\varphi[X,Y]\|},$$

$$= \frac{g(P[X,Y],P[X,Y])}{\|P[X,Y]\|\|\varphi[X,Y]\|},$$

$$= \frac{\|P[X,Y]\|}{\|\varphi[X,Y]\|} = constant, \tag{27}$$

then θ is constant . In this case, it is seen that $[X,Y] \in D_\theta$. Thus, D_θ is integrable.

Theorem 3 Let M be a pseudo slant submanifold of a nearly hyperbolic cosymplectic manifold $\widetilde{M}$. In this case $D_\mu \oplus \langle\xi\rangle$ is integrable.

Proof. Let M be a pseudo slant submanifold of a nearly hyperbolic cosymplectic manifold $\widetilde{M}$. If M is totally geodesic, then D_μ is integrable.

It can be shown that $[X,\xi] \in D_\mu$, for any vector field X in D_μ , and $\varphi[X,\xi] \in T^\perp M$, for a vector field Y in TM, from Eq. 7, it can be defined by

$$g(\varphi[X,\xi],Y) = g\left(\varphi\overline{\nabla}_X\xi,Y\right) - g\left(\varphi\overline{\nabla}_\xi X,Y\right), \tag{28}$$

$$g(\varphi[X,\xi],Y) = -g\left(\varphi\overline{\nabla}_\xi X,Y\right),$$

$$= g\left(\left(\overline{\nabla}_{\xi\varphi}\right)X - \overline{\nabla}_{\xi\varphi}X,Y\right), \tag{29}$$

for any vector field X in D_μ.

From Eq. 6 and 7, it can be seen that

$$\left(\overline{\nabla}_{\xi\varphi}\right)X + \varphi\overline{\nabla}_X\xi = 0 \tag{30}$$

$$\left(\overline{\nabla}_{\xi\varphi}\right)X = 0 \tag{31}$$

for any vector field X in D_μ. From Eq.7 , it can be defined by

$$\nabla_Y\xi = 0 \tag{32}$$

$$h(Y,\xi) = 0 \tag{33}$$

for any vector field Y in TM. If it uses Eq. 31 in Eq. 29, it can be get

$$g(\varphi[X,\xi],Y) = -g\left(\overline{\nabla}_{\xi\varphi}X,Y\right) \tag{34}$$

and from Eq. 9, 10 and 33.

$$g(\varphi[X,\xi],Y) = -g\left(A_{\varphi X}\xi + \nabla_\xi^\perp\varphi X,Y\right)$$

$$= g\left(A_{\varphi X}\xi,Y\right) = g(h(Y,\xi),\varphi X) = 0 \tag{35}$$

Then, $[X,\xi] \in D_\mu$ and in addition to $[X,\xi] \in D_\mu \oplus \langle\xi\rangle$. In this case, $D_\mu \oplus \langle\xi\rangle$ is always integrable.

Theorem 4 Let M be a pseudo slant submanifold of a nearly hyperbolic cosymplectic manifold $\widetilde{M}$. In this case $D_\theta \oplus \langle\xi\rangle$ is not integrable.

Proof. Let us assume M be a pseudo slant submanifold of a nearly hyperbolic cosymplectic manifold $\widetilde{M}$. We must show that $[X,\xi] \in D_\theta$ for any vector field X in D_θ. Let θ be an angle between $P[X,\xi]$ and $\varphi[X,\xi]$. Then, it can be written as

$$\cos\theta = \frac{g(P[X,\xi],\varphi[X,\xi])}{\|P[X,\xi]\|\|\varphi[X,\xi]\|} \tag{36}$$

for a vector field X in D_θ. From (2.6),

$$\left(\overline{\nabla}_X\varphi\right)Y + \varphi\overline{\nabla}_Y X = 0 \tag{37}$$

$$\overline{\nabla}_X\varphi Y - \varphi\overline{\nabla}_X Y + \varphi\overline{\nabla}_Y X = 0 \tag{38}$$

$$\overline{\nabla}_X\varphi Y = \varphi[X,Y] \tag{39}$$

for any vector field Y in TM and from this last above equation,

$$\varphi[X,\xi] = \overline{\nabla}_X\varphi\xi = 0 \tag{40}$$

for a vector field X in D_θ. If equation 36 in Eq. 40 use, it can be defined by

$$\cos\theta = 0, \tag{41}$$

thus $\theta = \left(\frac{\pi}{2}\right) + 2k\pi$, $(k\epsilon Z)$ (θ is not equal to $\left(\frac{\pi}{2}\right)$).

As a result of the definition of a pseudo slant submanifold of a nearly hyperbolic cosymplectic manifold; then it can be said that $D_\theta \oplus \langle\xi\rangle$ isn't integrable.

Theorem 5 Let M be a pseudo slant submanifold of a nearly hyperbolic cosymplectic manifold $\widetilde{M}$. Then $D_\theta \oplus D_\mu$ is integrable.

Proof. It is shown thatt $[X,Y] \in D_\theta \oplus D_\mu$ for any vector fields X, Y in $D_\theta \oplus D_\mu$. On the other hand,

$$\left(\overline{\nabla}_X g\right)(Y,\xi) = \overline{\nabla}_X g(Y,\xi) - g\left(\overline{\nabla}_X Y,\xi\right) - g\left(Y,\overline{\nabla}_X\xi\right)$$

$$= 0 \tag{42}$$

and from the last equation;

$$g\left(\overline{\nabla}_X Y,\xi\right) = 0 \tag{43}$$

for any vector fields X, Y in $D_\theta \oplus D_\mu$. In the same way, it can be written,

$$g\left(\overline{\nabla}_Y X,\xi\right) = 0 \tag{44}$$

for any vector fields X and Y in $D_\theta \oplus D_\mu$. If (43) and (44) use, it can be defined by

$$g([X,Y],\xi) = 0 \tag{45}$$

for any vector fields X, Y in $D_\theta \oplus D_\mu$. In this case, the proof is complete.

4. CONCLUSION

In the recent years, the geometry of the contact metric manifolds were studied. So the notion of pseudo slant submanifold on a nearly hyperbolic cosymplectic manifold were introduced. A lot of mathematicians with I have obtained some results on slant submanifolds on it. In this study, I have given some conditions for a slant submanifold of a nearly hyperbolic cosymplectic manifolds. It can be think that, the results which are obtained in this study are important for differential geometers which are dealing with nearly hyperbolic cosymplectic metric manifolds, the results which are stated in this study can be handled in different form.

DECLARATION OF ETHICAL STANDARDS

The author(s) of this article declare that the materials and methods used in this study do not require ethical committee permission and/or legal-special permission.

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