ALKU Journal of Science 2020, 2(3): 111-119
e-ISSN: 2667-7814

Original Article

On BHK-Algebras

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Özet

Bu çalışmada, d-cebirlerinin bir genellemesi olan BHK-cebirleri elde edilmiştir ve BHK-cebirleri ile d-cebirlerinin ilişkileri arasında bir araştırma yapılmıştır. Ayrıca, ikiz (twin) ideal tanımı verilmiş ve BHK bölüm cebirleri elde edilmiştir. Aynı zamanda, BHK-cebirlerinde türev tanımlanmış ve türevin ilgili özelikleri çalışılmıştır.

Anahtar Kelimeler: d-cebirleri, Bölüm Cebirleri, BHK-cebirleri, Türevler.

On BHK-Algebras

Abstract

In this work, the concept of BHK-algebras that is a generalization of d-algebras is established, and a survey of relations between BHK-algebras and d-algebras is made. Algebraical properties of BHK-algebras are derived and some discussions on the cases for a BHK-algebra to be a d-algebra is given. Additionally, twin ideal is given as a new concept and the quotient BHK-algebras is obtained. Also, the concept of derivation on BHK-algebras is defined and the features of the derivations are studied.

Keywords: d-algebras, Quotient Algebras, BHK-algebras, Derivations.

1. INTRODUCTION

BCK-algebras give a variety of algebras amongst whose subclasses can be seen the earlier implicational models of Henkin (1950), algebras of sets closed under set-substraction, and dual relatively pseudo-complemented upper semi-lattices. BCK and BCI- algebras were introduced by Y. Imai and K Iseki (1966, 1980). The concept of d-algebras as a salutary generality of BCK-algebras was presented by J. Neggers and H. K. Kim (1999) and in their work several connections between d-algebras and BCK algebras were included. Within this work, we present the conception of BHK-algebras as another beneficial generality of d-algebras and analyze relations between d-algebras and BHK-algebras. The term of a pseudo BCK- algebra as a content of BCK-algebra was risen by Georgescu and Iorgulescu (2001). The term of pseudo d-algebras as a generalization of d-algebras was studied by J.Neggers, Y.B. Jun and H.S. Kim (2009). The idea of pseudo d* -algebras was introduced by C. Ateş and A. Fırat and some relations between the pseudo BCK-algebras, the pseudo d* algebras and the pseudo d-algebras were investigated by them (2019). In this work,

Geliş/Received: 28 Tem 2020/28 July 20202
Kabul Ediliş/Accepted: 05 Kas 2020/05 Nov 2020

we give the judgement of ideals in BHK-algebras and describe the relevance between such ideals and congruences. The description of derivations emerging out analytic theory is much salutary in analyzing the nature and qualitification of algebraic systems. This concept was studied by several authors (Bell, H. E., and Kappe, L. C. , 1989) and (Kaya, K., 1988) in rings and near rings. The term of derivation in ring and near ring theory was applied to BCI-algebras by Jun and Xin (2004). Lastly, in this work we focus on derivations on BHK-algebras and obtain some properties of it.

2. BHK-ALGEBRAS

Definition 2.1. If the following axioms are satisfied for every è, â ϵ Æ where Æ is a non-empty set with a constant 0 and a binary operation * , then Æ is called a d-algebra.

(1) è * è = 0,

(2) 0 * è = 0,

(3) è * â = 0 = â * è if and only if è = â (Neggers, C. and Kim, H. S. ,1999).

Definition 2.2. Let (Æ; *, 0) be a d-algebra and è ϵ Æ. Define è * Æ := { è * a| a ϵ Æ }. Æ is said to be edge if for any è in Æ, è * Æ = { è, 0} (Neggers, C. and Kim, H. S. ,1999).

Lemma 2.3. Let (Æ; *, 0) be an edge d-algebra. Then è * 0 = è for any è ϵ Æ (Neggers, C. and Kim, H. S. ,1999).

