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A NOTE ON THE STABILITY OF SOLUTION FOR ELLIPTIC-SCHRÖDINGER TYPE NONLOCAL BOUNDARY VALUE PROBLEM

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Abstract. In the present article, a problem for an elliptic-Schrödinger equation with nonlocal boundary value condition is considered. The stability estimates are established for the solution of elliptic-Schrödinger problem. A theorem for stability of the solution of this problem and a conclusion section is presented.

1. introduction

In the present paper, the nonlocal boundary-value problem (NBVP)

$$\begin{cases} -\dfrac{d^2u(t)}{dt^2} + Au(t) = g(t)\,(0 \le t \le 1), \\ i\dfrac{du(t)}{dt} - Au(t) = f(t)\,(-1 \le t \le 0), \\ u(1) = u(-1) + \varphi \end{cases} \tag{1.1}$$

for differential equations of elliptic-Schrödinger type in a Hilbert space H with self-adjoint positive definite operator A is considered.

In the literature it is known that various NBVPs for the elliptic-Schrödinger equations can be reduced to the problem (1.1).

Whenever the following conditions are satisfied an abstract function $u(t)$ is called a solution of the problem (1.1):

i. $u(t)$ is twice continuously differentiable on the interval $(0, 1]$ and continuously differentiable on the segment $[-1, 1]$. The derivative at the endpoints of the segment are understood as the appropriate unilateral derivatives.

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ii. The element $u(t)$ belongs to $D(A)$ for all $t \in [-1,1]$, and the function $Au(t)$ is continuous on the segment $[-1,1]$.
iii. $u(t)$ satisfies the equations and nonlocal boundary condition (1.1).

There are also different type of works on elliptic and Schrödinger equations (see, for example, [13, 14, 15, 16] and references given therein).

Many scientists have been studied the methods of solutions of NBVPs for partial differential equations (PDEs) and PDEs of mixed type extensively (see, [1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12] and references given therein).

2. The main theorem on stability

In the present section the stability estimates for elliptic-Schrödinger equations are obtained.

Theorem 2.1. *Suppose that $\varphi \in D(A)$ and $f(0), g(0) \in H$. Let $f(t)$ be continuously differentiable and $g(t)$ be twice continuously differentiable functions on $[-1,0]$ and $[0,1]$, respectively. Then, there is a unique solution of the problem (1.1) and the following stability inequalities hold:*

$$\max_{-1\le t\le 1}\|u(t)\|_H \le M\left[\|\varphi\|_H + \max_{-1\le t\le 0}\|f(t)\|_H + \max_{0\le t\le 1}\left\|A^{-1/2}g(t)\right\|_H\right] \tag{2.1}$$

$$\max_{-1\le t\le 1}\left\|\frac{du(t)}{dt}\right\|_H + \max_{-1\le t\le 1}\left\|A^{1/2}u(t)\right\|_H \tag{2.2}$$
$$\le M\left[\left\|A^{1/2}\varphi\right\|_H + \max_{-1\le t\le 0}\left\|A^{1/2}f(t)\right\|_H + \max_{0\le t\le 1}\|g(t)\|_H\right]$$

$$\max_{-1\le t\le 0}\left\|\frac{du(t)}{dt}\right\|_H + \max_{0\le t\le 1}\left\|\frac{d^2u(t)}{dt^2}\right\|_H + \max_{-1\le t\le 1}\|Au(t)\|_H \tag{2.3}$$
$$\le M\left[\|A\varphi\|_H + \|g(0)\|_H + \|f(0)\|_H + \max_{0\le t\le 1}\|g'(t)\|_H + \max_{-1\le t\le 0}\|f'(t)\|\right],$$

where M is independent of not only $f(t)$, $t\in[-1,0]$ but also $g(t)$, $t\in[0,1]$ and φ.

