Conference Proceeding Science and Technology, 3(1), 2020, 68–76

Conference Proceeding of 3rd International E-Conference on Mathematical Advances and Applications (ICOMAA-2020)

ISSN: 2651-544X
http://dergipark.gov.tr/cpost

Effects of Neutrosophic Binomial Distribution on Double Acceptance Sampling Plans

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Abstract: Acceptance sampling plans (ASPs) offer to inspect a small set instead of all outputs in a production process. This approach minimizes the inspection cost dramatically and guarantees the output quality within a predefined risk ratio based on a small sample size. One type of ASPs named double acceptance sampling plans (DASPs) gives an ability to minimize the effect of the randomness on inspection results and reach a lower risk level with a small sample size. We know that ASPs use certain values while formulation and application procedures. However, it is also clear that the quality characteristics may not be certain in some real cases and they include some vagueness. So, we need some new techniques to model the uncertainty and manage human's evaluations. The fuzzy set theory (FST) is one of the most popular techniques to model the uncertainty in the engineering problems. Additionally, we know that the fuzzy extensions such as Neutrosophic sets (NSs) bring some advantages to manage these uncertainties. Generally, fuzzy DASPs are offered in the literature but they are formulated with α-cut approach to convert the problem into interval valued set problem. With the help of this conversion, it is enough to solve the problem with certain values for the upper and the lower limits of the intervals. However, the uncertainty is generally more complex in real life applications including human factor. NSs that include three terms; truthness (t), indeterminacy (i) and falsity (f) and cover inconsistent data cases are good representation of human thinking under uncertainty. In this study, DASPs are formulated and analyzed based on NSs by using binomial distribution. A numerical example is also presented to analyze the proposed sampling plans based on NSs.

Keywords: Acceptance double sampling plans, Binomial distribution, Fuzzy sets, Neutrosophic sets.

1 Introduction

Inspection processes can become too costly when all produced items are inspected. To minimize this cost, a small set of items are inspected in ASPs. The sample size (n) and allowed defective item count (c) are decided depending on the Producer's Risk (α) and Consumer's Risk (β). Producer's Risk is defined as the probability of rejecting a party while the defective ratio (p) is suitable with the producer's acceptable quality level and Consumer's Risk is defined as the probability of accepting a party while p is over the allowed proportion defective [1]. DASPs are used for reaching lower risks with small sample sizes [2].

Defective ratio of the parties and plan parameters are handled as certain values in standard ASPs. But assuming them as certain values does not represent the real outputs of the plan adequately while there are some uncertainty. Fuzzy logic is an efficient approach to model the uncertainty. The modeling of the uncertainty in FST is built on the term "membership function" by using a continuous variable (x) inside $[0, 1]$ interval; which is the main difference of the classical logic and fuzzy logic. Because in classical logic, statements are answered by using two absolute values; 0 and 1 [3]. Some studies have been performed to model the uncertainty of the real world applications by reformulating the ASPs with fuzzy defection information and fuzzy plan parameters.

Fuzzy sets (FSs) are extended for some specific cases to model the uncertainty better. NSs are extension of FSs having three functions for membership (truthiness), non-membership (falsity) and indeterminacy. This triple-term structure is more efficient in modeling uncertainty in case it is caused from multiple factors such as nature of the event, incomplete information etc. Having indeterminacy term at the same time with the membership and non-membership terms is the characteristic attribute of the NSs. There are no studies using any FS extensions in the formulation of the DASPs. This paper aims to design DASPs based on Binomial distribution while item defections are defined by using NSs. The rest of this paper has been organized as follows: Section 2 includes brief information about ASPs and DASPs. FST and NSs have been detailed into Section 3. Proposed DASPs for binomial distribution based on NSs has been analyzed into Section 4. A numerical example is detailed into Section 5. Section 6 indicates the obtained results and future research directions.

2 Acceptance Sampling Plans

ASP is a specific plan stating the sampling rules and acceptance criteria to decide on accepting or rejecting a party by inspecting a small set of items. ASPs can be classified in two groups. Variable ASPs focus on quality characteristics (i.e. height of an item) modeled with a statistical distribution while attribute ASPs are focusing on the defectiveness of the inspected items [2]. Attribute ASPs are formulated with binomial or poisson distributions. For binomial attribute ASPs, plan parameters are sample size (n) and maximum allowed defect count (c) which are decided to reach a specified quality level with minimum cost. Figure 1 demonstrates single ASPs [2].

