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Design of EWMA and CUSUM Control Charts Based On Type-2 Fuzzy Sets

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Abstract: A Shewhart control chart is a completely critical tool used for monitoring process' stability. The superiority of EWMA and CUSUM control charts over Shewhart control charts is their ability to handle small process shifts. Although fuzzy sets can manage uncertainties related to processes, the extensions of fuzzy sets can be used to improve the ability of these control charts. The main advantage of type-2 fuzzy sets is that they can effectively model uncertainty because the membership functions are not crisp. For these reasons, type-2 fuzzy sets are used to design of CUSUM and EWMA control charts. Therefore, in this study, the control limits and center lines in EWMA and CUSUM control charts have been re-formulated based on type-2 fuzzy sets. Furthermore, an illustrative example is provided to show the applicability of the proposed techniques.

Keywords: CUSUM, EWMA, Type-2 fuzzy sets, Quality control.

1 Introduction

With the increase in the variety of products and services in the market, quality has become one of the leading factors. Statistical Process Control (SPC) is a controlling methodology to analyze, monitor and improve the performance of a process. A Shewhart Control Chart (SCC), one of the most preferred techniques in statistical process control, is utilized to determine whether there are unusual sources of variability in a process or not. The systematic use of this technique is quite efficient to reduce such variations, as it will enable corrective measures to be taken to eliminate these unusual sources of variability [1–3]. The effectiveness of these control charts depends on the accuracy of the available data. However, the use of SCC has one drawback, which is that it disregards the information provided by the sequence and utilizes only the information given in the final sample observation. It consequently causes SCCs to be relatively insensitive to small process shifts [1]-[4]. For this reason, cumulative sum (CUSUM) [5], and exponentially weighted moving average (EWMA) control charts [6] have been proposed to overcome this drawback and to effectively handle such process shifts. In most of the real-world problems, there are uncertainties in the processes related to measurement systems or operators. The classical control charts are not enough to overcome these problems. Therefore, in this study, one of the fuzzy set theory (FST) extensions named type-2 fuzzy sets has been used to design of control charts. The main advantage of type-2 fuzzy sets, proposed by Zadeh [7], is that they can effectively model uncertainty even when the membership functions of them are not crisp [8]. Therefore, in this study, type-2 fuzzy sets are integrated into EWMA and CUSUM charts to design these control charts to be able to handle the uncertainties in the processes and measurements.

The rest of the paper is constructed as follows: studies related to fuzzy control charts are briefly summarized in Section 2. The fundamentals of type-2 fuzzy sets, EWMA and CUSUM control charts and the proposed charts based on type-2 fuzzy sets are given in Section 3. Finally, an illustrative example and concluding remarks are presented in Section 4 and 5, respectively.

2 Literature Review

The FST, proposed by Zadeh [9], have been commonly employed in many fields to deal with vague and imprecise information. FST, handling uncertainty by defining membership degrees, have been integrated to control charts in the literature. Gulbay and Kahraman [10] have developed alpha cut fuzzy control charts for both triangular (TFNs) and trapezoidal fuzzy numbers (TrFNs). According to their method, it is not necessary to use defuzzification to interpret the control chart. They exemplified their method by using the quality control processes of a large-scale toy production. Moraditadi and Avakhdarestani [11] developed a fuzzy I-MR control chart using triangular and trapezoidal fuzzy mode approaches. By illustrating their work for the automobile dashboard quality control process, they stated that TrFNs were more sensitive than TFNs in the fuzzy mode approach. Kaya and Kahraman [12] developed a fuzzy control chart using the fuzzy mode and fuzzy rule method with TFNs and TrFNs. In addition, specification limits and process capability indices were analyzed using TFNs and TrFNs. They applied the proposed chart and process capability indices to the piston production process. They had demonstrated that fuzzy charts and capability indices give more sensitive results. Erginel and Senturk [13] stated that EWMA and CUSUM control charts could detect small changes in the process well and for this purpose they developed fuzzy EWMA and CUSUM control charts by using TFNs in their work. They applied the proposed charts for plastic button production process. Erginel et al. [14] developed the interval valued type-2 fuzzy p-control chart in their studies, and they implemented the proposed chart for a company in the ceramic industry. Dos Santos Mendes et al. [15] integrated fuzzy logic to the X-R control chart using the α-level fuzzy midrange transformation. In this study, TFNs were used to represent uncertainties and fuzzy control chart was applied for measurement of the volume of the milk bags. It has been observed that the fuzzy control chart was more efficient than traditional control charts. Hesamian et al. [16] proposed a normal fuzzy random variable based EWMA control chart with fuzzy mean and non-fuzzy variance. Ercan-Teksen and Anagun [17] have developed a type-2 fuzzy c control chart using TrFNs for situations in real life where some data cannot be expressed with type-1 fuzzy numbers. They used ranking methods to create interval valued type-2 fuzzy c-control charts. Moreover, they created an intuitionistic fuzzy c-control chart using defuzzification methods for intuitionistic TrFNs [18]. Although EWMA and CUSUM control charts have been analyzed based on the FST before, it is the first time in this study that these control charts are integrated with type-2 fuzzy sets and the formulas of center line, upper control and lower control limits are restructured with type-2 trapezoidal fuzzy numbers.