Proof. First of all, we will obtain a formula for the solution of problem (1.1). It is known that there are unique solutions of the initial value problems

$$-\frac{d^2u(t)}{dt^2} + Au(t) = g(t), (0\le t\le 1), u(0)=u_0, u(1)=u_1, \tag{2.4}$$

and

$$i\frac{du(t)}{dt} - Au(t) = f(t), (-1\le t\le 0), u(0)=u_0 \tag{2.5}$$

that is,

$$u(t) = e^{-tA}u_0 - i\int_0^t e^{-i(t-s)A}f(s)\,ds, -1\le t\le 0 \tag{2.6}$$

and

$$u(t) = \left(I - e^{-2A^{1/2}}\right)^{-1}\left[\left(e^{-tA^{1/2}} - e^{-(-t+2)A^{1/2}}\right)u_0 \tag{2.7}$$
$$+\left(e^{-(1-t)A^{1/2}} - e^{-(t+1)A^{1/2}}\right)u_1\right]$$
$$+\left(I - e^{-2A^{1/2}}\right)^{-1}\left(e^{-(1-t)A^{1/2}} - e^{-(t+1)A^{1/2}}\right)$$

$$\times \int\limits_0^1 A^{-1/2} 2^{-1} \left(e^{-(1-s)A^{1/2}} - e^{-(s+1)A^{1/2}} \right) g(s) ds$$

$$- \int\limits_0^1 A^{-1/2} 2^{-1} \left(e^{-(t+s)A^{1/2}} - e^{-|t-s|A^{1/2}} \right) g(s) ds, 0 \le t \le 1,$$

respectively. Using formulas (2.6), (2.7) and nonlocal boundary condition

$$u(1) = u(-1) + \varphi,$$

we get

$$u(t) = \left(I - e^{-2A^{1/2}} \right)^{-1} \left[\left(e^{-tA^{1/2}} - e^{-(-t+2)A^{1/2}} \right) u_0 \right. \tag{2.8}$$

$$\left. + \left(e^{-(1-t)A^{1/2}} - e^{-(t+1)A^{1/2}} \right) \left(e^{iA} u_0 - i \int_0^{-1} e^{i(1+s)A} f(s) \, ds + \varphi \right) \right]$$

$$+ \left(I - e^{-2A^{1/2}} \right)^{-1} \left(e^{-(1-t)A^{1/2}} - e^{-(t+1)A^{1/2}} \right)$$

$$\times \frac{1}{2} \int_0^1 \left(e^{-(1-s)A^{1/2}} - e^{-(s+1)A^{1/2}} \right) A^{-1/2} g(s) \, ds$$

$$- \frac{1}{2} \int_0^1 \left(e^{-(t+s)A^{1/2}} - e^{-|t-s|A^{1/2}} \right) A^{-1/2} g(s) \, ds, 0 \le t \le 1.$$

Now, using the following

$$u'(0^+) = \frac{1}{i} \left[Au(0) + f(0) \right],$$

we obtain the operator equation

$$\left\{ \left(I - e^{-2A^{1/2}} \right) + i \left(I + e^{-2A^{1/2}} \right) A^{-1/2} - 2iA^{-1/2} e^{-\left(A^{1/2} - iA \right)} \right\} u_0$$

$$= i \left\{ \left[-2A^{-1/2} e^{-A^{1/2}} \left(i \int_0^{-1} e^{iA(-1+s)} f(s) \, ds + \varphi \right) \right] \right.$$

$$+ A^{-1} e^{-A^{1/2}} \int_0^1 \left(e^{-(1-s)A^{1/2}} - e^{-(s+1)A^{1/2}} \right) g(s) \, ds$$

$$\left. + i \left(I - e^{-2A^{1/2}} \right) A^{-1} f(0) + A^{-1} \left(I - e^{-2A^{1/2}} \right) \int_0^1 e^{-sA^{1/2}} g(s) \, ds \right\}.$$

Here, the operator

$$\left(I - e^{-2A^{1/2}} \right) + iA^{-1/2} \left(I + e^{-2A^{1/2}} \right) - 2iA^{-1/2} e^{-\left(A^{1/2} - iA \right)}$$

has an inverse,

$$T = \left[\left(I - e^{-2A^{1/2}} \right) + iA^{-1/2} \left(I + e^{-2A^{1/2}} \right) - 2iA^{-1/2} e^{-\left(A^{1/2} - iA \right)} \right]^{-1}$$

and

$$\left\| A^{-1/2} T \right\|_{H \to H} \le M \tag{2.9}$$

holds.