When observed defective item count is d, the probability of accepting a party (P_a) is formulated for the binomial distribution as shown in Eq.(1) [2].

$$P_a = P\{d \leqslant c\} = \sum_{d=0}^{c} \binom{n}{d} .p^d.(1-p)^{n-d}. \tag{1}$$

The average outgoing quality (AOQ) is the long-term average fraction defective that the consumer will encounter. For a given incoming item quality p, AOQ is calculated as shown as in Eq.(2) [2].

$$AOQ = P_a.p. \tag{2}$$

Average number of inspection per party (ATI) while 100% inspection is performed for rejected parties is calculated as shown in Eq. (3) [2].

$$ATI = n + (1 + P_a)(N - n). \tag{3}$$

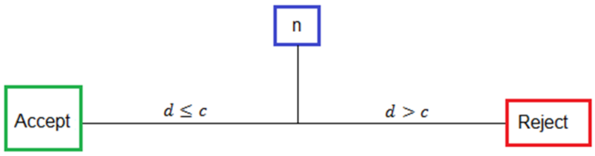

Fig. 1: Representation of Single Acceptance Sampling Plans

Operating characteristic (OC) is demonstrated as a a curve between defect ratio (p) and probability of party accepting (Pa). An example OC curve for a single ASP is shown in Figure 2. To decrease risks, n should be increased and c should be decreased according to the OC graph, but it increases the inspection cost. The best approach is to minimize the cost while satisfying the acceptable quality level. For this reason, plan parameters (n and c) are determined by using OC curve, producer's risk (α) and consumer's risk (β). ASPs should have P_a near as α [2].

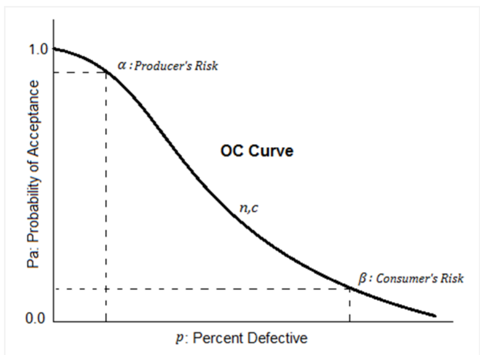

Fig. 2: OC Curve for a Single Acceptance Sampling Plan

In this study P_a, AOQ, ATI and OC parameters will be analyzed within the scope of NSs, sticking to the binomial distribution.

2.1 Double Acceptance Sampling Plans

DASPs are two-step ASPs which are used for reaching lower risks with less inspection. They have two set of plan parameters. If the first threshold value for defect count (c_1) is not exceeded in first sampling step inside n_1 items, the party is accepted. But if the defect count exceeds c_1 but does not exceed the second threshold value (c_2), the party is not rejected and the second inspection step is applied for a new sample having n_2 items. DASPs can be demonstrated as shown in Fig. 3 [2].

For the following definitions, P_a can be calculated as in Eq. (4):

p : Defective probability of items
$1-p$: Non-defective probability of items
P_a : Acceptance probability of party
n_1 : Sample size for the first step
n_2 : Sample size for the second step
c_1 : Max. allowed defective item count for the first step
c_2 : Max. allowed defective item count for the second step

$$P_a = P\{d_1 \leqslant c_1\} + P\{d_1 + d_2 \leqslant c_2 | c_1 < d_1 \leqslant c_2\}$$

$$P_a = \sum_{d_1=0}^{c_1} \binom{n_1}{d_1} .p^{d_1}.(1-p)^{n_1-d_1} + \sum_{d_1=c_1+1}^{c_2} \left(\binom{n_1}{d_1} .p^{d_1}.(1-p)^{n_1-d_1}. \left[\sum_{d_2=0}^{c_2-d_1} \binom{n_2}{d_2} .p^{d_2}.(1-p)^{n_2-d_2} \right] \right). \tag{4}$$

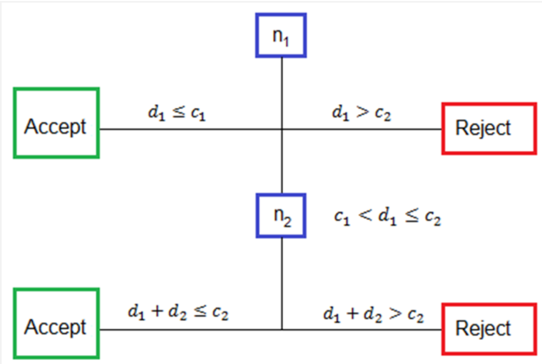

Fig. 3: Representation of Double Acceptance Sampling Plans

DASPs have four plan parameters; n_1, n_2, c_1 and c_2 so the OC curve is dependent to these four parameters. The second step of the plan means giving a second acceptance chance to a party so the OC curve becomes steeper in comparison to ASPs. It can be represented with a curve illustrated as the red curve in Fig. 4.

