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Existence and Exponential Stability of a Class of Impulsive Neutral Stochastic Integrodifferential Equations with Poisson Jumps

K. Ramkumar [1], **K. Ravikumar** [2], **E. M. Elsayed** [3*]
[1,2] PSG College of Arts and Science, Department of Mathematics
Coimbatore, India, ramkumarkpsg@gmail.com, ravikumarkpsg@gmail.com
[3] Mansoura University, Faculty of Science, Department of Mathematics
Mansoura, Egypt

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Abstract: In this paper by employing the fractional power of operators and semi group theory we obtain some new criteria ensuring the existence and exponential stability of a class of impulsive neutral stochastic integrodifferential equations with Poisson jumps. We use fixed point strategy to establish some new sufficient conditions that ensure the exponential stability of mild solution in the mean square moment by utilizing an impulsive integral inequality.

Keywords: Existence, exponential stability, impulsive system, stochastic integrodifferential equations, poisson jumps.

1. Introduction

Stochastic differential equations have been investigated as mathematical models to describe the dynamical behavior of real life phenomena. It is essential to take into account the environmental disturbances as well as the time delay while constructing realistic models in the area of engineering, biology, etc. In the past few decades many authors studies on quantitative and qualitative properties of neutral stochastic functional differential equations were carried out see [1, 2, 4, 6–8, 13] and references therein.

Impulsive differential equations thrive to be a promising area and have gained much attention among the researchers due to their potential application in various fields such as orbital transfer of satellite, dosage supply in pharmacokinetics, etc. It is worth mentioning that many real world systems are subjected to stochastic abrupt changes, and therefore it is necessary to investigate them using impulsive stochastic differential equations. Several authors have investigated the neutral stochastic integrodifferential equation with impulsive effects, refer to [5, 6, 10–12].

Furthermore, several practical systems (such as sudden price variations [jumps] due to mar-

*Correspondence: emmelsayed@yahoo.com
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ket crashes, earthquakes, hurricanes, epidemics, and so on) experiences some jump type stochastic perturbations. The sample paths are not being continuous. Thus it is seize considering stochastic processes with jumps in describing the models. Generally, the jump models are derived from poisson random measure. The sample paths of systems being right continuous possess left limits. In the recent trend, researchers are focusing more on the theory and applications of impulsive stochastic functional differential equations with poisson jumps. Precisely, existence and stability results on impulsive stochastic functional differential equations with poisson jumps are found in [2–5] and the references therein. Successively, few works have been reported in the study of stochastic differential equations with poisson jumps, refer to [2, 10, 12–14].

However, motivated by the above consideration, the aim of this paper is to establish the results on existence and exponential stability of mild solution of impulsive neutral stochastic integrodifferential equations with Poisson jumps of the form:

$$\begin{aligned} d\left[x(t)+q(t,x_t)\right] &= \left[Ax(t)+f(t,x_t)+\int_0^t g(t,s,x_s)ds\right]dt+\sigma(t,x_t)dw(t) \\ &+ \int_{\mathcal{U}} h(t,x_t,u)\widetilde{N}(dt,du), \quad t\in[0,b], \quad t\neq t_k, \qquad (1) \\ \Delta x(t_k) &= I_k(x(t_k^-)), \quad t=t_k, \quad k=1,2,..., \qquad (2) \\ x(t) &= \varphi(t)\in\mathscr{PC}([-r,0];\mathcal{X}), \quad -r\le t\le 0. \qquad (3) \end{aligned}$$

where A is the infinitesimal generator of an analytic semigroup $(S(t))$, $t\geq 0$ of bounded linear operators in a Hilbert space $\mathcal{X}$. The functions $q,f:[0,+\infty)\times\mathscr{PC}\to\mathcal{X}$, $\sigma:[0,+\infty)\times\mathscr{PC}\to\mathcal{L}_2^0(\mathcal{Y},\mathcal{X})$, $h:[0,+\infty)\times\mathscr{PC}\times\mathcal{U}\to\mathcal{X}$ and $I_k:\mathcal{X}\to\mathcal{X}$ are appropriate functions. The impulsive moments t_k satisfy the condition $0<t_1<t_2<\cdots<t_k<\cdots, \lim_{k\to\infty}t_k=\infty$, $\Delta x(t_k)=x(t_k^+)-x(t_k^-)$ is the jump size of the state x at t_k. For $\varphi\in\mathscr{PC}$, $\|\varphi\|_{\mathscr{PC}}=\sup_{s\in[-r,0]}\|\varphi(s)\|<+\infty$, where $\mathscr{PC}=\big\{\varphi:[-r,0]\to\mathcal{X}:\varphi(t)$ is continuous everywhere except a finite number of points $\bar{t}$ at which $\varphi(\bar{t}^-),\varphi(\bar{t}^+)$ exist and $\varphi(\bar{t}^-)=\varphi(\bar{t}^+)\ \big\}$. For any $t\in[0,b]$ and any continuous functions x, the element of $\mathscr{PC}$ is defined by $x_t(\theta)=x(t+\theta)$.