It is needed to obtain a formula for $u(0)$ for the solution of problem (1.1). To do this we must show that T is a bounded operator. Let $A^{1/2} = B$. Since

$$A^{-1/2} T = \left[\left(A^{1/2} - A^{1/2} e^{-2A^{1/2}} \right) + i \left(I + e^{-2A^{1/2}} \right) - 2ie^{-\left(A^{1/2} - iA \right)} \right]^{-1}$$

$$= \left[\left(B - Be^{-2B}\right) + i\left(I + e^{-2B}\right) - 2ie^{-\left(B - iB^2\right)} \right]^{-1}$$

we have that

$$\left\| A^{-1/2} T \right\|_{H \to H} \leq \sup_{\delta \leq \mu < \infty} \frac{1}{\mu - \mu e^{-2\mu} + i\left(1 + e^{-2\mu}\right) - 2ie^{-\mu} e^{-i\mu^2}}.$$

Using Euler formula, we get

$$\beta(\mu) = \mu - \mu e^{-2\mu} + 2e^{-\mu}\sin\mu^2 + i\left(I + e^{-2\mu} - 2e^{-\mu}\cos\mu^2\right).$$

Taking absolute value of $\beta(\mu)$, we obtain

$$|\beta(\mu)| = \sqrt{\begin{array}{l} \mu^2 + \mu^2 e^{-4\mu} + 4e^{-2\mu}\sin^2\mu^2 - 2\mu^2 e^{-2\mu} + 4\mu e^{-\mu}\sin\mu^2 - 4\mu e^{-3\mu}\sin\mu^2 \\ +1 + e^{-4\mu} + 4e^{-2\mu}\cos^2\mu^2 + 2e^{-2\mu} - 4e^{-\mu}\cos\mu^2 - 4e^{-3\mu}\cos\mu^2 \end{array}}$$

or

$$|\beta(\mu)| = \sqrt{\begin{array}{l} 1 + \mu^2 + \left(1 + \mu^2\right)e^{-\mu} + 4e^{-2\mu} + 2\left(1 - \mu^2\right)e^{-2\mu} \\ +4\sqrt{1+\mu^2}e^{-\mu}\left[\mu\left(\sqrt{1+\mu^2}\right)^{-1}\sin\mu^2 - \left(\sqrt{1+\mu^2}\right)^{-1}\cos\mu^2\right] \\ -4\sqrt{1+\mu^2}e^{-3\mu}\left[\mu\left(\sqrt{1+\mu^2}\right)^{-1}\sin\mu^2 + \left(\sqrt{1+\mu^2}\right)^{-1}\cos\mu^2\right] \end{array}}.$$

Choosing $\dfrac{\mu}{\sqrt{1+\mu^2}} = \sin\alpha$ and $\dfrac{1}{\sqrt{1+\mu^2}} = \cos\alpha$, we can write

$$|\beta(\mu)| = \sqrt{\begin{array}{l} \left(1 + \mu^2\right) + \left(1 + \mu^2\right)e^{-4\mu} + 4e^{-2\mu} + 2\left(1 - \mu^2\right)e^{-2\mu} \\ -4\sqrt{1+\mu^2}e^{-\mu}\cos\left(\mu^2 + \alpha\right) - 4\sqrt{1+\mu^2}e^{-3\mu}\cos\left(\mu^2 - \alpha\right) \end{array}}$$

$$\geq \sqrt{1 + \mu^2 + (1 + \mu^2)e^{-4\mu} + 2(1 - \mu^2)e^{-2\mu}4\sqrt{1+\mu^2}e^{-\mu} - 4\sqrt{1+\mu^2}e^{-3\mu}}.$$

Now, let

$$\psi(\mu) = \mu^2 + \left(1 + \mu^2\right)e^{-4\mu} + 2\left(1 - \mu^2\right)e^{-2\mu}4\sqrt{1+\mu^2}e^{-\mu} - 4\sqrt{1+\mu^2}e^{-3\mu}.$$

Hence, it is enough to prove the inequality $\psi(\mu) > 0$ for $\mu \geq \delta$. It is seen that for sufficiently large δ the inequality holds. Therefore,

$$\left\| A^{-1/2} T \right\|_{H \to H} \leq 1.$$

This means that $A^{-1/2}T$ is bounded. So,

$$u(0) = A^{-1/2}T\left(I - e^{-2A^{1/2}}\right)\left\{ i\int_0^1 A^{-1/2}e^{-sA^{1/2}}g(s)\,ds - A^{-1/2}f(0)\right\} \quad (2.10)$$

$$+Te^{-A^{1/2}}A^{-1/2}\left\{2\int_0^{-1} e^{iA(-1+s)}f(s)\,ds - 2i\varphi + i\int_0^1 \right.$$

$$\left. \times \left(e^{-(1-s)A^{1/2}} - e^{-(s+1)A^{1/2}}\right)A^{-1/2}g(s)\,ds\right\}.$$

Finally, we have formulas (2.6), (2.8) and (2.10) for the solution of the nonlocal boundary value problem (1.1).

