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On Soft Locally Path Connected Spaces

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Abstract: In this work, we give the definition of a soft locally path connected space and obtain some results.

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1. Introduction

The problems in daily life have many uncertainties. Classical "true-false" logic is not sufficient for uncertain, relative situations. Scientists in many sciences such as economics, sociology, engineering and medicine are dealing with the complexity of modeling inaccurate information. This situation has prompted scientists to develop very different mathematical modeling and tools to overcome uncertainties. As a result of these studies, different theories have been developed for the solution of uncertain types of problems.

The most common method used in uncertainty related problems is the fuzzy theory developed by Zadeh in 1965 [14]. While the accuracy value of a proposition is 0 or 1 by Aristo logic, in fuzzy logic, the accuracy value is all numbers in $[0,1]$. That is, a fuzzy set is considered as a function that matches each element in a set with a real number in the unit interval $[0,1]$. This function is called membership function. Also, since the construction of the membership function is completely individual, many membership functions for each case can be created. So, different results for the same problem appear.

Due to this disadvantage of the membership function, in 1999, it was introduced the concept of a soft set by Molodtsov as a new mathematical tool for uncertainties, seeing the need for a theory independent of the membership function [8]. Soft set theory was studied in many different fields of mathematics [4, 9–11].

The topological examination of the soft set theory was made by Shabir and Naz in 2011

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[13]. In that paper, Shabir and Naz defined the basic topological concepts such as soft open set, soft closed set, soft subspace, soft interior point, soft closure, soft neighborhood of a soft point over an initial universe with a fixed set of parameters. After Shabir and Naz gave the definition of soft topological space, the studies on soft topological spaces have increased. The concept of soft connectedness was first introduced in the literature by Peyghan et al., in 2013 [12]. In this study, a soft connected space is defined as not being able to write as a union of two non-empty soft sets which are soft disjoint and soft open. Peyghan et al., also presented the concept of soft local connected space in this study. They have also obtained important results related to the soft connectness and soft local connectedness. In 2014, Al-Khafaj and Mahmood obtained significant results related to the soft connected sets and soft disconnected sets [2]. The soft topological version of the concept of path, which is the main element of the path connectedness, was given by Bayramov et al., in 2013 [5]. For this purpose, firstly, by creating a soft topology on the unit interval $I = [0,1]$ and defining the concept of soft unit space, they proved that soft unit interval is a soft connected space. With the presentation of a soft unit interval, the concept of a soft path connected space is first presented in this study. This study which investigated the relationships between soft path connected spaces and soft connected spaces can be considered as the beginning of the soft homotopy theory. In this study, we aim to advance the studies on soft connectedness. For this purpose, we give the definition of a soft locally path connected space. We get some characterizations about soft locally path connectedness.

2. Preliminaries

We are here going to give fundamental concepts about the soft sets and soft topological spaces. For this aim, we will consider these concepts as two subsections: Soft sets and soft topological spaces.

2.1. Soft Sets

We here present some basic information about soft set theory. For details of the information in this section, we refer the reader to Molodtsov' s paper.

In this section, we will denote an initial universe and its power set by U and $P(U)$, respectively. Also E will denote a set of parameters.

Definition 2.1 *[8] Let U be an initial universe and $A \subset E$ be a subset of parameters. Then, a pair (F, A) is said to be a soft set over U, where F is a map given by $F : A \to P(U)$.*

In other words, a soft set over U is a parameterized family of subsets of the universe U. We will denote the set of all soft sets over U by $S(U)$ or $S(U, E)$.

Definition 2.2 *[8] A soft set (F, A) over U is called a null soft set, denoted by Φ, if $F(e) = \varnothing$*

for all $e \in A$.

Definition 2.3 *[8] A soft set* (F,A) *over* U *is called an absolute soft set, denoted by* $\widetilde{U}$*, if* $F(e) = U$ *for all* $e \in A$.

Definition 2.4 *[8] Let* (F,A) *and* (G,B) *be two soft sets over* U*. Then* (F,A) *is subset of* (G,B)*, denoted by* $(F,A)\widetilde{\subseteq}(G,B)$ *if* $A \subset B$ *and for all* $e \in A$*,* $F(e) \subset G(e)$.