2. Preliminaries

Let $\mathcal{X}$, $\mathcal{Y}$ be real separable Hilbert spaces and $\mathcal{L}(\mathcal{Y},\mathcal{X})$ be the space of bounded linear operators mapping $\mathcal{Y}$ into $\mathcal{X}$. Let $(\Omega,\Im,\mathbb{P})$ be a complete probability space with an increasing right continuous family $\{\Im_t\}_{t\geq 0}$ of complete sub σ algebra of $\Im$. Let $\{w(t):t\geq 0\}$ denote a $\mathcal{Y}$-valued Wiener process defined on the probability space $(\Omega,\Im,\mathbb{P})$ with covariance operator Q, that is $\mathbb{E}<w(t),x>_{\mathcal{Y}}<w(s),y>_{\mathcal{Y}}=(t\wedge s)<Qx,y>_{\mathcal{Y}}$, for all $x,y\in\mathcal{Y}$, where Q is a positive, self-adjoint, trace class operator on $\mathcal{Y}$. We assume that there exists a complete orthonormal system $\{e_i\}_{i\geq 1}$ in

$\mathcal{Y}$, a bounded sequence of non-negative real numbers λ_i such that $Qe_i = \lambda_i e_i$, $i = 1,2,...,$ and a sequence $\{\beta_i\}_{i\geq 1}$ of independent Brownian motions such that $\langle w(t), e\rangle = \sum_{n=1}^{\infty} \sqrt{\lambda_i}\,\langle e_i, e\rangle\,\beta_i(t)$, $e \in \mathcal{Y}$, and $\mathfrak{I}_t = \mathfrak{I}_t^w$, where $\mathfrak{I}_t^w$ is the sigma algebra generated by $\{w(s) : 0 \leq s \leq t\}$. Let $\mathcal{L}_2^0 = \mathcal{L}_2(\mathcal{Y}_0, \mathcal{X})$ denote the space of all Hilbert-Schmidt operators from $\mathcal{Y}_0$ into $\mathcal{X}$. It turns out to be a separable Hilbert space equipped with the norm $\|\zeta\|_{\mathcal{L}_2^0}^2 = \operatorname{tr}\left((\zeta Q^{\frac{1}{2}})(\zeta Q^{\frac{1}{2}})^*\right)$ for any $\zeta \in \mathcal{L}_2^0$. Clearly for any bounded operators $\zeta \in \mathcal{L}(\mathrm{Y},\mathrm{X})$ this norm reduces to $\|\zeta\|_{\mathcal{L}_2^0}^2 = \operatorname{tr}(\zeta Q \zeta^*)$.

Suppose $\{P(t), t \geq 0\}$ is a σ-finite stationary $\mathfrak{I}_t$-adapted Poisson point process taking values in measurable space $(\mathrm{U}, \mathscr{B}(\mathrm{U}))$. The random measure N_p defined by $N_p((0,t] \times \Lambda) := \sum_{s\in(0,t]} 1_{\Lambda}(P(s))$ for $\Lambda \in \mathscr{B}(\mathrm{U})$ is called the Poisson random measure induced by $P(\cdot)$, thus, we can define the measure $\widetilde{N}$ by $\widetilde{N}(dt,dz) = N_P(dt,dz) - v(dz)dt$, where v is the characteristic measure of N_P, which is called the compensated Poisson random measure.

Let us state some notations and basic facts about the theory of semi groups and fractional power operators. Let $A : \mathcal{D}(A) \to \mathcal{X}$ be the infinitesimal generator of an analytic semigroup, $(S(t))$, $t \geq 0$, of bounded linear operators on $\mathcal{X}$. For the theory of strongly continuous semigroup, we refer to [9]. We will point out here some notations and properties that will be used in this work. It is well known that there exists $M \geq 1$ and $\lambda \in \mathbb{R}$ such that

$$\|S(t)\| \quad \leq \quad Me^{\lambda t}, \quad t \geq 0.$$

If $(S(t))$, $t \geq 0$, is a uniformly bounded and analytic semigroup so as $0 \in \rho(A)$, where $\rho(A)$ being the resolvent set of A, then it is possible to define the fractional power $(-A)^{\zeta}$ for $0 < \zeta \leq 1$, as a closed linear operator on its domain $\mathcal{D}(-A)^{\zeta}$. Furthermore, the subspace $\mathcal{D}(-A)^{\zeta}$ is dense in $\mathbb{X}$, and the assertion

$$\|h\|_{\zeta} \quad = \quad \left\|(-A)^{\zeta} h\right\|$$

interprets a norm in $\mathcal{D}(-A)^{\zeta}$. If $\mathcal{X}_{\zeta}$ denotes the space $\mathcal{D}(-A)^{\zeta}$ endowed with the norm $\|\cdot\|_{\zeta}$, then the following properties are well known in see [9].

Lemma 2.1 *[9] Suppose the following conditions hold:*

(1) If $0 < \zeta \leq 1$, then $\mathcal{X}_{\zeta}$ is a Banach space.

(2) If $0 < \beta \leq \zeta$, then the injection $\mathcal{X}_{\zeta} \hookrightarrow \mathcal{X}_{\beta}$.