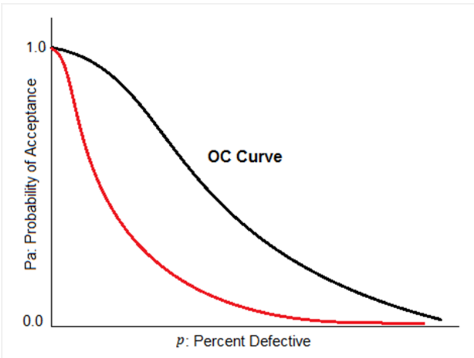

Fig. 4: OC Curve for a Double Sampling Plan

3 Fuzzy Sets

Classical logic is built on binary absolute output vales; 0 and 1. The mathematical approach that is built on the classical logic, analyzes the events under certainty. It is easy to apply but it may become inadequate in modeling the real world applications in case of uncertainty. Fuzzy logic is used for modeling the uncertainty by using a membership degree concept that is presented with a decimal number between 0 and 1. FS, is a class of continuous membership values. The uncertainty level is represented with a membership degree: 1 means full membership and 0 means full non-membership. It can be said that, a member of an FS can be partially a member and partially a non-member. An FS having membership function $\mu_{\tilde{A}}$ can be represented mathematically as in Eq. (5) in universal space X. Non-membership degree is represented as $1 - \mu_{\tilde{A}}$ for a set member x [3].

$$\tilde{A} = \{(x, \mu_{\tilde{A}}(x) | x \in X)\}. \tag{5}$$

If the membership is defined with a function, it can be represented with a curve in a graph. Fig. 5 shows membership functions for both classical logic and fuzzy logic.

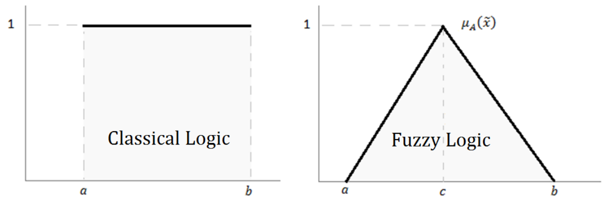

Fig. 5: Membership Functions for Classical Logic and Fuzzy Logic

Even though the fuzzy logic ensures to model the uncertainty close to the reality, it needs modifications for better modeling in some real world cases. This situation leaded up the development of FS extensions. Basic extensions are classified as follows by Kahraman et all. [4].

- *Type-n FSs*: The FSs having uncertainty on membership function. Most popular one is Type-2 FSs (T2FSs).
- *Interval Valued FSs*: Membership function is presented as an interval valued number for these FSs.
- *Intuitionistic FSs (IFSs)*: Sum of membership and non-membership is less than or equal to 1 for these FSs. Neutrosophic sets (NSs) is generalized form of IFS.
- *Fuzzy Multi-Sets*: Membership function is presented with a membership count and membership order is decided with the descending order of the membership counts.
- *Non-Stationary FSs*: The membership function is defined as a continuous region depending on time. It has no common application area.
- *Hesitant FSs (HFSs)*: These are the FSs, having multiple membership degree for each set element.

3.1 Neutrosophic Sets

In some cases, non-membership degree cannot be found by using $1-\mu_{\tilde{A}}$ equation. This is named as incomplete information case and classical fuzzy logic cannot be applicable for this case. IFSs were offered to handle the incomplete information case. IFSs have independent membership $\mu(x)$ and non-membership $\nu(x)$ functions satisfying $\mu(x)+\nu(x)\leqslant 1$ [5]. NSs are the generalized form of IFSs. In IFSs, the term $1-(\mu(x)+\nu(x))$ is dependent to membership and non-membership values but in NSs the uncertainty caused by incomplete information is named as "indeterminacy" and represented with an independent term. The independency of the terms makes possible to use inconsistent data in modeling. From this point of view, NSs can be represented by using the terms membership/truthiness (t), non-membership/falsity (f) and indeterminacy (i) as in Eq. (6) [6].

$$\begin{gathered}(t,i,f)=(truthiness, indeterminacy, falsity)\\ 0\leqslant t+i+f\leqslant 3,\quad t,i,f\in[0,1]\end{gathered}\tag{6}$$