3 Methodology

In this section, after fundamentals of type-2 fuzzy sets, EWMA and CUSUM control charts are given, the proposed charts, type-2 fuzzy EWMA and type-2 fuzzy CUSUM, are presented.

3.1 Type-2 Fuzzy Sets

Type-2 fuzzy sets were developed by Lotfi Zadeh [7] because the type-1 fuzzy sets are inadequate in some areas and modeling. In Type-1 fuzzy sets, membership functions are assigned by decision-makers are crisp. However, in type-2 fuzzy sets, decision makers assign a membership degree to the membership function as well. In this way, modeling can be carried out much more effectively [8]. Type-2 fuzzy sets are especially quite useful if decision makers cannot fully determine the membership functions. Thus, using type-2 fuzzy sets in decision-making problems provides more flexibility [19]. Type-2 fuzzy sets can be expressed in different forms such as interval-valued, triangular, and trapezoidal fuzzy numbers. Mathematical representation of type-2 trapezoidal fuzzy sets is as follows:

$$\tilde{\tilde{A}}_i = \{a^U_{i_1}, a^U_{i_2}, a^U_{i_3}, a^U_{i_4}; H_1(\tilde{A}^U), H_2(\tilde{A}^U)), (a^L_{i_1}, a^L_{i_2}, a^L_{i_3}, a^L_{i_4}; H_1(\tilde{A}^L), H_2(\tilde{A}^L))\}. \tag{1}$$

Type-2 trapezoidal fuzzy numbers consists of variables belonging to the lower and upper components of the relevant number [20]. As $\tilde{\tilde{A}}_i$ is a trapezoidal fuzzy number, $\tilde{A}^U_i$ and $\tilde{A}^L_i$ represent the upper and lower component of the fuzzy number, respectively. The upper and lower membership functions of this fuzzy number are represented with $\mu_{\tilde{A}^U_i}$ and $\mu_{\tilde{A}^L_i}$, respectively. The heights of the upper component in a type-2 trapezoidal fuzzy number are $H_1(\tilde{A}^U_i)$ and $H_2(\tilde{A}^U_i)$ whereas the heights of the lower component in a type-2 trapezoidal fuzzy number are $H_1(\tilde{A}^L_i)$ and $H_2(\tilde{A}^L_i)$. Graphical representation of type-2 trapezoidal fuzzy numbers is shown in Figure 1 [21].

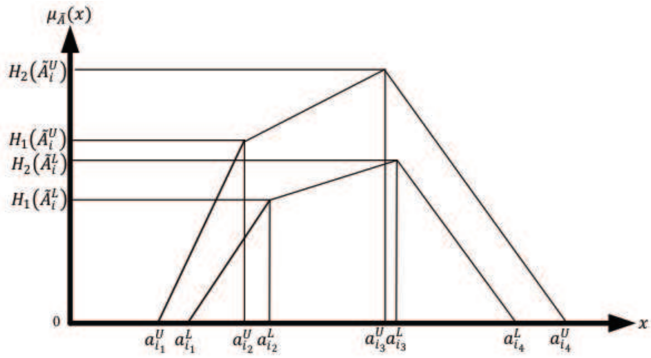

Fig. 1: Type-2 Trapezoidal Fuzzy Number Graphical Representation

Arithmetical operations for type-2 trapezoidal fuzzy numbers defined in Eqs. 2-5 are shown in the following equations [22].