(3) There exists $M_{\zeta} > 0$ such that

$$\left\|(-A)^{\zeta} S(t)\right\| \quad \leq \quad M_{\zeta} t^{\zeta} e^{-\lambda t}, \quad t > 0, \quad \lambda > 0.$$

Definition 2.2 *An $\mathcal{X}$-valued stochastic process $x(t)$, $t \in [-r, b]$ is called a mild solution of system (1)-(3) if*

(1) $x(\cdot) \in \mathscr{PC}([-r, b]; \mathcal{L}(\Omega, \mathcal{X}))$,

(2) $x(t) = \varphi(t)$ for $t \in [-r, 0]$,

(3) For $t \in [0, b]$, $x(t)$ satisfies the following integral equation

$$\begin{aligned} x(t) &= S(t)[\varphi(0) - q(0, \varphi)] + q(t, x_t) + \int_0^t AS(t-s)q(s, x_s)ds + \int_0^t S(t-s)f(s, x_s)ds \\ &+ \int_0^t \int_0^s S(t-s)g(s, \tau, x_\tau)d\tau ds + \int_0^t S(t-s)\sigma(s, x_s)dw(s) \\ &+ \int_0^t \int_{\mathcal{U}} S(t-s)h(s, x_s, u)\widetilde{N}(ds, du) + \sum_{0<t_k<t} S(t-t_k)I_k(x(t_k^-)). \end{aligned} \tag{4}$$

3. Existence Results

In this section, we first formulate and prove sufficient conditions for the existence and uniqueness of a mild solution of system (1)-(3) by using the fixed point theory. To guarantee the existence and uniqueness of the solution, we impose some hypotheses:

(H1) A is the infinitesimal generator of an analytic semigroup $(S(t))_{t\geq 0}$, of bounded linear operators on $\mathcal{X}$ such that $0 \in \rho(A)$, the resolventset of $-A$, and $S(t)$ is uniformly bounded

$$\|S(t)\| \leq M \quad \text{and} \quad \left\|(-A)^{1-\zeta}S(t)S(t)\right\| \leq \frac{M_{1-\zeta}}{t^{1-\zeta}}$$

for some constants $M, M_{1-\zeta}$ and every $t \in [0, T]$.

(H2) For all $t \in [0, b]$ there exist constants $\frac{1}{2} < \infty < 1$ and $k_1 > 0$ such that, for $\varphi_i \in \mathscr{PC}$, $i = 1, 2$ the $\mathcal{X}_\alpha$-valued function $q : [0, +\infty) \times \mathscr{PC} \to \mathcal{X}$ satisfies the condition

$$\|(-A)^\alpha q(t, \varphi_1) - (-A)^\alpha q(t, \varphi_2)\| \quad \leq \quad k_1 \|\varphi_1 - \varphi_2\|.$$

Also, $\bar{k}_1 = \sup_{t\in[0,b]} \|(-A)^\alpha q(t, 0)\|$.

(H3) $(-A)^\alpha q$ is a continuous function in the quadratic mean sense.

$$\lim_{t\to s} \mathbb{E}\,\|(-A)^\alpha q(t, \varphi) - (-A)^\alpha q(s, \varphi)\|^2 \quad = \quad 0.$$

(H4) The mapping $f : [0, +\infty) \times \mathscr{PC} \to \mathcal{X}$, $\sigma : [0, +\infty) \times \mathscr{PC} \to \mathcal{L}(\mathcal{Y}, \mathcal{X})$ and $h : [0, +\infty) \times \mathscr{PC} \times \mathcal{U} \to \mathcal{X}$ satisfies the following Lipschitz condition for all $t \in [0, b]$, $k_2, k_3, k_4 > 0$ such that $\varphi_i \in \mathscr{PC}$,

$i = 1, 2$

$$\|f(t,\varphi_1) - f(t,\varphi_2)\| \leq k_2 \|\varphi_1 - \varphi_2\|, \qquad \|\sigma(t,\varphi_1) - \sigma(t,\varphi_2)\| \leq k_3 \|\varphi_1 - \varphi_2\|.$$

$$\int_{\mathcal{U}} \|h(t,\varphi_1,u) - h(t,\varphi_2,u)\|^2 v(du) \vee \left(\int_{\mathcal{U}} \|h(t,\varphi_1,u) - h(t,\varphi_2,u)\|^4 v(du)^{1/2}\right) \leq k_4 \|\varphi_1 - \varphi_2\|.$$

$$\left(\int_{\mathcal{U}} \|h(t,\varphi,u)\|^4 v(du)^{1/2}\right) \leq k_4 \|\varphi\|.$$

Here $\bar{k}_2 = \sup_{t\in[0,b]} \|f(t,0)\|$, $\bar{k}_3 = \sup_{t\in[0,b]} \|\sigma(t,0)\|$, $\bar{k}_4 = \sup_{t\in[0,b]} \|h(t,0,u)\|$.

(H5) The mapping $g : [0,+\infty)\times[0,+\infty)\times\mathscr{PC} \to \mathcal{X}$ satisfies the Lipschitz condition for all $t \in [0,b]$, $k_5 > 0$ such that $\varphi_i \in \mathscr{PC}$, $i = 1, 2$

$$\left\| \int_0^t [g(t,s,\varphi_1) - g(t,s,\varphi_2)]\, ds \right\| \leq k_5 \|\varphi_1 - \varphi_2\|.$$

Here $\bar{k}_5 = \sup_{t\in[0,b]} \|g(t,s,0)\|$.