NSs can be used to formulate three-state events. Assume that winning chance in a match is asked to a audience of a football team and he estimated % 60 winning chances while another audience is estimating %70 winning chances to opposite team and an expert is estimating %30 changes for tie. This is an inconsistent information case because the total probability is bigger than %100. The case can be easily modeled with NSs [6].
Eq. (6) shows that NSs are the generalized form of FSs for complete, incomplete, consistent and inconsistent information cases. It should be emphasized that the indeterminacy is considered as a separate term in formulation by only NSs overall the FS extensions [6]. So it can be said that, if a fuzzy model does not include membership, non-membership and indeterminacy terms at the same time, it is not an exactly NS formulation. It is a model for a sub-set of the NSs. Indeterminacy term is the characteristic feature of NSs.
Truthiness, indeterminacy and falsity can be interval-valued numbers. These type of NSs are named as Interval Neutrosophic Sets (INSs) and they are represented with three interval-valued numbers. Summation of the biggest upper limits (sup) of these three intervals must between 0 and 3. Representation of INSs is shown in Eq. (7) [7].

$$\begin{gathered}x=\langle[T_{x_L},T_{x_U}],[I_{x_L},I_{x_U}],[F_{x_L},F_{x_U}]\rangle\\ 0\leqslant\sup T_x+\sup I_x+\sup F_x\leqslant 3\\ T_x,I_x,F_x\in[0,1].\end{gathered}\tag{7}$$

3.2 Neutrosophic Binomial Distribution

Neutrosophic Binomial Distribution (NBD) is suggested by Smarandache [8]. The classical Binomial distribution is extended by adding indeterminacy related to the probabilistic experiment. For a binomial event, some trials can result with indeterminate results while some of them are resulting with determinate results (i.e. success or failure). This partially indeterminacy depends on the problem should be solved on the expert's point of view. So, indeterminacy threshold (th) is defined as "number of trials whose outcome is indeterminate". The indeterminate results exceeding th will belong to the indeterminate part, while the others are belonging to the determinate part. For the following definitions, probability values can be calculated as in Eq. (8) [8].
$P(S)$: Chance a particular trial results in a success
$P(F)$: Chance a particular trial results in a failure, for both S and F different from indeterminacy
$P(I)$: Chance a particular trial results in indeterminacy
T_x: Chance of x successes, and $n-x$ failures and indeterminacies but such that the number of indeterminacies is less than or equal to th
F_x: Chance of y successes, with $y\neq x$ and $n-y$ failures and indeterminacies but such that the number of indeterminacies is less than or equal to th

I_x: Chance of z indeterminacies, where z is strictly greater than th

$$T_x = \frac{n!}{x!}.P(S)^x.\sum_{k=0}^{th}\frac{P(I)^k.P(F)^{n-x-k}}{k!(n-x-k)!}$$
$$F_x = \sum_{\substack{y=0\\y\neq x}}^{n} T_y = \sum_{\substack{y=0\\y\neq x}}^{n}\frac{n!}{y!}.P(S)^y.\left[\sum_{k=0}^{th}\frac{P(I)^k.P(F)^{n-y-k}}{k!(n-y-k)!}\right] \tag{8}$$
$$I_x = \sum_{z=th+1}^{n}\frac{n!}{z!}.P(I)^z.\left[\sum_{k=0}^{n-z}\frac{P(S)^k.P(F)^{n-z-k}}{k!(n-z-k)!}\right].$$

If the probability values have inconsistency, they should be normalized by dividing each of them with total probability to satisfy Eq. (9)

$$t+i+f=1. \tag{9}$$

3.3 Fuzzy Sets in Acceptance Sampling

Some ASPs have been offered in the literature on attribute single ASPs with the help of traditional FSs. Jamkhaneh et all. [9] and Kahraman et all. [10] have offered fuzzy single ASPs for binomial and poisson distributions. They formulated the ASPs by converting into interval valued sets by using $\alpha - cut$ technique. In $\alpha - cut$, a threshold value is decided in terms of membership degree and the FS is cut from this level horizontally. The upper side is assumed as full member and the lower membership degrees are assumed as non-member [11]. After this transformation, the problem is solved twice with the upper limit and the lower limit of the intervals. There are no studies about fuzzy extensions (such as T2FS, IFS, HFS etc.) except NSs on ASPs. Aslam [12] has offered single attribute ASPs for NBD. Parties having interval-valued specification measurement values are used and these specification values are converted to Neutrosophic defectiveness information by using the standard deviation threshold values as a pre-step of the ASP. The offered ASPs are organized on NBD but differently they just attended to the defect count and there is no threshold value for indeterminacy. Isik and Kaya [13] have developed Neutrosophic single ASPs for poisson distribution. The developed plan has an additional plan parameter (I) as indeterminacy threshold. Kahraman et all. [10] have offered fuzzy DASPs for binomial and poisson distributions and again $\alpha - cut$ approach is used. There is no study on DASPs with NSs or other FSs extensions.