Definition 2.4. (Æ; *; 0) as an algebra of the form (2; 0) is called a BHK-algebra if for every è, â ϵ Æ:

(BHK1) 0 * è = 0,

(BHK2) è * 0 = è,

(BHK3) è * â = 0 = â * è if and only if è = â.

Every edge d algebra is a BHK-algebra but the converse does not hold in general.

Example 2.5. Let Æ = {0, a, b} be a set in which the operation * is defined as follows:

Table 1. A BHK-algebra

$*$	0	a	b
0	0	0	0
a	a	0	b
b	b	b	b

Then Æ is a BHK-algebra but not a d-algebra.

A relation " ≺ " on Æ where (Æ; *, 0) be a BHK-algebra is defined as " è ≺ â if and only if è * â = 0" for any è, â ϵ Æ. Attention should be given that this relation is only antisymmetric.

Proposition 2.6. For every è, â, æ ϵ Æ where Æ is a BHK-algebra

(1) if è * 0 = 0 then è = 0,

(2) è * (è * (0 * è))= è * è,

(3) if è * (â * æ) = (â * è) * æ then è * è = 0, and then Æ is an edge d-algebra.

Proof. (1) It is clear from (BHK2).

(2) With (BHK1) and (BHK2)

è * (è * (0* è))= (è * (è *0)) = è * è

(3) Let è * (â * æ) = (â * è) * æ for every è, â, æ ϵ Æ. Using (BHK1) 0 = 0*(è * è) = (è *0)* è = è * è.

Hence è * è = 0.

Proposition 2.7. Let â, è be any two elements in a BHK-algebra Æ. If â ≺ è then we have the following

(1) è * (â * è) = è.

(2) â * (è * (â * è)) = 0.

(3) â * (â * (â * è)) = â * â.

(4) (â * è) * è = 0.

Proof. Let Æ be a BHK-algebra where è, â ϵ Æ.

(1) Given è ϵ Æ è * 0 = è by (BHK2) . If â ≺ è then è * (â * è) = è for any è ϵ Æ.

(2) Given è, â ϵ Æ consider â * (è * (â * è)). If â ≺ è we have â * è = 0 and by using (BHK2) we get â * (è * (â * è)) = â * (è * 0) = â * è = 0.

(3) Given è, â ϵ Æ consider â * (â * (â * è)). If â ≺ è we have â * è = 0 and by using (BHK2) we get â * (â * (â * è)) = â * (â * 0) = â * â.

(4) Let è, â be contained in Æ. By (BHK1) we have 0 * è = 0 and if â ≺ è it can be obtained that (â * è) * è = 0.

Definition 2.8. For a BHK-algebra Æ if for every è, â, æ ϵ Æ

(è * â) * æ = (è * æ) * (â * æ); (æ * (è * â) = (æ * è) * (æ * â); respectively)

is satisfied then Æ is called right distributive (or left distributive, respectively).

Let Æ = {0, a, b} be a BHK-algebra in which the operation * is defined as follows

Table 2. A right distributive BHK-algebra

*	0	a	b
0	0	0	0
a	a	0	a
b	b	b	b

It is clear that Æ is right distributive.

Proposition 2.9. If è ≺ â for a right distributive BHK-algebra Æ then è * æ ≺ â * æ for every è, â, æ ϵ Æ.

Proof. If è ≺ â, then è * â = 0 and we get by (BHK1) (è * æ) * (â * æ) = (è * â) * æ = 0 * æ = 0. Therefore, è * æ ≺ â * æ.

Note that if è ≺ â then æ * è ≺ æ * â may not be true since for example a ≺ a but b * a ⊀ b * a for the elements a, b ϵ Æ given above as a right distributive BHK-Algebra in Table 2.

Proposition 2.10. The induced relation " è ≺ â " is a transitive relation for a right distributive BHK-algebra Æ.