(H6) The impulsive function $I_k : \mathcal{X} \to \mathcal{X}$ is continuous and there exist positive numbers q_k, $k = 1, 2, \ldots$ such that $\sum_{k=1}^{\infty} q_k < \infty$

$$\|I_k(\varphi_1) - I_k(\varphi_2)\| \leq q_k \|\varphi_1 - \varphi_2\|, \ldots, \|I_k(0)\| = 0 \quad \text{for} \quad \varphi_1, \varphi_2 \in \mathscr{PC}.$$

Theorem 3.1 *Assume that* ***(H1)-(H6)*** *are satisfied for all $\varphi \in \mathscr{PC}$, $b > 0$ and*

$$\frac{6M^2 \sum_{k=1}^{\infty} q_k^2}{(1-k)^2} < 1, \tag{5}$$

where $k = k_1 \left\|(-A)^{-\beta}\right\|$. Then system (1)-(3) has a unique mild solution on $[-r, b]$.

Proof For $b > 0$, the Banach space of all continuous functions from $[-r,b]$ into $\mathcal{L}^2(\Omega,\mathcal{X})$ stands for the set $\Gamma_b = \mathscr{PC}([-r,b];\mathcal{L}^2(\Omega,\mathcal{X}))$ equipped with the norm

$$\|\phi\|^2_{\Gamma_b} = \sup_{s\in[-r,b]} \mathbb{E}\|\phi\|^2.$$

Denote $\hat{\Gamma}_b = \{x \in \Gamma_b : x(\tau) = \varphi(\tau) \text{ for } \tau \in [-r, 0]\}$, which is a closed subset of Γ_b provided with the norm $\|\cdot\|_{\Gamma_b}$. Now we define $\mathcal{G} : \hat{\Gamma}_b \to \hat{\Gamma}_b$ by

$$
\begin{aligned}
(\mathcal{G}x)(t) &= \varphi(t), \quad t \in [-r, 0], \\
(\mathcal{G}x)(t) &= S(t)\left[\varphi(0) - q(0,\varphi)\right] + q(t, x_t) + \int_0^t AS(t-s)q(s, x_s)ds + \int_0^t S(t-s)f(s, x_s)ds \\
&+ \int_0^t \int_0^s S(t-s)g(s, \tau, x_\tau)d\tau ds + \int_0^t S(t-s)\sigma(s, x_s)dw(s) \\
&+ \int_0^t \int_{\mathcal{U}} S(t-s)h(s, x_s, u)\widetilde{N}(ds, du) + \sum_{0<t_k<t} S(t-t_k)I_k(x(t_k^-)).
\end{aligned}
$$

Now, to prove the existence of mild solutions of (1)-(3), it is sufficient to show that $\mathcal{G}$ has a fixed point.

Step 1: We claim that $t \to (\mathcal{G}x)(t)$ is continuous on the interval $[0, b]$.

Let $x \in \hat{\Gamma}_b$, $0 < t < b$, and let $|\gamma|$ be sufficiently small. Then we have

$$
\mathbb{E}\left\|(\mathcal{G}x)(t+\gamma) - (\mathcal{G}x)(t)\right\|^2 \le 8\left\|S(t+\gamma) - S(t)\left[\varphi(0) + q(0,\varphi)\right]\right\|^2 + 8\sum_{i=1}^{7}\left\|\mathcal{K}_i(t+\gamma) - \mathcal{K}_i(t)\right\|^2.
$$

Then employing the Lebesgue dominated theorem and the strong continuity of $S(t)$ implies that

$$
\lim_{\gamma \to 0}\left\|S(t+\gamma) - S(t)\right\|^2 \mathbb{E}\left\|\left[\varphi(0) + q(0,\varphi)\right]\right\|^2 \to 0.
$$

Next, it is well known that $(-A)^{-\beta}$ is bounded,

$$
\mathbb{E}\left\|\mathcal{K}_1(t+\gamma) - \mathcal{K}_1(t)\right\|^2 \le \left\|(-A)^{-\beta}\right\|^2 \mathbb{E}\left\|(-A)^{-\beta}q(t+\gamma, x_{t+\gamma}) - (-A)^{-\beta}q(t, x_t)\right\|^2.
$$

By assumption **(H3)**, we obtain that $\lim_{\gamma\to 0}\mathbb{E}\left\|\mathcal{K}_1(t+\gamma) - \mathcal{K}_1(t)\right\|^2 \to 0$.