Now, proofs of estimates (2.1), (2.2) and (2.3) will be given. Firstly, we consider (2.1). Using formula (2.10), we get

$$\|u(0)\|_H \leq \left\| A^{-1/2}T \right\|_{H \to H} \left\| I - e^{-2A^{1/2}} \right\|_{H \to H} \quad (2.11)$$

$$\times\left\{|i|\int_0^1\left\|e^{-sA^{1/2}}\right\|_{H\to H}\left\|A^{-1/2}g(s)\right\|_H ds+\left\|A^{-1/2}\right\|_{H\to H}\|f(0)\|_H\right\}$$

$$+\left\|e^{-A^{1/2}}\right\|_{H\to H}\left\|A^{-1/2}T\right\|_{H\to H}\left\{2\int_0^{-1}\left\|e^{iA(-1+s)}\right\|_{H\to H}\|f(s)\|_H ds+2|i|\,\|\varphi\|_H\right.$$

$$\left.+|i|\int_0^1\left(\left\|e^{-(1-s)A^{1/2}}\right\|_{H\to H}+\left\|e^{-(s+1)A^{1/2}}\right\|_{H\to H}\right)\left\|A^{-1/2}g(s)\right\|_H ds\right\}$$

or

$$\|u(0)\|_H\le M\left[\|\varphi\|_H+\max_{-1\le t\le 0}\|f(t)\|_H+\max_{0\le t\le 1}\left\|A^{-1/2}g(t)\right\|_H\right]. \tag{2.12}$$

Then, using formulas (2.6) and (2.8), we obtain for $-1\le t\le 0$

$$\|u(t)\|_H\le M\left[\|\varphi\|_H+\max_{-1\le t\le 0}\|f(t)\|_H+\max_{0\le t\le 1}\left\|A^{-1/2}g(t)\right\|_H\right] \tag{2.13}$$

and for $0\le t\le 1$

$$\|u(t)\|_H\le M\left[\|u(0)\|_H+\max_{-1\le t\le 0}\|f(t)\|_H+\|\varphi\|_H+\max_{0\le t\le 1}\left\|A^{-1/2}g(t)\right\|_H\right]. \tag{2.14}$$

Therefore, using estimates (2.12), (2.13) and (2.14), we complete proof of inequality (2.1).

Secondly, the proof of the estimate (2.2) will be obtained. Applying $A^{1/2}$ to (2.10) and taking the norm of it, we can write

$$\left\|A^{1/2}u(0)\right\|_H\le M\left[\left\|A^{1/2}\varphi\right\|_H+\max_{-1\le t\le 0}\left\|A^{1/2}f(t)\right\|_H+\max_{0\le t\le 1}\|g(t)\|_H\right]. \tag{2.15}$$

After that, applying $A^{1/2}$ to (2.6) and (2.8) and taking norm of them, we obtain

$$\left\|A^{1/2}u(t)\right\|_H\le\left\|A^{1/2}u(0)\right\|_H+\max_{-1\le t\le 0}\left\|A^{1/2}f(t)\right\|_H,-1\le t\le 0 \tag{2.16}$$

and

$$\left\|A^{1/2}u(t)\right\|_H\le M\left[\left\|A^{1/2}u(0)\right\|_H+\max_{-1\le t\le 0}\left\|A^{1/2}f(t)\right\|_H\right. \tag{2.17}$$
$$\left.+\left\|A^{1/2}\varphi\right\|_H+\max_{0\le t\le 1}\|g(t)\|_H\right],0\le t\le 1.$$

Combining estimates (2.15), (2.16) and (2.17), we obtain (2.2).