4 Neutrosophic Binomial Double Acceptance Sampling Plan

In this study, DASPs having certain plan parameters and Neutrosophic defectiveness are offered for binomial distribution. The formulation is developed based on NBD that is offered by Smarandache [8]. As shown in the outline given in Fig. 6, the offered plans have indeterminacy threshold values as plan parameters. The inspection process has indeterminacy outcome in addition to acceptance and rejection. It should be noticed that defectiveness case is positioned as dominant to indeterminacy case in the inspection cycle. In order to make this assumption clear, it can be explained with another words. If a sample has defective items more than c and more indeterminate items than I at the same time, the party is rejected without paying attention to indeterminate items.

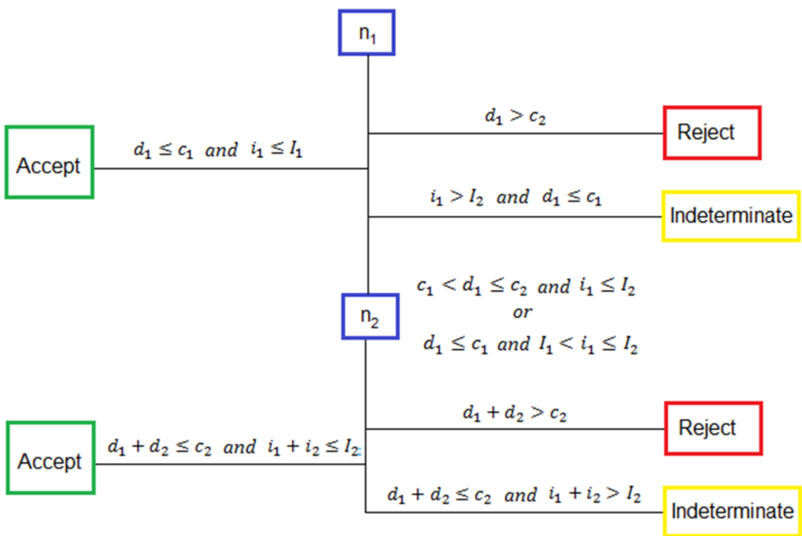

Fig. 6: Representation of Neutrosophic Binomial Double Acceptance Sampling Plans

If an NS has inconsistency ($P(S) + P(F) + P(I) \neq 1$), the probability values should be normalized by dividing each of them with total probability to satisfy Eq. (9) before or after the calculations as offered by Smarandache [8]. To ensure calculation simplicity, pre-normalization is preferred in this study. The following definitions are used in formulations.

A: NS for item defect such that $A = \{t, i, f\} = \{P(S), P(I), P(F)\}$
$P(S)$: Non-defective probability of the items
$P(I)$: Indeterminacy probability of the items
$P(F)$: Defective probability of the items
B: NS for test result such that $B = \{t, i, f\} = \{P_a, P_i, P_r\}$
P_a: Acceptance probability of the party
P_i: Indeterminacy probability of the party
P_r: Rejection probability of the party
n_1: Sample size for the first step
n_2: Sample size for the second step
c_1: Max. allowed defective item count for the first step
c_2: Max. allowed defective item count for the second step
I_1: Max. allowed indeterminate item count for the first step
I_2: Max. allowed indeterminate item count for the second step
P_a^I: Acceptance probability of the party in the first step
P_a^{II}: Acceptance probability of the party in the second step
P^I: Termination probability of the sampling in the first step

Similar to the classical DASPs, the assumptions given in Eq. (10) are made while formulating the plans.

$$c_1 < c_2, I_1 < I_2, c_1 + I_1 < n_1, c_2 + I_2 < n_2. \tag{10}$$

Party acceptance probability (P_a) is consist of the sum of the probabilities of three cases: Acceptance in the first step, acceptance in the second step when the second step is required because of the defective item count, acceptance in the second step when the second step is required because of the indeterminate item count. P_a is calculated as shown in Eq. (11).