Definition 2.5 *[8] Let* (F,A) *and* (G,B) *be two soft sets over a common universe* U*. Let* $C = A \cup B$*. Then, the union of* (F,A) *and* (G,B) *is a soft set* (H,C) *defined as follows: for all* $e \in C$,

$$H(e) = \begin{cases} F(e) & , \text{ if } e \in A - B \\ G(e) & , \text{ if } e \in B - A \\ F(e) \cup G(e) & , \text{ if } e \in A \cap B \end{cases}$$

Then we denote it by $(F,A)\widetilde{\cup}(G,B) = (H,C)$.

Definition 2.6 *[8] Let* (F,A) *and* (G,B) *be soft sets over* U*. The intersection of* (F,A) *and* (G,B) *is a soft set* $(H,C) = (F,A)\widetilde{\cap}(G,B)$ *defined by* $H(e) = F(e) \cap G(e)$ *for all* $e \in C = A \cap B$.

Definition 2.7 *[8] A soft set* (F,E) *is called a soft point, denoted by* (x_e, E)*, if for each element* $e \in E$*,* $F(e) = \{x\}$ *and* $F(e') = \varnothing$ *for all element* $e' \in E - \{e\}$.

Definition 2.8 *[6] The difference* (H,E) *of two soft sets* (F,E) *and* (G,E) *over* U*, denoted by* $(F,E)\widetilde{\setminus}(G,E)$*, is defined as* $H(e) = F(e) \setminus G(e)$*, for all* $e \in E$.

Definition 2.9 *[4] Let* $S(X,E)$ *and* $S(Y,E^{'})$ *be families of the all soft sets defined over the universe* X *and* Y*, respectively. Let* $u : X \to Y$ *and* $p : E \to E^{'}$ *be two maps. Then, the soft map* $f_{up} : S(X,E) \to S(Y,E^{'})$ *is defined as follows:*

(i) *Let* $A \subset E$ *and let* $(F,A) \in S(X,E)$*. Then,* $f_{up}((F,A))$ *is an element of* $S(Y,E^{'})$ *such that*

$$f_{up}(F,A)(a^{'}) = \bigcup_{a \in p^{-1}(a' \cap A)} F(a)$$

for $a^{'} \in p(A)$.

(ii) *Let* $(G,A^{'}) \in S(Y,E^{'})$*. Then,* $f_{up}^{-1}((G,A^{'}))$ *in an element of* $S(X,E)$ *such that* $f_{up}^{-1}((G,A^{'}))(a) = u^{-1}(G(p(a)))$ *for* $a \in p^{-1}(A^{'})$.

2.2. Soft Topological Spaces

In this section, we are going to recall fundamental concepts on soft topological spaces.

Definition 2.10 *[13] Let $E \neq \varnothing$ be a set of parameters and X an initial universe. Let (F,E) be a soft set over X and let us take any $x \in X$. If $x \in F(\alpha)$ for all $\alpha \in E$, then it is called that x belongs to (F,E) and it is denoted by $x \widetilde{\in} (F,E)$. Also, if $x \notin F(\alpha)$ for some $\alpha \in E$, then it is called that x doesn't belongs to (F,E) and it is written $x \widetilde{\notin} (F,E)$.*

Definition 2.11 *[1] Let $x \in X$. Then the notation (x,E) denotes the soft set for which $x(\alpha) = \{x\}$ for all $\alpha \in E$, and it is called soft singleton set.*

Definition 2.12 *[13] Let (F,E) be a soft set over X and Y be a non-empty subset of X. A sub soft set of (F,E) over Y is defined by ${}^{Y}F(\alpha) = Y \cap F(\alpha)$ for $\alpha \in E$ and is denoted by $({}^{Y}F,E)$.*

Definition 2.13 *[13] Let τ be a family of soft sets over X. If the following conditions hold, then the family τ is called a soft topology over X:*

(i) $\Phi, \widetilde{X} \in \tau$,

(ii) $\widetilde{\bigcup}_{i \in I} (F_i, E) \in \tau$ *for* $(F_i, E) \in \tau$, $i \in I$,

(iii) $\widetilde{\bigcap}_{i=1}^{n} (F_i, E) \in \tau$ *for* $(F_i, E) \in \tau$, $i = \{1, 2, \ldots, n\}$.

Then, the triplet (X,τ,E) is called soft topological space (S.T.S.).