Next, for the term $\mathcal{K}_2$, using **(H2)**, Holder's inequality, we get

$$
\begin{aligned}
\mathbb{E}\left\|\mathcal{K}_2(t+\gamma) - \mathcal{K}_2(t)\right\|^2 &\le 2\mathbb{E}\left\|\int_0^t \left[S(t+\gamma-s) - S(t-s)\right](-A)^{1-\beta}(-A)^{\beta}q(s, x_s)ds\right\|^2 \\
&+ 2\mathbb{E}\left\|\int_0^t \left[S(t+\gamma-s)\right](-A)^{1-\beta}(-A)^{\beta}q(s, x_s)ds\right\|^2 \\
&\le 2t\int_0^t \left\|S(t+\gamma-s) - S(t-s)\right\|^2 \left\|(-A)^{1-\beta}\right\|^2 \mathbb{E}\left[k_1\left\|x_s\right\|^2 + \bar{k_1}\right] \\
&+ 2h\int_0^{t+\gamma} \left\|S(t+\gamma-s)\right\|^2 \left\|(-A)^{1-\beta}\right\|^2 \mathbb{E}\left[k_1\left\|x_s\right\|^2 + \bar{k_1}\right] \\
&\to 0 \text{ as } |\gamma| \to 0.
\end{aligned}
$$

A similar computation gives us $\mathbb{E}\|\mathcal{K}_i(t+\gamma)-\mathcal{K}_2(i)\|^2 \to 0$ as $|\gamma|\to 0$ for $i=3,4,5,7$. Similarly,

$$\begin{aligned}\mathbb{E}\|\mathcal{K}_6(t+\gamma)-\mathcal{K}_6(t)\|^2 &\leq 2\mathbb{E}\left\|\int_0^t\int_{\mathcal{U}}[S(t+\gamma-s)-S(t-s)]h(s,x_s,u)\widetilde{N}(ds,du)\right\|^2\\ &+ 2\mathbb{E}\left\|\int_0^{t+\gamma}\int_{\mathcal{U}}[S(t+\gamma-s)]h(s,x_s,u)\widetilde{N}(ds,du)\right\|^2\\ &\to 0 \text{ as } |\gamma|\to 0.\end{aligned}$$

Hence, the above arguments imply that $t\to(\mathcal{G}x)(t)$ is continuous on the interval $[0,b]$.

Step 2: Next, we verify that $\mathcal{G}$ is a contraction mapping in $\hat{\Gamma}_{b_1}$ with some $b_1\leq b$ to be specified later. Let $x,y\in\hat{\Gamma}_b$ and $t\in[0,b]$.Then we have

$$\begin{aligned}\mathbb{E}\|\mathcal{G}(x(t))-\mathcal{G}(y(t))\|^2 &\leq \frac{1}{k}\mathbb{E}\left\|(-A)^{-\beta}\right\|^2\left\|(-A)^{\beta}q(t,x_t)-q(t,y_t)\right\|^2\\ &+ \frac{6}{1-k}\mathbb{E}\left\|\int_0^t(-A)^{1-\beta}S(t-s)(-A)^{\beta}[q(s,x_s)-q(s,y_s)]ds\right\|^2\\ &+ \frac{6}{1-k}\mathbb{E}\left\|\int_0^t S(t-s)(-A)^{\beta}[f(s,x_s)-f(s,y_s)]ds\right\|^2\\ &+ \frac{6}{1-k}\mathbb{E}\left\|\int_0^t S(t-s)(-A)^{\beta}[\sigma(s,x_s)-\sigma(s,y_s)]ds\right\|^2\\ &+ \frac{6}{1-k}\mathbb{E}\left\|\int_0^t\int_0^s S(t-s)[g(s,\tau,x_\tau)-g(s,\tau,y_\tau)]d\tau ds\right\|^2\\ &+ \frac{6}{1-k}\mathbb{E}\left\|\int_0^t\int_{\mathcal{U}}S(t-s)[h(s,x_s,u)-h(s,y_s,u)]\widetilde{N}(ds,du)\right\|^2\\ &+ \frac{6}{1-k}\mathbb{E}\left\|\sum_{0<t_k<t}S(t-t_k)[I_k(x(t_k))-I_k(y(t_k))]\right\|^2.\end{aligned}$$

By using Holder's inequality, together with assumptions **(H1)-(H6)**, we get

$$\begin{aligned}\mathbb{E}\|\mathcal{G}(x(t))-\mathcal{G}(y(t))\|^2 &\leq k\mathbb{E}\|x-y\|^2+\frac{6}{1-k}\Bigg[M_{1-\beta}^2k_1^2\left(\frac{t^{2\beta-1}}{2\beta-1}\right)\\ &+ M^2\left(t(k_2^2+k_5^2)+k_3^2+k_4^2\right)\Bigg]\int_0^t\mathbb{E}\|x-y\|_s^2ds+\frac{6}{1-k}M^2\sum_{k=1}^{\infty}q_k^2\mathbb{E}\|x-y\|_t^2.\end{aligned}$$

Hence,

$$\sup_{s\in[-r,t]}\mathbb{E}\|\mathcal{G}(x(s))-\mathcal{G}(y(s))\|^2 \leq \rho(t)\sup_{s\in[-r,t]}\mathbb{E}\|x(s)-y(s)\|^2,$$

where $\rho(t) = k + \frac{6}{1-k}\left[M_{1-\beta}^2 k_1^2\left(\frac{t^{2\beta-1}}{2\beta-1}\right) + tM^2\left(t(k_2^2+k_5^2)+k_3^2+k_4^2\right)\right] + \frac{6}{1-k}M^2\sum_{k=1}^{\infty} q_k^2$ by (5), we have $\rho(0) = k + \frac{6}{1-k}M^2\sum_{k=1}^{\infty} q_k^2 = \frac{6M^2\sum_{k=1}^{\infty} q_k^2}{(1-k)^2} < 1$. Then there prevails $0 < b_1 \leq b$ such that $0 < \rho(T_1) < 1$ and $\mathcal{G}$ is a contraction mapping on $\hat{\Gamma}_{b_1}$ and thus has a unique fixed point, being a mild solution of equation (1)-(3) on $[-\tau, b_1]$. By repeating a similar process the solution can be extended to the entire interval $[-\tau, b]$ in finitely many steps. This completes the proof. □