$$\begin{aligned}
P_a &= P\{d_1 \leqslant c_1, i_1 \leqslant I_1\} \\
&\quad + P\{d_1 + d_2 \leqslant c_2, i_1 + i_2 \leqslant I_2 \mid c_1 < d_1 \leqslant c_2, i_1 \leqslant I_2\} \\
&\quad + P\{d_1 + d_2 \leqslant c_2, i_1 + i_2 \leqslant I_2 \mid d_1 \leqslant c_1, I_1 < i_1 \leqslant I_2\}
\end{aligned}$$

$$\begin{aligned}
P_a &= \sum_{d_1=0}^{c_1} \left(\binom{n_1}{d_1} .P(F)^{d_1} . \left[\sum_{i_1=0}^{I_1} \binom{n_1 - d_1}{i_1} .P(I)^{i_1} .P(S)^{n_1 - i_1 - d_1} \right] \right) \\
&\quad + \sum_{d_1=c_1+1}^{c_2} \left(\binom{n_1}{d_1} .P(F)^{d_1} . \left[\sum_{i_1=0}^{I_2} \left(\binom{n_1 - d_1}{i_1} .P(I)^{i_1} .P(S)^{n_1 - i_1 - d_1} . \right.\right.\right. \\
&\quad \left.\left.\left. \left[\sum_{d_2=0}^{c_2 - d_1} \left(\binom{n_2}{d_2} .P(F)^{d_2} . \left[\sum_{i_2=0}^{I_2 - i_1} \binom{n_2 - d_2}{i_2} .P(I)^{i_2} .P(S)^{n_2 - i_2 - d_2} \right] \right) \right] \right) \right] \right) \\
&\quad + \sum_{d_1=0}^{c_1} \left(\binom{n_1}{d_1} .P(F)^{d_1} . \left[\sum_{i_1=I_1+1}^{I_2} \left(\binom{n_1 - d_1}{i_1} .P(I)^{i_1} .P(S)^{n_1 - i_1 - d_1} . \right.\right.\right. \\
&\quad \left.\left.\left. \left[\sum_{d_2=0}^{c_2 - d_1} \left(\binom{n_2}{d_2} .P(F)^{d_2} . \left[\sum_{i_2=0}^{I_2 - i_1} \binom{n_2 - d_2}{i_2} .P(I)^{i_2} .P(S)^{n_2 - i_2 - d_2} \right] \right) \right] \right) \right] \right)
\end{aligned} \tag{11}$$

Similar to P_a, party rejection probability (P_r) is consist of the sum of the probabilities of three cases: Rejection in the first step, rejection in the second step when the second step is required because of the defective item count, rejection in the second step when the second step is required because of the indeterminate item count. P_r is calculated as shown in Eq. (12).

$$
\begin{aligned}
P_r &= P\{d_1 > c_2\} \\
&\quad + P\{d_1 + d_2 > c_2 \mid c_1 < d_1 \leqslant c_2, i_1 \leqslant I_2\} \\
&\quad + P\{d_1 + d_2 > c_2 \mid d_1 \leqslant c_1, I_1 < i_1 \leqslant I_2\}
\end{aligned}
$$

$$
\begin{aligned}
P_r = &\sum_{d_1=c_2+1}^{n_1} \left(\binom{n_1}{d_1} .P(F)^{d_1} . \left[\sum_{i_1=0}^{n_1-d_1} \binom{n_1-d_1}{i_1} .P(I)^{i_1} .P(S)^{n_1-i_1-d_1} \right] \right) \\
&+ \sum_{d_1=c_1+1}^{c_2} \left(\binom{n_1}{d_1} .P(F)^{d_1} . \left[\sum_{i_1=0}^{I_2} \left(\binom{n_1-d_1}{i_1} .P(I)^{i_1} .P(S)^{n_1-i_1-d_1} . \right.\right.\right. \\
&\left[\sum_{d_2=c_2-d_1+1}^{n_2} \left(\binom{n_2}{d_2} .P(F)^{d_2} . \left[\sum_{i_2=0}^{n_2-d_2} \binom{n_2-d_2}{i_2} .P(I)^{i_2} .P(S)^{n_2-i_2-d_2} \right] \right) \right] \left.\left.\left. \right) \right] \right) \\
&+ \sum_{d_1=0}^{c_1} \left(\binom{n_1}{d_1} .P(F)^{d_1} . \left[\sum_{i_1=I_1+1}^{I_2} \left(\binom{n_1-d_1}{i_1} .P(I)^{i_1} .P(S)^{n_1-i_1-d_1} . \right.\right.\right. \\
&\left[\sum_{d_2=c_2-d_1+1}^{n_2} \left(\binom{n_2}{d_2} .P(F)^{d_2} . \left[\sum_{i_2=0}^{n_2-d_2} \binom{n_2-d_2}{i_2} .P(I)^{i_2} .P(S)^{n_2-i_2-d_2} \right] \right) \right] \left.\left.\left. \right) \right] \right)
\end{aligned} \tag{12}
$$