- Addition:

$$\begin{aligned}\tilde{\tilde{A}}_1 \oplus \tilde{\tilde{A}}_2 = ((&a^U_{1_1} + a^U_{2_1}, a^U_{1_2} + a^U_{2_2}, a^U_{1_3} + a^U_{2_3}, a^U_{1_4} + a^U_{2_4}; min(H_1(\tilde{A}^U_1), H_1(\tilde{A}^U_2)), min(H_2(\tilde{A}^U_1), H_2(\tilde{A}^U_2))),\\ &(a^L_{1_1} + a^L_{2_1}, a^L_{1_2} + a^L_{2_2}, a^L_{1_3} + a^L_{2_3}, a^L_{1_4} + a^L_{2_4}; min(H_1(\tilde{A}^L_1), H_1(\tilde{A}^L_2)), min(H_2(\tilde{A}^L_1), H_2(\tilde{A}^L_2)))).\end{aligned} \tag{2}$$

- Subtraction:

$$\begin{aligned}\tilde{\tilde{A}}_1 \ominus \tilde{\tilde{A}}_2 = ((&a^U_{1_1} - a^U_{2_4}, a^U_{1_2} - a^U_{2_3}, a^U_{1_3} - a^U_{2_2}, a^U_{1_4} - a^U_{2_1}; min(H_1(\tilde{A}^U_1), H_1(\tilde{A}^U_2)), min(H_2(\tilde{A}^U_1), H_2(\tilde{A}^U_2))),\\ &(a^L_{1_1} - a^L_{2_4}, a^L_{1_2} - a^L_{2_3}, a^L_{1_3} - a^L_{2_2}, a^L_{1_4} - a^L_{2_1}; min(H_1(\tilde{A}^L_1), H_1(\tilde{A}^L_2)), min(H_2(\tilde{A}^L_1), H_2(\tilde{A}^L_2)))).\end{aligned} \tag{3}$$

- Multiplication:

$$\begin{aligned}\tilde{\tilde{A}}_1 \otimes \tilde{\tilde{A}}_2 = ((&a^U_{1_1} \times a^U_{2_1}, a^U_{1_2} \times a^U_{2_2}, a^U_{1_3} \times a^U_{2_3}, a^U_{1_4} \times a^U_{2_4}; min(H_1(\tilde{A}^U_1), H_1(\tilde{A}^U_2)), min(H_2(\tilde{A}^U_1), H_2(\tilde{A}^U_2))),\\ &(a^L_{1_1} \times a^L_{2_1}, a^L_{1_2} \times a^L_{2_2}, a^L_{1_3} \times a^L_{2_3}, a^L_{1_4} \times a^L_{2_4}; min(H_1(\tilde{A}^L_1), H_1(\tilde{A}^L_2)), min(H_2(\tilde{A}^L_1), H_2(\tilde{A}^L_2)))).\end{aligned} \tag{4}$$

- Multiplication with a constant:

$$k \otimes \tilde{\tilde{A}}_1 = ((k \times a_{1_1}^U, k \times a_{1_2}^U, k \times a_{1_3}^U, k \times a_{1_4}^U; H_1(\tilde{A}_1^U), H_2(\tilde{A}_1^U)), \\ (k \times a_{1_1}^L, k \times a_{1_2}^L, k \times a_{1_3}^L, k \times a_{1_4}^L; H_1(\tilde{A}_1^L), H_2(\tilde{A}_1^L))) \tag{5}$$

where $k > 0$ is satisfied.

Moreover, the formula given in the Eq. 6 was developed by Kahraman et al. [23] for the defuzzification of type-2 trapezoidal fuzzy numbers.