Proof. If è ≺ â and â ≺ æ for any è, â, æ ϵ Æ then

è * æ = (è * æ) * 0 = (è * æ) * (â * æ) = (è * â) * æ = 0 * æ = 0

Showing that è ≺ æ.

Proposition 2.11. Let Æ be a BHK- algebra under the circumstance: (æ * è) * (æ * â) = â * è for every è, â, æ ϵ Æ. If è ≺ â then æ * â ≺ æ * è.

Proof. If è ≺ â then è * â = 0. (æ * â) * (æ * è) = è * â = 0. And so æ * â ≺ æ * è.

Proposition 2.12. If ((æ * è) * (â * è)) * (æ * â) = 0 for every è, â, æ ϵ Æ where Æ is a BHK-algebra then è * è = 0.

Proof. Putting è = â = 0 in ((æ * è) * (â * è)) * (æ * â) = 0 we get

((æ * 0) * (0 * 0)) * (æ * 0) = (æ * 0) * (æ * 0) = æ * æ = 0.

Corollary 2.13. A BHK-algebra Æ with the property ((æ * è) * (â * è)) * (æ * â) = 0 for every è, â, æ ϵ Æ is a d-algebra. But the converse of this statement is not true in general. If Æ is an edge d-algebra then it is a BHK-algebra.

The algebra defined in the following table

Table 3. A BHK-algebra that is a d-algebra.

$*$	0	a	b
0	0	0	0
a	a	0	0
b	b	a	0

satisfies the properties è * 0 = è, 0 * è = 0, è ≺ â and â ≺ è ⇒ è = â, è * è = 0 and ((æ * è) * (â * è)) * (æ * â) = 0 for any element â, è, æ in the given Table 3. It is a BHK-algebra that is a d-algebra.

The algebra defined in the following, for any element â, è, æ in the given Table 4

Table 4. An algebra where ((æ * è) * (â * è)) * (æ * â) = 0 is not satisfied.

$*$	0	a	b
0	0	0	0
a	b	0	b
b	a	a	0

satisfies the properties è * 0 = è, 0 * è = 0, è ≺ â and â ≺ è ⇒ è = â, but ((æ * è) * (â * è)) * (æ * â) = 0 is not satisfied for ((a * b) * (0 * b)) * (a * 0) = b.

The algebra defined in the following table, for any element â, è, æ in the given Table 5

Table 5. An algebra where 0 * è = 0 is not satisfied

$*$	0	a	b
0	0	0	b
a	a	0	b
b	b	0	0

satisfies è * 0 = è, è ≺ â and â ≺ è ⇒ è = â, ((æ * è) * (â * è)) * (æ * â) = 0 but 0 * è = 0 is not satisfied since 0* b = b.

The algebra defined in the following table, for any element â, è, æ in the given Table 6

Table 6. An algebra where è ≺ â and â ≺ è ⇒ è = â is not satisfied.

$*$	0	a	b
0	0	0	0
a	a	0	0
b	b	0	0

satisfies 0 * è = 0, è * 0 = 0, ((æ * è) * (â * è)) * (æ * â) = 0 but è ≺ â and â ≺ è ⇒ è = â is not satisfied since a ≺b and b≺a but a≠b.

From now on, Æ will denote a BHK-algebra.

Definition 2.14. A non-empty subset A of Æ is called a BHK-subalgebra of Æ, if è * â ϵ A whenever è, â ϵ A.

Definition 2.15. For a subset $\mathscr{J}$ of Æ, if for any è, â ϵ Æ

(1) 0 ϵ $\mathscr{J}$,

(2) â ϵ $\mathscr{J}$ and è * â ϵ $\mathscr{J}$ imply è ϵ $\mathscr{J}$.

are satisfied then it is called an ideal of Æ.

Apparently, {0} and Æ are ideals of Æ. They are called as zero ideal and trivial ideal in the given order.