4. Stability Analysis

In this section, to initiate adequate conditions securing the exponential decay to zero in mean square for mild solution of equation (1)-(3), we need to state the following additional assumptions:

(H7) $(S(t))_{t\geq 0}$ satisfies the following conditions in addition to **(H1)**. There exists

$$\lambda > 0,\ M > 0 \ \text{ such that } \ \|S(t)\| \leq Me^{-\lambda t} \ \ for \ \ all \ \ t \geq 0,$$

we note that the semigroup is exponentially stable.

(H8) There exist non-negative real numbers $N_1, N_2, N_3, N_4, N_5 \geq 0$ and continuous functions $\eta_1, \eta_2, \eta_3, \eta_4, \eta_5 : [0, +\infty) \longrightarrow \mathbb{R}_+$ such that for all $t \geq 0$ and $x, y \in \mathbb{X}$.

1. $\left\|(-A)^{\zeta} p(t,x)\right\|^2 \leq N_1 \|x\|^2 + \eta_1(t)$,
2. $\|f(t,x)\|^2 \leq N_2 \|x\|^2 + \eta_2(t)$,
3. $\|\sigma(t,x)\|^2 \leq N_3 \|x\|^2 + \eta_3(t)$,
4. $\left\|\int_0^t g(t,s,\varphi)dt\right\| \leq N_4 \|x\|^2 + \eta_4(t)$,
5. $\int_{\mathcal{U}} \|h(t,x,u)\|^2 v(du) \vee \left(\int_{\mathcal{U}} \|h(t,x,u)\|^2 v(du)\right)^{\frac{1}{2}} \leq N_5 \|x\|^2 + \eta_5(t)$.

(H9) There exist non-negative real numbers $Q_j \geq 0$, $j = 1,2,3,4,5$ such that

$$\xi_j(t) \leq Q_j e^{-\lambda t} \ \ for \ \ all \ \ t \geq 0, \quad j = 1,2,3,4,5.$$

Lemma 4.1 *Let $N : [-\tau, +\infty) \longrightarrow [0, +\infty)$ be a function and suppose that there exist some constants $\gamma > 0$, $\lambda_i > 0$ $(i = 1,2,3)$ such that*

$$N(t) \ \leq \ \lambda_1 e^{-\gamma t} + \lambda_2 \sup_{\theta\in[-\tau,0]} N(t+\theta) + \lambda_3 \int_0^t e^{-\gamma(t-s)} \sup_{\theta\in[-\tau,0]} N(s+\theta) ds, \quad t \geq 0$$

and

$$N(t) \ \leq \ \lambda_1 e^{-\gamma t}, \qquad t \in [-\tau, 0].$$

If $\lambda_2 + \frac{\lambda_3}{\gamma} < 1$. *Then, we have* $N(t) \le Me^{-\mu t}$ $(t \ge -\tau)$, *where* μ *is a positive root of the algebra equation* $\lambda_2 + \frac{\lambda_3}{\gamma} e^{\mu\tau} = 1$ *and* $M = \max\left\{\frac{\lambda_t(\gamma-\mu)}{\lambda_3 e^{\mu\tau}}, \lambda_1\right\}$.

Theorem 4.2 *Assume that* ***(H7)-(H9)*** *and the following inequality holds*

$$\frac{7\lambda^{1-2\beta}2^{2(1-\beta)}M_{1-\beta}^2 M^2\Gamma(2\beta-1)N_1/\lambda + 7M^2[N_2+N_3+N_4+N_5]/\lambda^2 + 7M^2\sum_{k=1}^{\infty} q_k)^2}{(1-k)^2} < 1, \qquad (6)$$

where $k = \sqrt{N_1}\left\|(-A)^{-\beta}\right\|$. *Then the mild solution of system (1)-(3) is exponentially stable in the mean square moment.*

Proof By inequality (6) theere is $\epsilon > 0$ small enough such that

$$k + \frac{7\lambda^{1-2\beta}2^{2(1-\beta)}M_{1-\beta}^2 M^2\Gamma(2\beta-1)N_1}{(\lambda-\epsilon)(1-k)} + \frac{7M^2[N_2+N_3+N_4+N_5]}{\lambda(\lambda-\epsilon)(1-k)} + \frac{7M^{2(\sum_{k=1}^{\infty} q_k)^2}}{1-k} < 1.$$