Definition 2.14 *[13] Let (X,τ,E) be an S.T.S. Each element (F,E) of τ is called a soft open set. Also, for any soft set (F,E), if $(F,E)' \in \tau$, then (F,E) is called soft closed set.*

The following theorem states the relation between a soft topological space and a topological space.

Theorem 2.15 *[13] Let (X,τ,E) be S.T.S. Then the family $\tau_\alpha = \{F(\alpha) \mid (F,E) \in \tau\}$ for each $\alpha \in E$ determines a topology on X.*

Definition 2.16 *[13] Let X be an initial universe and E be a set of parameters.*

(i) *If $\tau = \{\Phi, \widetilde{X}\}$, then τ is called soft indiscrete topology over X, and the triplet (X,τ,E) is called soft indiscrete topological space over X.*

(ii) *If τ consists of all soft sets which can be defined over X, then τ is called soft discrete topology over X and the triplet (X,τ,E) is called soft discrete topological space over X.*

Definition 2.17 *[13] Let (X,τ,E) be S.T.S. and let us take any soft set (F,E) over X. The soft intersection of all soft closed sets which contains (F,E) is called soft closure of (F,E) and it is denoted by $\overline{(F,E)}$. Obviously, $\overline{(F,E)}$ is the smallest soft closed set that contains (F,E).*

Definition 2.18 *[13] Let (X,τ,E) be S.T.S. and let us take any soft set (F,E) over X. For all $\alpha \in E$, let us denote the closure of $F(\alpha)$ in τ_α by $\overline{F(\alpha)}$. Then, the soft set $(\overline{F},E)$ defined by $\overline{F}(\alpha)=\overline{F(\alpha)}$ for all $\alpha \in E$, is called relative closure of (F,E).*

Definition 2.19 *[13] Let (X,τ,E) be S.T.S. Let us take any soft set (G,E) over X and any element $x \in X$. If there is a soft open set (F,E) such that $x\tilde{\in}(F,E)\tilde{\subset}(G,E)$, the element x is called a soft interior point of (G,E). In this case, we said that (G,E) is a soft neighborhood of x.*

Definition 2.20 *[7] Let (X,τ,E) be S.T.S. Then, a family $\mathfrak{B} \subset \tau$ is called a base of (X,τ,E) if every non-null soft open set $(F,E) \in \tau$ can be written as a union of some elements of $\mathfrak{B}$.*

Definition 2.21 *[7] Let (X,τ,E) be S.T.S. and $x\tilde{\in}X$. For every soft neighborhood (F,A) of x, if there exists a soft set $(G,B)\tilde{\in}\mathfrak{S}_{(x)}$ such that $(G,B)\tilde{\subset}(F,A)$, then the family $\mathfrak{S}_{(x)}$ is called a soft neighborhoods system of x.*

Definition 2.22 *[13] Let (X,τ,E) be S.T.S. and $\varnothing \neq Y \subset X$. Then a family*

$$\tau_Y = \left\{ ({}^{Y}F,E) = (Y,E)\tilde{\cap}(F,E) : (F,E) \in \tau \right\}$$

is called soft relative topology on Y, and the triplet (Y,τ_Y,E) is called soft subspace of (X,τ,E).

Definition 2.23 *[15] Let (X,τ,E) and (Y,σ,E') be two S.T.S. over X and Y, respectively. Then, for any soft point $x_e\tilde{\in}\widetilde{X}$, the soft map $f_{up}: S(X,E) \to S(Y,E')$ is called soft up-continuous at x_e if for every $(G,E') \in \mathfrak{S}^{\sigma}_{(f_{up}(x_e))}$, there is $(F,E) \in \mathfrak{S}^{\tau}_{(x_e)}$ such that $f_{up}(F,E)\tilde{\subset}(G,E')$.*

If f_{up} is soft up-continuous at each soft point of X, then f_{up} is called soft up-continuous on X.

Remark 2.24 *In terms of notation simplicity, we will be contented with just notation f instead of a soft up-continuous map f_{up}.*

According to this remark, let us restatement the definition of a soft continuous map as follows in order to get rid of notation confusion later in the article.

Definition 2.25 *Let* (X,τ,E) *and* (Y,σ,E') *be two S.T.S. and let* $f:S(X,E)\to S(Y,E')$ *be a soft map. Then,* f *is soft continuous if and only if for every soft set* (F,E) *over* X *and every soft neighborhood* (G,E') *of* $f(F,E))$ *there exists a soft neighborhood* (H,E) *of* (F,E) *such that* $f((H,E))\tilde{\subseteq}(G,E')$.