An ideal $\mathscr{J}$ is called proper if $\mathscr{J} \neq$ Æ.

In Example 2.5 $\mathscr{J}_1$ = {0, a} is an ideal but $\mathscr{J}_2$ = {0, b} is not for a * b = b ϵ $\mathscr{J}_2$ and b ϵ $\mathscr{J}_2$ yet a $\notin \mathscr{J}_2$.

Proposition 2.16. If è ϵ $\mathscr{J}$ and â ≺ è for an ideal $\mathscr{J}$ of Æ then â ϵ $\mathscr{J}$.

Proof. If è ϵ $\mathscr{J}$ and â ≺ è in that case â * è = 0 ϵ $\mathscr{J}$. For e è ϵ $\mathscr{J}$ and $\mathscr{J}$ is an ideal, clearly â ϵ $\mathscr{J}$.

Proposition 2.17. $\mathscr{J}$ is an ideal of Æ if and only if è * â ϵ $\mathscr{J}$ and è $\notin \mathscr{J}$ implies â $\notin \mathscr{J}$.

Proof. Condition is necessary. Let $\mathscr{J}$ be an ideal of Æ. Suppose è * â ϵ $\mathscr{J}$ and è $\notin \mathscr{J}$ but â ϵ $\mathscr{J}$. Then this contradicts the fact that $\mathscr{J}$ is an ideal of Æ.

Condition is sufficient Assume the hypothesis. Then, if è * â ϵ $\mathscr{J}$ and â ϵ $\mathscr{J}$, then the contrapositive of the hypothesis yields è ϵ $\mathscr{J}$. Hence, $\mathscr{J}$ is an ideal of Æ.

Definition 2.18. A mapping υ from Æ1 into Æ2 is called a BHK-homomorphism if for every $è_1$, $è_2$ ϵ Æ1, υ ($è_1$ * $è_2$) = υ ($è_1$) * υ ($è_2$).

Example 2.19. Consider the operation * is defined on Æ = {0, a, b} here in below:

Table 7. A BHK-algebra

$*$	0	a	b
0	0	0	0
a	a	a	a
b	b	b	0

The mapping υ defined from Æ into Æ as $\upsilon(è) = \{\begin{matrix} 0 & è = 0, b \\ a & è = a \end{matrix}\}$ is a BHK-homomorphism.

† 1. In general, for a υ homomorphism of a BHK-algebra Æ υ (0) = 0 need not to be hold since for every element è ϵ Æ, è * è = 0 is not hold for every BHK-algebra Æ. For example, consider the given example. Let Æ be the BHK- algebra given in Example 2.19. The homomorphism υ defined on Æ as

$$\upsilon(è) = \{\begin{matrix} 0 & è = a, b \\ a & è = 0 \end{matrix}\}$$

is a BHK-homomorphism where υ (0) ≠0.

Theorem 2.20. Let υ : $Æ_1 \rightarrow Æ_2$ be a BHK-homomorphism for BHK-algebras $Æ_1$ and $Æ_2$. Then the following holds:

The set υ (A) is a subalgebra of $Æ_2$ for any subalgebra A of $Æ_1$. In particular, Im(υ) is a subalgebra of Æ2,

The set $υ^{-1}$(A) is a subalgebra of $Æ_1$ for any subalgebra A of $Æ_2$.

Proof. 1) Let υ(a), υ(b)∈ υ (A) where a,b∈ A. Then υ(a*b)=υ(a)*υ(b). Since is a A subalgebra of $Æ_1$ we have a*b ∈ A. We have υ(a*b)∈ υ (A) by definition. Thus υ(a)*υ(b) ∈ υ (A). So υ (A) is a subalgebra of $Æ_2$.