Let $\mu = \lambda - \epsilon$ and $x(t)$ be the mild solution of (1)-(3). Then for $t \ge 0$,

$$\begin{aligned}
\mathbb{E}\|x\|^2 &\le \frac{1}{k}\mathbb{E}\|q(t,x_t)\|^2 + \frac{7}{1-k}\mathbb{E}\Bigg[\|S(t)[\varphi(0)+q(0,\varphi)]\|^2 \\
&+ \left\|\int_0^t AS(t-s)q(s,x_s)ds\right\|^2 + \left\|\int_0^t S(t-s)f(s,x_s)ds\right\|^2 \\
&+ \left\|\int_0^t S(t-s)\sigma(s,x_s)dw(s)\right\|^2 + \left\|\int_0^t \int_0^s S(t-s)g(s,\tau,x_\tau)d\tau ds\right\|^2 \\
&+ \left\|\int_0^t \int_{\mathcal{U}} h(s,x_s,u)\widetilde{N}(ds,du)\right\|^2 + \left\|\sum_{0<t_k<t} S(t-t_k)I_k(x(t_k))\right\|^2\Bigg] \\
&\le \sum_{i=1}^{8} G_i(t).
\end{aligned}$$

From assumptions **(H7)-(H9)**, we get

$$\begin{aligned}
G_1(t) &= \frac{1}{k}\mathbb{E}\|q(t,x_t)\|^2 \\
&\le \frac{\left\|(-A)^{-\beta}\right\|^2}{k}\left[N_1\mathbb{E}\|x\|^2 + \eta_1(t)\right] \\
&\le k\mathbb{E}\|x_t\|^2 + S_1 e^{-\lambda t},
\end{aligned}$$

where $S_1 = \frac{\|(-A)^{-\beta}\|^2}{k} Q_1$. From **(H7)-(H9)**, we get

$$\begin{aligned}
G_2(t) &= \frac{14}{1-k}\mathbb{E}\|S(t)\varphi(0)\|^2 + \frac{14}{1-k}\mathbb{E}\|S(t)q(0,\varphi)\|^2 \\
&\le S_2 e^{-\lambda t},
\end{aligned}$$

where $\frac{14M^2}{1-k}\left[\mathbb{E}\left\|\varphi(0)\right\|^2+\left\|(-A)^{-\beta}\right\|^2\left\{N_1\mathbb{E}\left\|\varphi\right\|^2+Q_2\right\}\right]$.

Using Holder's inequality and **(H7)-(H9)**, we get

$$\begin{aligned} G_3(t) &= \frac{7}{1-k}\mathbb{E}\left\|\int_0^t (-A)^{1-\beta}T(\frac{t-s}{2})T(\frac{t-s}{2})(-A)^{\beta}q(s,x_s)ds\right\|^2 \\ &\leq \frac{7\lambda^{1-2\beta}2^{2(1-\beta)}M_{1-\beta}^2M^2\Gamma(2\beta-1)N_1}{1-k}\int_0^t e^{-\lambda(t-s)}\mathbb{E}\left\|x_s\right\|^2ds+S_3e^{-\eta t}, \end{aligned}$$

where $S_3=\frac{7\lambda^{1-2\beta}2^{2(1-\beta)}M_{1-\beta}^2M^2\Gamma(2\beta-1)N_1}{1-k}\frac{Q_1}{\lambda-\eta}$.

By assumptions **(H7)-(H9)**, applying the Holder's inequality, we get

$$\begin{aligned} G_4(t) &= \frac{7}{1-k}\mathbb{E}\left(\int_0^t Me^{-\lambda(t-s)}\left\|f(s,x_s)\right\|ds\right)^2 \\ &\leq \frac{7M^2N_2}{\lambda(1-k)}\int_0^t e^{-\lambda(t-s)}\mathbb{E}\left\|x_s\right\|^2ds+S_4e^{-\lambda t}, \end{aligned}$$

where $S_4=\frac{7M^2}{\lambda(1-k)}\frac{Q_2}{\lambda-\eta}$.

Similarly,

$$\begin{aligned} G_5(t) &= \frac{7}{1-k}\left(\int_0^t e^{-\lambda(t-s)}\mathbb{E}\left\|\sigma(s,x_s)\right\|ds\right)^2 \\ &\leq \frac{7M^2N_3}{\lambda(1-k)}\int_0^t e^{-\lambda(t-s)}\mathbb{E}\left\|x\right\|_s^2+S_5e^{-\lambda t}, \end{aligned}$$

where $S_5=\frac{7M^2}{\lambda(1-k)}\frac{Q_3}{\lambda-\eta}$.

Also,

$$\begin{aligned} G_6(t) &= \frac{7}{1-k}\mathbb{E}\left(\int_0^t\int_0^s Me^{-\lambda(t-s)}\left\|h(s,\tau,x_\tau)d\tau\right\|ds\right)^2 \\ &\leq \frac{7M^2N_4}{\lambda(1-k)}\int_0^t e^{-\lambda(t-s)}\mathbb{E}\left\|x_s\right\|^2ds+S_6e^{-\lambda t}, \end{aligned}$$

where $S_6 = \frac{7M^2}{\lambda(1-k)} \frac{Q_4}{\lambda-\eta}$.