As equivalent to Definition 2.25,

Definition 2.26 *Let* (X,τ,E) *and* (Y,σ,E') *be two S.T.S., let* $f:(X,\tau,E)\to(Y,\sigma,E)$ *be a soft map and* $x_e\tilde{\in}X$. *Then* f *is soft continuous at* x_e *iff for every soft neighborhood* (G,E) *containing* $f(x_e)$ *there is a soft neighborhood* (F,E) *of* x_e *in* X *such that* $f((F,E))\tilde{\subseteq}(G,E)$.

3. Types of Connectedness on Soft Topological Spaces

We will here give the types of connectedness for soft topological spaces. These types are soft connectedness, soft locally connectedness, soft path connectedness. Also, in the last section, we present the concept of soft locally path connectedness and obtain some characterizations about it.

3.1. Soft Connectedness

Definition 3.1 *[6] Let* (X,σ,E) *be a S.T.S. and* $(F,E),(G,E)\in S(X,E)$. *If* $(F,E)\tilde{\cap}(G,E)=\Phi$, *then the soft sets* (F,E) *and* (G,E) *are called soft disjoint sets.*

Definition 3.2 *[12] Let* (X,σ,E) *be a S.T.S. and let* $(F,E),(G,E)\in S(X,E)$ *be non-null soft sets. If* $(F,E)\tilde{\cap}(G,E)=\Phi$ *and* $(F,E)\tilde{\cup}(G,E)=\tilde{X}$, *then the pair* $\{(F,E),(G,E)\}$ *is called a soft separation of* $\tilde{X}$.

After these definitions, we can define a soft connected space.

Definition 3.3 *[12] An S.T.S.* (X,σ,E) *is called soft connected, if there is not a separation which consist of soft open sets of* (X,σ,E). *Otherwise,* (X,σ,E) *is said to be soft disconnected.*

In other words, if an S.T.S. can be written as union of two non-null separated soft sets, it is called a soft disconnected space, otherwise is called soft connected space.

Example 3.4 *[12] While any soft indiscrete space is soft connected, a soft discrete space is a soft disconnected space.*

Theorem 3.5 *[12] An S.T.S.* (X,σ,E) *is soft connected iff the unique soft sets that both soft open and soft closed are* $\tilde{X}$ *and* Φ.

Definition 3.6 *[6] Let (F,E) be a soft set in S.T.S. (X,σ,E). If soft subspace $((F,E),\sigma_{(F,E)},E)$ is soft connected, it is called soft connected set in (X,σ,E).*

Definition 3.7 *[7] Let (X,σ,E) be an S.T.S. and let $(F,A)\in S(X,E)$ be a soft connected set. If there is no larger a soft connected soft subspace that includes the soft subspace $((F,A),\sigma_{(F,A)},A)$, then (F,A) is called a soft component of (X,σ,E). In other words, the largest soft connected soft subspace of (X,σ,E) is called soft component of (X,σ,E).*

Definition 3.8 *[7] Let (X,σ,E) be an S.T.S. and let $x_e\widetilde{\in}X$ be any soft point. The soft component which is contain the soft point x_e is called soft component of x_e and it is denoted by C_{x_e}. In other words, the union of all soft connected subsets containing x_e is called soft component of the soft point x_e. Clearly, C_{x_e} is the largest soft connected subset containing x_e.*

3.2. Soft Locally Connectedness

Definition 3.9 *[2] An S.T.S. (X,σ,E) is called soft locally connected at soft point x_e if for every soft open set (F,E) containing x, there exists a soft connected soft open (G,E) containing x contained in (F,E). If (X,σ,E) is soft locally connected at each of its soft points, it is said simply to be soft locally connected.*

Example 3.10 *[2] Soft indiscrete and soft discrete topological spaces are soft locally connected spaces.*

Remark 3.11 *A soft connected space is soft locally connected, but the opposite is usually not true.*

Theorem 3.12 *[2] Let (X,σ,E) be an S.T.S. and let $\varnothing\neq Y\subset X$. Then (X,σ,E) is soft locally connected iff each component of (Y,σ_Y,E) is element of σ.*

Proof Let (X,σ,E) be soft locally connected, let $x_e\widetilde{\in}Y$ be any soft point and let C be soft component of x_e in (Y,σ_Y,E). Since (X,σ,E) is soft locally connected, there is a soft connected soft open set (G,E) containing x_e in σ such that $(G,E)\widetilde{\subset}C$, and also it is clear that $C=\widetilde{\bigcup}_{x_e\in C}(G,E)$. Thus, C is soft open in (X,σ,E).