2) Recall that, by definition, $υ^{-1}$(A)={a∈ Æ1| υ(a)∈ A }. Suppose that two elements a, a' of $υ^{-1}$(A) are given. We must show that a *a'∈$υ^{-1}$(A). Clearly, υ(a), υ(a')∈ A and we must check that υ(a *a')∈ A. But υ(a *a')= υ(a)*υ(a')∈ A. Since A is a subalgebra of $Æ_2$ and υ(a), υ(a')∈ A. It follows that a *a'∈$υ^{-1}$(A).

Definition 2.21. An ideal 𝒥 of Æ is called twin ideal if è * è ϵ 𝒥 for every è ϵ Æ.

In Example 2.5 $𝒥_1$ = {0, a} is an ideal that is not a twin ideal but the ideal 𝒥 = {0, b} of Æ in Example 2.25 is a twin ideal.

Definition 2.22. An ideal 𝒥 of Æ is called a transitive ideal if è * â ϵ 𝒥 and â * æ ϵ 𝒥 imply è * æ ϵ 𝒥 for every è, â, æ ϵ Æ.

The ideal 𝒥 = {0, a} in the BHK-algebra Æ given with the Cayley Table below is a transitive ideal.

Table 8. A BHK-algebra

*	0	a	b
0	0	0	0
a	a	0	a
b	b	b	b

Denitfiion 2.23. An ideal 𝒥 of Æ is entitled a congruent ideal if è * â ϵ 𝒥 and â * è ϵ 𝒥 imply (è * æ)*(â * æ) ϵ 𝒥 and (æ * è) * (æ * â) ϵ 𝒥 for every è, â, æ ϵ Æ.

The ideal 𝒥 = {0, a} in the BHK-algebra Æ given with the Cayley Table below

Table 9. A BHK-algebra

*	0	a	b
0	0	0	0
a	a	0	a
b	b	b	0

is a congruent ideal.

Definition 2.24. An ideal 𝒥 of Æ is entitled a powerful ideal whenever 𝒥 is twin, transitive and congruent ideal.

Example 2.25. Let Æ = {0, a, b} be a set where * is qualified here in below:

Table 10. A BHK-algebra

$*$	0	a	b
0	0	0	0
a	a	0	a
b	b	0	b

It can be seen that 𝒥 = {0, b} is a powerful ideal.

Proposition 2.26. Describe a relation on Æ by è ⌒ â iff è * â ϵ 𝒥 and â * è ϵ 𝒥 for all è, â ϵ 𝒥 where 𝒥 is a powerful ideal of Æ. Then ⌒ is a congruence relation on Æ.

Proof. ⌒ is reflexive since by Definition 2.21, è * è ϵ 𝒥 for every è ϵ Æ, that is è ⌒ è. Apparently, ⌒ is symmetric. For any è, â, æ ϵ Æ let è ⌒ â and â ⌒ æ. At that time, è * â ϵ 𝒥, â * è ϵ 𝒥 and â * æ ϵ 𝒥, æ * â ϵ 𝒥. By Definition 2.22 we have è * æ ϵ 𝒥, æ * è ϵ 𝒥. Hence è ⌒ æ. Therefore, ⌒ is transitive. Hence ⌒ is an equivalence relation.
Let è ⌒ â and ð ⌒ ñ for any è, â, ð, ñ ϵ Æ. Then è * â, â * è ϵ 𝒥, ð * ñ, ñ * ð ϵ 𝒥. Then by the Definition 2.23 we have

(è * ð) * (â * ð) ϵ 𝒥

(â * ñ) * (è * ñ) ϵ 𝒥

and

(â * ð) * (â *ñ) ϵ 𝒥

(è * ñ) * (è * ð) ϵ 𝒥

Hence, by Definition 2.23, (è * ð) * (â * ð) ϵ 𝒥 and (â * ð) * (â * ñ) ϵ 𝒥 we have (è * ð) * (â * ñ), and by (â * ñ) * (è * ñ) ϵ 𝒥 and (è * ñ) * (è * ð) ϵ 𝒥 we have (â * ñ) * (è * ð) ϵ 𝒥 in other words (è * ð) * (â * ñ). That is to say ⌒ is a congruence relation on Æ.