Using **(H7)-(H9)**, we get

$$\begin{aligned} G_7(t) &= \frac{7}{1-k}\mathbb{E}\left(\left\| \int_0^t \int_{\mathcal{U}} S(t-s)h(s,x_s,u)\widetilde{N}(ds,du)\right\|\right)^2 \\ &\leq \frac{7}{1-k}M^2\bigg(\int_0^t \int_{\mathcal{U}} e^{-2\lambda(t-s)}\Big[\mathbb{E}\left\|h(s,x_s,u)\right\|^2 v(du) \\ &+ \left(\mathbb{E}\left\|h(s,x_s,u)\right\|^2 v(du)\right)^{1/2}\Big]ds\bigg) \\ &\leq \frac{7M^2}{\lambda(1-k)}N_5 \int_0^t e^{-\lambda(t-s)}\mathbb{E}\left\|x\right\|_s^2 ds + S_7 e^{-\lambda t}, \end{aligned}$$

where $S_7 = \frac{7M^2}{\lambda(1-k)} \frac{Q_5}{\lambda-\eta}$.

By using **(H6)**, one can get

$$\begin{aligned} G_8(t) &= \frac{7M^2}{1-k}\sum_{k=1}^{\infty} q_k^2 e^{-2\lambda(t-t_k)}\mathbb{E}\left\|x(t_k)\right\|^2 \\ &\leq \frac{7M^2}{1-k}\sum_{k=1}^{\infty} q_k^2 e^{\mu(t-t_k)}\mathbb{E}\left\|x(t_k)\right\|^2. \end{aligned}$$

The above inequalities together with Lemma 4.1 imply that

$$\mathbb{E}\left\|x(t)\right\|^2 \leq \delta e^{\mu t} \quad t \in [-r,0]$$

and for each $t \geq 0$,

$$\begin{aligned} \mathbb{E}\left\|x(t)\right\|^2 &\leq \delta e^{\mu t} + k \sup_{-r\leq\theta\leq 0} \mathbb{E}\left\|x(t+\theta)\right\|^2 + \hat{k}\int_0^t e^{-\mu(t-s)} \sup_{-r\leq\theta\leq 0} \mathbb{E}\left\|x(t+\theta)\right\|^2 ds \\ &+ \sum_{k=1}^{\infty} q_k e^{-\mu(t-t_k)}\mathbb{E}\left\|x(t_k^-)\right\|^2, \end{aligned}$$

where

$$\hat{k} = \frac{7\lambda^{1-2\beta}2^{2(1-\beta)}M_{1-\beta}^2 M^2\Gamma(2\beta-1)N_1}{1-k} + \frac{7M^2[N_2+N_3+N_4+N_5]}{\lambda(1-k)}$$

and

$$\delta = \max\left(\sum_{k=1}^{7} S_i, \sup_{-r\leq\theta\leq 0} \mathbb{E}\left\|\varphi(\theta)\right\|^2\right).$$

The mild solution of system (1)-(3) is exponentially stable in mean square moment, since $k + \frac{\hat{k}}{\mu} + \sum_{k=1}^{\infty} q_k < 1$ and by Lemma 4.1 there exist two positive constants S and θ such that $\mathbb{E}\left\|x(t)\right\|^2 \leq$

$Se^{-\theta t}$ for any $t \geq -r$, where $\theta > 0$. This ensures the exponential stability of mild solution in mean square. Hence the proof. □

Remark 4.3 *If the impulsive moments* $\Delta x(t_k) = I_k = 0$, $k = 1, 2, \ldots$ *then system (1)-(3) reduces to the following form*

$$\begin{aligned} d[x(t) + q(t, x_t)] &= \left[Ax(t) + f(t, x_t) + \int_0^t g(t, s, x_s)ds\right]dt + \sigma(t, x_t)dw(t) \\ &+ \int_{\mathcal{U}} h(t, x_t, u)\widetilde{N}(dt, du), \quad t \in [0, b], \quad t \neq t_k, \qquad (7) \\ x(t) &= \varphi(t) \in \mathscr{PC}([-r, 0]; \mathcal{X}), \quad -r \leq t \leq 0, \qquad (8) \end{aligned}$$

where the operators A, q, f, σ, h *are defined as before. Hence* $\mathscr{C} = \mathscr{C}([-r, 0]; \mathcal{X})$ *is endowed with the norm* $\|\varphi\|_{\mathscr{C}} = \sup_{\theta \in [-r, 0]} \|\varphi(\theta)\|$.

We can easily deduce the following corollary by using the same technique as in Theorem 4.2.

Corollary 4.4 *Assume that* ***(H7)-(H9)*** *hold and*

$$\frac{6\lambda^{1-2\beta}2^{2(1-\beta)}M_{1-\beta}^2 M^2 \Gamma(2\beta - 1)N_1/\lambda + 6M^2[N_2 + N_3 + N_4 + N_5]/\lambda^2}{(1-k)^2} < 1.$$

Then the mild solution of system (7)-(8) is exponentially stable in mean square moment.

5. Conclusion

In this paper by employing the fractional power of operators and semi group theory we obtain some new criteria ensuring the existence and exponential stability of a class of impulsive neutral stochastic integrodifferential equations with Poisson jumps. We use fixed point strategy to establish some new sufficient conditions that ensure the exponential stability of mild solution in the mean square moment by utilizing an impulsive integral inequality. Hence, in near future, we would like to extend this precious problem to the impulsive neutral stochastic integrodifferential equations with inclusions.

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