- Defuzzification:

$$DTrAT(\tilde{\tilde{A}}) = \frac{1}{2}\left[\left(\frac{a_{i_1}^U + min(H_1(\tilde{A}_i^U), H_2(\tilde{A}_i^U)) \times a_{i_2}^U + max(H_1(\tilde{A}_i^U), H_2(\tilde{A}_i^U)) \times a_{i_3}^U + a_{i_4}^U}{4}\right) + \\ \left(\frac{a_{i_1}^L + min(H_1(\tilde{A}_i^L), H_2(\tilde{A}_i^L)) \times a_{i_2}^L + max(H_1(\tilde{A}_i^L), H_2(\tilde{A}_i^L)) \times a_{i_3}^L + a_{i_4}^L}{4}\right)\right] \tag{6}$$

3.2 EWMA Control Chart

The EWMA (Exponentially Weighted Moving Average) control chart was first developed by Roberts [6]. The EWMA control chart can also be used to estimate the next observation value. In the EWMA control chart, if the data is suitable for normal distribution, the chart gives appropriate results [24]. Exponential average of period t, Z_t, is calculated as follows [1, 25]:

$$Z_t = \lambda \bar{X}_t + (1 - \lambda) Z_{t-1} \tag{7}$$

where the λ value specified in the equation represents the exponential smoothing value. In the case of $t = 1$, $Z_0 = \mu_0$ holds where $\mu_0 = \frac{\sum_1^k \bar{X}_j}{k}$.

Accordingly, limits are calculated as follows:

$$UCL = \mu_0 + L\sigma\sqrt{(\lambda/((2-\lambda))}, \tag{8}$$

$$CL = \mu_0, \tag{9}$$

$$LCL = \mu_0 - L\sigma\sqrt{(\lambda/((2-\lambda))} \tag{10}$$

where UCL, CL and LCL represent upper control limit, center line and lower control limit, respectively.

3.3 EWMA Control Chart based on Type-2 Fuzzy Sets

With respect to type-2 fuzzy sets, the EWMA control charts can be obtained as in Eqs. 12-14. $\tilde{\tilde{Z}}_0$ value is obtained as follows:

$$\tilde{\tilde{Z}}_0 = \tilde{\tilde{\mu}}_0 = ((\bar{X}_{a_1^U}, \bar{X}_{a_2^U}, \bar{X}_{a_3^U}, \bar{X}_{a_4^U}; min(H_1(\tilde{A}^U)), min(H_2(\tilde{A}^U))), (\bar{X}_{a_1^L}, \bar{X}_{a_2^L}, \bar{X}_{a_3^L}, \bar{X}_{a_4^L}; min(H_1(\tilde{A}^L)), min(H_2(\tilde{A}^L)))) \tag{11}$$

where the observations are expressed in terms of type-2 trapezoidal fuzzy numbers as $\tilde{\tilde{A}} = \{X_{a_1^U}, X_{a_2^U}, X_{a_3^U}, X_{a_4^U}; H_1(\tilde{A}^U), H_2(\tilde{A}^U)),$ $(X_{a_1^L}, X_{a_2^L}, X_{a_3^L}, X_{a_4^L}; H_1(\tilde{A}^L), H_2(\tilde{A}^L))\}$. Then, limits of EWMA control chart are obtained with type-2 trapezoidal fuzzy numbers by utilizing the Eqs. 8-10 as follows:

$$U\tilde{\tilde{C}}L = ((\bar{X}_{a_1^U}, \bar{X}_{a_2^U}, \bar{X}_{a_3^U}, \bar{X}_{a_4^U}; min(H_1(\tilde{A}^U)), min(H_2(\tilde{A}^U))), (\bar{X}_{a_1^L}, \bar{X}_{a_2^L}, \bar{X}_{a_3^L}, \bar{X}_{a_4^L}; min(H_1(\tilde{A}^L)), min(H_2(\tilde{A}^L)))) \\ + L\sigma\sqrt{\frac{\lambda}{(2-\lambda)}(1-(1-\lambda))^{2i}}, \tag{12}$$

$$\tilde{\tilde{C}}L = ((\bar{X}_{a_1^U}, \bar{X}_{a_2^U}, \bar{X}_{a_3^U}, \bar{X}_{a_4^U}; min(H_1(\tilde{A}^U)), min(H_2(\tilde{A}^U))), (\bar{X}_{a_1^L}, \bar{X}_{a_2^L}, \bar{X}_{a_3^L}, \bar{X}_{a_4^L}; min(H_1(\tilde{A}^L)), min(H_2(\tilde{A}^L)))), \tag{13}$$

$$L\tilde{\tilde{C}}L = ((\bar{X}_{a_1^U}, \bar{X}_{a_2^U}, \bar{X}_{a_3^U}, \bar{X}_{a_4^U}; min(H_1(\tilde{A}^U)), min(H_2(\tilde{A}^U))), (\bar{X}_{a_1^L}, \bar{X}_{a_2^L}, \bar{X}_{a_3^L}, \bar{X}_{a_4^L}; min(H_1(\tilde{A}^L)), min(H_2(\tilde{A}^L)))) \\ - L\sigma\sqrt{\frac{\lambda}{(2-\lambda)}(1-(1-\lambda))^{2i}}. \tag{14}$$