Notate Æ /𝒥 = {[è]$_{𝒥}$ | è ϵ Æ } where [è]$_{𝒥}$ = { â ϵ Æ | è ⌒ â }. Whenever [è]$_{𝒥}$ ⁎ [â]$_{𝒥}$ = [è * â]$_{𝒥}$ is depicted then ⁎ is well defined, since * is a congruence relation.

Theorem 2.27. Let 𝒥 be a powerful ideal of a BHK-algebra Æ. Then (Æ/𝒥; ⁎; [0]$_{𝒥}$) is a BHK-algebra that is termed the quotient BHK-algebra of Æ obtained with 𝒥.

Proof. Note that [0]$_{𝒥}$ = { è ϵ Æ | è ⌒ 0} = { è ϵ Æ | è * 0; 0 * è ϵ 𝒥 } = { è ϵ Æ | è, 0 ϵ 𝒥 } = 𝒥. So the proof is obvious since checking the axioms is trivial.

Example 2.28. Let 𝒥 be a powerful ideal of a BHK-algebra in Example 2.25. Then (Æ /𝒥; ⁎; [0]𝒥) is a BHK-algebra whose Cayley Table is given as

Table 11. A (Æ/𝒥; ⁎; [0]$_{𝒥}$) BHK-algebra.

⁎	[0]$_{𝒥}$	[a]$_{𝒥}$
[0]$_{𝒥}$	[0]$_{𝒥}$	[0]$_{𝒥}$
[a]$_{𝒥}$	[a]$_{𝒥}$	[0]$_{𝒥}$

Proposition 2.29. Let $\mathscr{J}$ be a twin ideal of a BHK-algebra Æ. Then the mapping φ : Æ → Æ/$\mathscr{J}$, given by φ(è) = [è]$\mathscr{J}$, is a surjective homomorphism and $Ker_\varphi = \mathscr{J}$.

Proof. As * is a congruence relation the operation • on Æ/$\mathscr{J}$ obatined with $[è]_{\mathscr{J}}$ • $[â]_{\mathscr{J}}$ = $[è * â]_{\mathscr{J}}$ is well defined. For every è, â ϵ Æ, we have φ (è * â) = $[è * â]_{\mathscr{J}}$ = $[è]_{\mathscr{J}}$ • $[â]_{\mathscr{J}}$ = φ (è) • φ (â). So φ is a BHK-homomorphism. For φ(Æ) = { φ (è)| è ϵ Æ} = {[è]$_{\mathscr{J}}$| è ϵ Æ } = Æ/$\mathscr{J}$ we get φ is surjective. Additionally, Ker_φ = { è ϵ Æ| φ(è) = $\mathscr{J}$ } = { è ϵ Æ |[è]$_{\mathscr{J}}$ = $\mathscr{J}$ } = { è ϵ Æ |[è]$_{\mathscr{J}}$ = $[0]_{\mathscr{J}}$ } = { è ϵ Æ | è ϵ $\mathscr{J}$ } = $\mathscr{J}$. Therefore, $Ker_\varphi = \mathscr{J}$.

3. DERIVATIONS ON BHK-AlGEBRAS

For è, â ϵ Æ, è ^ â = â * (â * è) is set out for the rest of the study.

Definition 3.1. A mapping Ɋ: Æ→ Æ is a left-right derivation (brifley (l,r)- derivation) of Æ if for all è, â ϵ Æ it satisfies:

Ɋ (è * â) = (Ɋ (è) * â) ^ (è * Ɋ (â))

If Ɋ satisfies for all è, â ϵ Æ

Ɋ (è * â) = (è * Ɋ (â)) ^ (Ɋ (è) * â)

Ɋ is called a right-left derivation (brifley, (r, l)-derivation) of Æ.