Conversely, let C be soft component of x_e in (Y,σ_Y,E) and C be soft open in (X,σ,E). Then, by definition of a soft locally connected space it is clear that (X,σ,E) is soft locally connected.

□

Corollary 3.13 *[2] The soft components of a soft locally connected space are both soft open and soft closed.*

3.3. Soft Path Connectedness

In this subsection, unless otherwise stated we will take $E = IN$ as the set of parameters and we will denote the set of rational numbers on the unit interval $I = [0,1]$ by $\{0, \lambda_1, \lambda_2, ..., 1\}$. For $e \in IN$, $\lambda_e \in Q \cap I$ and $\delta > 0$, let us consider the soft set $G_\delta : E \to P(I)$ defined by $G_\delta(e) = (\lambda_e - \delta, \lambda_e + \delta)$. Then the collection $\mathcal{B} = \{(G_\delta, E) \mid \delta > 0\}$ forms a soft base of soft topology on I. The soft topology obtained in this way and denoted by σ_I is called soft topology generated by $\mathcal{B}$.

Definition 3.14 *[5] An S.T.S. (I, σ_I, E) is called unit soft interval.*

Definition 3.15 *[5] Let $(X, \sigma, E^{'})$ be any S.T.S. and let (I, σ_I, E) be a unit soft interval. A soft continuous map $f = (f,p) : (I, \sigma_I, E) \to (X, \sigma, E^{'})$ is called a soft path, where $f : I \longrightarrow X$ and $p : E \to E'$. The soft sets $\left\{f(0)_{p(n)}\right\}_{n \in E}$ and $\left\{f(1)_{p(n)}\right\}_{n \in E}$ are called initial and final of the soft path f, respectively.*

It is clear that for each $n \in E$ the map $f : (I, \sigma_I) \longrightarrow \left(X, \sigma_{p(n)}\right)$ is a path from $f(0)_{p(n)}$ to $f(1)_{p(n)}$ in the topological space $(X, \sigma_{p(n)})$.

Definition 3.16 *[5] Let $(X, \sigma, E^{'})$ be an S.T.S. For all soft points $x_{e_1'}$, $y_{e_2'} \widetilde{\in} X$ if there exists a soft path $f = (f,p) : (I, \sigma_I, E) \to (X, \sigma, E^{'})$ such that $p(n_1) = e_1'$, $p(n_2) = e_2'$, $f(0) = x$ and $f(1) = y$, then $(X, \sigma, E^{'})$ is called soft path connected space.*

Theorem 3.17 *[5] The image of a soft path connected pace under a soft continuous map is also soft path connected.*

Theorem 3.18 *[5] The soft unit interval (I, σ_I, E) is both soft path connected and soft connected.*

Proposition 3.19 *Let $(X, \sigma, E^{'})$ be an S.T.S. and let $x = x_{e_1'}$, $y = y_{e_2'}$ be soft points in $(X, \sigma, E^{'})$. If $(f,p) : (I, \sigma_I, E) \to (X, \sigma, E^{'})$ is a soft path from x to y, then a soft map $(g,r) : (I, \sigma_I, E) \to (X, \sigma, E^{'})$ defined by $(g,r)(t) = (f,p)(1-t), t \in I$ is a soft path from y to x. That is, the soft map (g,r) determines a soft path in the opposite direction of the soft map (f,p).*

Proof The proof is straightforward. □

Proposition 3.20 *Let $(X, \sigma, E^{'})$ be an S.T.S. and let $x = x_{e_1'}$, $y = y_{e_2'}$ and $z = z_{e_3'}$ be soft points in $(X, \sigma, E^{'})$. If $(f,p) : (I, \sigma_I, E) \to (X, \sigma, E^{'})$ is a soft path from x to y and $(g,r) : (I, \sigma_I, E) \to$*

$(X,\sigma,E^{'})$ is a soft path from y to z, then the soft map $(h,\omega):(I,\sigma_I,E)\to(X,\sigma,E')$ defined as

$$(h,s)=\begin{cases}(f,p)(2t), & 0\le t\le \frac{1}{2}\\ (g,r)(2t-1), & \frac{1}{2}\le t\le 1\end{cases}$$

is a soft path from x to z. It is called product of the soft paths (f,p) and (g,r).