Thirdly, the proof of the estimate (2.3) will be obtained. Using formula (2.6) and integration by parts, we get for $-1\le t\le 0$

$$u(t)=e^{-tA}u_0-A^{-1}\left\{\left[f(t)-e^{-itA}f(0)\right]-\int_0^t e^{-i(t-s)}f'(s)\,ds\right\}. \tag{2.18}$$

Using formula (2.8) and integration by parts, we get for $0\le t\le 1$

$$u(t)=\left(I-e^{-2A^{1/2}}\right)^{-1}\left[\left(e^{-tA^{1/2}}-e^{-(-t+2)A^{1/2}}\right)u_0\right. \tag{2.19}$$
$$+\left(e^{-(1-t)A^{1/2}}-e^{-(t+1)A^{1/2}}\right)$$
$$\left.\times\left(e^{iA}u_0-A^{-1}\left\{\left[f(-1)-e^{iA}f(0)\right]-\int_0^{-1}e^{i(1+s)A}f'(s)\,ds+\varphi\right\}\right)\right]$$

$$+\left(I-e^{-2A^{1/2}}\right)^{-1}\left(e^{-(1-t)A^{1/2}}-e^{-(t+1)A^{1/2}}\right)$$

$$\times\frac{1}{2}A^{-1}\left\{\left(I-e^{-2A^{1/2}}\right)g\left(1\right)-\int_{0}^{1}\left(e^{-(1-s)A^{1/2}}-e^{-(s+1)A^{1/2}}\right)g'\left(s\right)ds\right\}$$

$$-\frac{1}{2}A^{-1}\left\{\left[e^{-(1+t)A^{1/2}}-e^{-|1-t|A^{1/2}}\right]g\left(1\right)\right.$$

$$\left.-\int_{0}^{1}\left(e^{-(t+s)A^{1/2}}-e^{-|t-s|A^{1/2}}\right)g'\left(s\right)ds\right\}.$$

Lastly, using (2.10) and integration by parts, we get

$$u\left(0\right)=A^{-1/2}T\left(I-e^{-2A^{1/2}}\right) \tag{2.20}$$

$$\times\left\{-iA^{-1}\left[\left(e^{-A^{1/2}}g\left(1\right)-g\left(0\right)\right)-\int_{0}^{1}e^{-sA^{1/2}}g'\left(s\right)ds\right]-A^{-1/2}f\left(0\right)\right\}$$

$$+A^{-1/2}Te^{-A^{1/2}}\left\{2A^{-1}\left[\left(e^{-2iA}f\left(-1\right)-e^{-iA}f\left(0\right)\right)-\int_{0}^{-1}e^{iA(-1+s)}f'\left(s\right)ds\right]-2i\varphi\right.$$

$$\left.+\frac{iA^{-1}}{2}\left[\left(I-e^{-2A^{1/2}}\right)g\left(1\right)-\int_{0}^{1}\left(e^{-(1-s)A^{1/2}}-e^{-(s+1)A^{1/2}}\right)g'\left(s\right)ds\right]\right\}.$$

Here, we can write

$$g\left(1\right)=g\left(0\right)+\int_{0}^{1}g'\left(s\right)ds \quad \text{and} \quad f\left(-1\right)=f\left(0\right)+\int_{-1}^{0}f'\left(s\right)ds.$$

Now, applying the operator A to formulas (2.18), (2.19), (2.20) and taking their norm, we obtain

$$\left\|Au\left(0\right)\right\|_{H}\leq M\left[\left\|A\varphi\right\|_{H}+\left\|g\left(0\right)\right\|_{H}+\left\|f\left(0\right)\right\|_{H}+\max_{0\leq t\leq 1}\left\|g'\left(t\right)\right\|_{H}\right. \tag{2.21}$$

$$\left.+\max_{-1\leq t\leq 0}\left\|f'\left(t\right)\right\|_{H}\right],$$

$$\left\|Au\left(t\right)\right\|_{H}\leq M\left[\left\|Au_{0}\right\|_{H}+\left\|f\left(0\right)\right\|_{H}+\max_{-1\leq t\leq 0}\left\|f'\left(t\right)\right\|_{H}\right],-1\leq t\leq 0, \tag{2.22}$$

$$\left\|Au\left(t\right)\right\|_{H}\leq M\left[\left\|Au_{0}\right\|_{H}+\left\|\varphi\right\|_{H}+\left\|g\left(0\right)\right\|_{H}+\left\|f\left(0\right)\right\|_{H}\right. \tag{2.23}$$

$$\left.+\max_{0\leq t\leq 1}\left\|g'\left(t\right)\right\|_{H}+\max_{-1\leq t\leq 0}\left\|f'\left(t\right)\right\|_{H}\right],0\leq t\leq 1.$$

Combining estimates (2.21), (2.22) and (2.23), we obtain inequality (2.3). This completes the proof of the main theorem.

3. Conclusion

In conclusion, the stability estimates for the solution of problem (1.1) for the elliptic-Schrödinger equation are established. Note that some results of this paper, without proof, were presented in [17, 18].

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