Similar to P_a and P_r, indeterminacy probability (P_i) is consist of the sum of the probabilities of three cases: Indeterminacy in the first step, indeterminacy in the second step when the second step is required because of the defective item count, indeterminacy in the second step when the second step is required because of the indeterminate item count. P_i is calculated as shown in Eq. (13).

$$
\begin{aligned}
P_i &= P\{i_1 > I_2, d_1 \leqslant c_1\} \\
&\quad + P\{d_1 + d_2 \leqslant c_2, i_1 + i_2 > I_2 \mid c_1 < d_1 \leqslant c_2, i_1 \leqslant I_2\} \\
&\quad + P\{d_1 + d_2 \leqslant c_2, i_1 + i_2 > I_2 \mid d_1 \leqslant c_1, I_1 < i_1 \leqslant I_2\} \\
P_i = &\sum_{d_1=0}^{c_2} \left(\binom{n_1}{d_1} .P(F)^{d_1} . \left[\sum_{i_1=I_2+1}^{n_1-d_1} \binom{n_1-d_1}{i_1} .P(I)^{i_1} .P(S)^{n_1-i_1-d_1} \right] \right) \\
&+ \sum_{d_1=c_1+1}^{c_2} \left(\binom{n_1}{d_1} .P(F)^{d_1} . \left[\sum_{i_1=0}^{I_2} \left(\binom{n_1-d_1}{i_1} .P(I)^{i_1} .P(S)^{n_1-i_1-d_1} . \right.\right.\right. \\
&\left[\sum_{d_2=0}^{c_2-d_1} \left(\binom{n_2}{d_2} .P(F)^{d_2} . \left[\sum_{i_2=I_2-i_1+1}^{n_2-d_2} \binom{n_2-d_2}{i_2} .P(I)^{i_2} .P(S)^{n_2-i_2-d_2} \right] \right) \right] \left.\left.\left. \right) \right] \right) \\
&+ \sum_{d_1=0}^{c_1} \left(\binom{n_1}{d_1} .P(F)^{d_1} . \left[\sum_{i_1=I_1+1}^{I_2} \left(\binom{n_1-d_1}{i_1} .P(I)^{i_1} .P(S)^{n_1-i_1-d_1} . \right.\right.\right. \\
&\left[\sum_{d_2=0}^{c_2-d_1} \left(\binom{n_2}{d_2} .P(F)^{d_2} . \left[\sum_{i_2=I_2-i_1+1}^{n_2-d_2} \binom{n_2-d_2}{i_2} .P(I)^{i_2} .P(S)^{n_2-i_2-d_2} \right] \right) \right] \left.\left.\left. \right) \right] \right)
\end{aligned} \tag{13}
$$

OC is formed as a surface depending on $P(F)$, $P(I)$ and P_a. Fig. 7 shows OC surface for a plan having parameters $n_1 = 30$, $n_2 = 50$, $c_1 = 2$, $c_2 = 4$, $I_1 = 2$, $I_2 = 4$ for the probability pairs satisfying $P(F) \in [0.01, 0.1]$ and $P(I) \in [0.01, 0.1]$. The axes show the normalized probability values in the inconsistent information case. The obtained OC surface is formed steeper than the OC surface found for single Neutrosophic ASPs in [13].

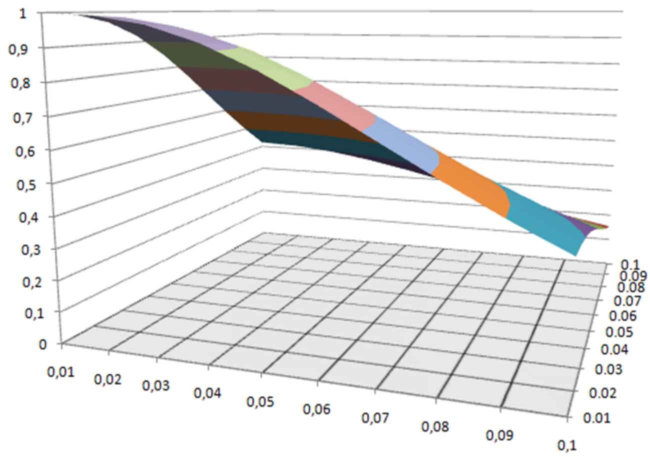

Fig. 7: OC Surface for an Example Plan

AOQ is a function of $P(F)$ and P_a. Similar to Eq.(2), it is calculated as shown as in Eq.(14).