Proof The proof is straightforward. □

Theorem 3.21 *Let a relation $\mathcal{R}$ on an S.T.S. $(X,\sigma,E^{'})$ be given as follows: for all soft points x,y*

$$\text{“ } x\mathcal{R}y \Longleftrightarrow \textit{there is a soft path between } x \textit{ and } y \text{ ”}$$

Then the relation $\mathcal{R}$ is an equivalence relation.

Proof It follows from Theorem 3.18, Propositions 3.19 and 3.20. □

Definition 3.22 *The equivalence classes according to the equivalence relation defined above on an S.T.S. $(X,\sigma,E^{'})$ are called soft path components of $(X,\sigma,E^{'})$.*

3.4. Soft Locally Path Connectedness

Definition 3.23 *For all soft point $x=x_{e'}$ in an S.T.S. (X,σ,E'), if there is a soft neighborhoods system consisting of soft path connected soft sets, then (X,σ,E') is called soft locally path connected space.*

That is, (X,σ,E') is soft locally path connected if and only if there exists a soft path connected soft neighborhood (G,B) such that

$$\forall x\tilde{\in}\widetilde{X} \text{ and } \forall(F,A)\tilde{\in}\mathfrak{S}_{(x)},\ \exists(G,B)\ \ni\ x\tilde{\in}(G,B)\tilde{\subseteq}(F,A).$$

Theorem 3.24 *An S.T.S. $\left(X,\sigma,E^{'}\right)$ is soft locally path connected if and only if the soft path component of every soft open set is soft open in $\left(X,\sigma,E^{'}\right)$.*

Proof Let S.T.S. $\left(X,\sigma,E^{'}\right)$ be soft locally path connected, let $(F,A)\tilde{\subseteq}X$ be a soft open set and C a soft path component containing (F,A). Let us prove that C is soft open set. Let us take any soft point $x\tilde{\in}C$. Since $\left(X,\sigma,E^{'}\right)$ is the soft locally path connected space, there exists a soft path connected neighborhood (G,B) of x such that $x\tilde{\in}(G,B)\tilde{\subseteq}(F,A)$. But, since the largest soft path connected set containing x is C, we have $x\tilde{\in}(G,B)\tilde{\subseteq}C$. Thus, C is soft open.

Conversely, we suppose that the soft path components of every soft open set are soft open. Let us show that $\left(X,\sigma,E'\right)$ is soft locally path connected space. Let us take any soft point $x\tilde{\in}\widetilde{X}$ and let (F,A) be a soft open neighborhood of x. Let C be the soft path component of (F,A) containing x. Thus, C is a soft open neighborhood of x contained in (F,A). Consequently, $\left(X,\sigma,E'\right)$ is soft locally path connected space. □

Theorem 3.25 *The soft components and soft path components of a soft open set in a soft locally path connected topological space are same.*

Proof Let (F,A) be a soft open set in a soft locally path connected space $\left(X,\sigma,E'\right)$. We will show that soft components and soft path components of (F,A) are the same. Let C be a soft component of (F,A), and let D be a soft path component of C. By Theorem 3.12, $C\tilde{\subseteq}(F,A)$ is a soft open set. Since a soft path component of a soft open set in a soft locally path connected space is a soft open, $D\tilde{\subseteq}C$ is soft open. At the same time, $D\tilde{\subseteq}C$ is soft closed. But, since C is soft connected, we have $D\tilde{=}C$. Therefore, C is a soft path connected component. □

Let us state two main results of this theorem.

Corollary 3.26 *The soft components and soft path components of a soft locally path connected space are same.*

Corollary 3.27 *A soft topological space which is both soft connected and soft locally path connected is a soft path connected space.*

4. Conclusion

In this paper, we have presented the concept of soft locally path connectedness. We have obtained two results about the soft locally path connected spaces. This work is a beginning of next our papers about the coverings and liftings of soft topological spaces.

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