Additionally, if Ɋ is a (l,r) and (r, l)-derivation at the same time it is called a derivation of Æ.

Definition 3.2. A derivation of Æ is called as regular if Ɋ (0) = 0.

Example 3.3. Let Æ be given in Example 2.5. A mapping Ɋ: Æ → Æ described by

$$Ɋ(è) = \begin{cases} 0 & è = 0, a \\ b & è = b \end{cases} \tag{1}$$

is a (l; r)-derivation. Ɋ is not a (r; l)-derivation of Æ since Ɋ (a* b)=b whereas (a* Ɋ(b)) ^ (Ɋ(a) *b) =(a*b) ^ (0*b)=b^0=0*(0*b)=0*0=0

And a mapping Ɋ* : Æ → Æ by Ɋ*(è) = b determined for every è ϵ Æ.
It is easily seen that Ɋ*is a (r; l)-derivation but not a (l; r)-derivation of Æ since Ɋ*(0*a)= Ɋ*(0)=b where (Ɋ*(0) *a) ^ (0 * Ɋ*(a))=(b*a) ^ (0* b)=b^0=0* (0*b)=0.

Proposition 3.4. A (l; r) derivation Ɋ of Æ is regular.

Proof. Let Ɋ be a (l; r) derivation of Æ and for every è ϵ Æ by (BHK1)

Ɋ (0) = Ɋ (0 * è) = (Ɋ (0) * è) ^ (0 * Ɋ (è)) = (Ɋ (0) * è) ^ 0 = 0.

Proposition 3.5. Let Ɋ be a self map of Æ.

1. Ɋ (è) = Ɋ (è) ^ è for every è ϵ Æ where Ɋ given as (l; r)-derivation of Æ.

2. Ɋ (è) = (è * Ɋ (0)) ^ Ɋ (è) for every è ϵ Æ where Ɋ is a (r; l)-derivation of Æ.

Proof. 1. Let Ɋ be a (l; r)-derivation of Æ. Then
Ɋ (è) = Ɋ (è * 0) = (Ɋ (è) * 0) ^ (è * Ɋ (0)) = Ɋ (è) ^ (è * 0) = Ɋ (è) ^ è.

2. Let Ɑ] be a (r; l)-derivation of Æ. Then

$$Ɑ] (è) = Ɑ] (è * 0) = (è * Ɑ] (0)) \wedge (Ɑ] (è) * 0) = (è * Ɑ] (0)) \wedge Ɑ] (è).$$

Proposition 3.6. For every è, â ϵ Æ and Ɑ] as a regular derivation of Æ $Ɑ]^{-1}(0)$ = { è ϵ Æ | Ɑ] (è) = 0} is a BHK-subalgebra of Æ.

Proof. Since Ɑ] is regular $Ɑ]^{-1}(0) \neq \emptyset$. Let è, â ϵ $Ɑ]^{-1}$. Since Ɑ] (è * â) = (è * Ɑ] (â))^(Ɑ] (è)* â) = (è *0)^(0* â) = è ^ 0 = 0 we have è * â ϵ $Ɑ]^{-1}$. Therefore, $Ɑ]^{-1}$ is a BHK-subalgebra of Æ.

Definition 3.7. For any map f: Æ→ Æ on Æ a set $Ʞ_f$ is qualified as $Ʞ_f$ = { è ϵ Æ |f(è) = 0} .

Proposition 3.8. $Ʞ_{Ɑ]}$ for a derivation Ɑ] of Æ is a subalgebra of Æ.

Proof. Let è, â ϵ Æ such that è, â ϵ $Ʞ_{Ɑ]}$. Then by the definition of Ɑ]

$$Ɑ] (è * â) = (è * Ɑ] (â)) \wedge (Ɑ] (è) * â) = (è * 0) \wedge (0 * â) = è \wedge 0 = 0.$$

Thus, è * â ϵ $Ʞ_{Ɑ]}$.

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