$$AOQ = P_a.P(F). \tag{14}$$

While calculating ATI, two different strategies can be followed. In pessimistic strategy, all the items are tested in case of indeterminacy. $ATI_{pessimistic}$ is calculated as shown as in Eq.(15). If the business allows to not test in case of indeterminacy, optimistic strategy is followed. $ATI_{optimistic}$ is calculated as shown as in Eq.(16).

$$\begin{aligned} ATI_{pessimistic} &= n_1 P_a^I + (n_1 + n_2) P_a^{II} + N(P_r + P_i), \\ &= n_1 P_a^I + (n_1 + n_2)(P_a - P_a^I) + N(P_r + P_i). \end{aligned} \tag{15}$$

$$\begin{aligned} ATI_{optimistic} &= n_1 P_a^I + (n_1 + n_2) P_a^{II} + N P_r, \\ &= n_1 P_a^I + (n_1 + n_2)(P_a - P_a^I) + N P_r. \end{aligned} \tag{16}$$

ASN is calculated like the standard ASPs as shown in Eq.(17). But, there are three outcomes of the plan. For this reason, P^I is calculated as $P_a^I + P_r^I + P_i^I$ instead of $P_a^I + P_r^I$.

$$\begin{aligned} ASN &= n_1 + n_2(1 - P^I) \\ &= n_1 + n_2(1 - (P_a^I + P_r^I + P_i^I)). \end{aligned} \tag{17}$$

4.1 A Numerical Example

A computer program is built to calculate P_a, P_r, P_i, AOQ, ATI and ASN values to demonstrate the proposed plans based on example data. The obtained results have been summarized in Table 1.

Table 1 Numerical Example Results

Parameters										Results							
N	n_1	n_2	C_1	C_2	I_1	I_2	$P(S)$	$P(F)$	$P(I)$	P_a	P_r	P_i	$P_a + P_r + P_i$	AOQ	ATI_O	ATI_P	ASN
1000	30	50	3	5	2	4	0.95	0.05	0.05	0.825	0.061	0.114	1.000	0.039	87	202	40
1000	35	50	2	5	2	5	0.83	0.03	0.04	0.857	0.041	0.102	1.000	0.029	78	180	49
1000	35	60	3	6	2	3	0.95	0.03	0.02	0.968	0.007	0.025	1.000	0.029	42	67	38
500	35	60	2	6	3	5	0.94	0.06	0.02	0.792	0.205	0.003	1.000	0.047	138	140	55
500	30	40	4	8	5	10	0.9	0.03	0.02	1.000	0.000	0.000	1.000	0.032	30	30	30
1000	30	50	3	5	3	5	0.9	0.05	0.05	0.903	0.056	0.041	1.000	0.045	84	126	36

It should be noted that the total probability is equal to 1 in all rows because the probabilities were normalized before calculations to satisfy Eq. (9). Solutions are reached for both consistent and inconsistent data cases. If defective and/or indeterminate item threshold values are increased, party acceptance probability increases. In inconsistent data case, relative greatness of non-defective probability of the items over defective and indeterminate probabilities has a positive effect on party acceptance probability. If indeterminacy probability is equal to defectiveness probabilitiy and threshold values are equal to each other ($c = I$, $P(F) = P(I)$), party rejection probability is observed bigger than party indeterminacy probability ($P_r > P_i$) as a result of the dominancy of the rejection over indeterminacy. Party size does not affect P_a, P_r, P_i and AOQ values while plan parameters are not changing.

5 Conclusion

ASP is a certain set of rules for the inspection process to reach a specified quality level statistically. It offers to inspect small set of items to reduce the costs in terms of money, effort and time. DASPs are used to catch a higher quality level without increasing the sample size too much. ASPs procure clear statistical output measurements by using the mass quality metrics such as defective ratio of the items. Unfortunately, the mass quality metrics may not be known as certainly in some cases. Traditional fuzzy logic was used in some studies to formulate the DASPs under uncertainty but there is no study using FS extensions. NSs have a big advantage on modeling real case problems having uncertainty related with human factors. In this study, DASPs having certain plan parameters and neutrosophic defection statuses are offered for binomial distribution at the first time. As a future study, formulations can be extended for poisson distribution, a similary study can be conducted based on interval NSs or another study can be organized for the plans having neutrosophic plan parameters.

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