3.4 CUSUM Control Chart

The cumulative sum, CUSUM, was developed by Page [5]. CUSUM control charts are basically created based on the sum of the cumulative differences between the sample mean and the targeted value [26]. CUSUM control charts can detect sudden changes that occur in the process and its time can be determined in tabular CUSUM [1, 4]. The cumulative total of these values is calculated as follows [26]:

$$S_t = \sum_{i=1}^{t}(x_i - k) = S_{t+1} + (x_1 - k) \tag{15}$$

where k value represents the target value In tabular CUSUM, S_i^U shows deviations above the target and S_i^L demonstrates deviations below the target. C_i^U and C_i^L calculate the deviations from the target cumulatively and are called upper and lower CUSUM, respectively [1].

S_i^U and S_i^L values are calculated as follows:

$$S_i^U = x_i - (k + K), \tag{16}$$

$$S_i^L = (k - K) - x_i, \tag{17}$$

where the K value is called the reference value (or tolerance value).

By taking C_0^L and C_0^U values as 0, C_i^U and C_i^L values are calculated as follows:

$$C_i^U = max(0, C_{i-1}^U + S_i^U), \tag{18}$$

$$C_i^L = max(0, C_{i-1}^L + S_i^L). \tag{19}$$

The first period when either $C_i^U > H$ or $C_i^L > H$ is satisfied, where H is the recommended value of the decision interval, indicates the process is out of control. The probable time of the shift is determined as follows: N^U and N^L values show the number of sequential observations where C_i^U and C_i^L are not equal to zero. After determining the N^U value in the first period when C_i^U is greater than H and the period i in which this value is observed, it is concluded that the process was last under control at the time $i - N^U$ [1].

3.5 CUSUM Control Chart based on Type-2 Fuzzy Sets

Based on type-2 fuzzy sets, CUSUM control charts are re-designed, in which $\tilde{\tilde{k}}$ value is obtained as in Eq. 20:

$$\tilde{\tilde{k}} = \tilde{\tilde{\mu}}_0 = ((\bar{X}_{a_1^U}, \bar{X}_{a_2^U}, \bar{X}_{a_3^U}, \bar{X}_{a_4^U}; min(H_1(\tilde{A}^U)), min(H_2(\tilde{A}^U))), (\bar{X}_{a_1^L}, \bar{X}_{a_2^L}, \bar{X}_{a_3^L}, \bar{X}_{a_4^L}; min(H_1(\tilde{A}^L)), min(H_2(\tilde{A}^L)))). \tag{20}$$

Since the observations are expressed in terms of type-2 trapezoidal fuzzy numbers as $\tilde{\tilde{A}}_i = ((X_{a_1^U}, X_{a_2^U}, X_{a_3^U}, X_{a_4^U}; H_1(\tilde{A}^U), H_2(\tilde{A}^U)),$ $(X_{a_1^L}, X_{a_2^L}, X_{a_3^L}, X_{a_4^L}; H_1(\tilde{A}^L), H_2(\tilde{A}^L)))$.

D_i^U and D_i^L values are obtained by defuzzifying S_i^U and S_i^L values by using Eq. 6. Then, by taking initial C_0^L and C_0^U values as equal to 0, C_i^U and C_i^L are calculated as $C_i^U = max(0, D_i^U + C_{i-1}^U)$ and $C_i^L = max(0, D_i^L + C_{i-1}^L)$. The first period when either $C_i^U > H$ or $C_i^L > H$ is satisfied indicates the process is out of control. The probable time of the shift is determined as $i - N^U$ as in the crisp tabular CUSUM.

4 A Descriptive Example

The inner diameter of the samples taken from the bearing production process was measured [1]. Approximate values were recorded for the last three digits of the measurements. 24 samples are analyzed. Since the measurements are approximate values, they are expressed with linguistic terms as given in Table 1. The corresponding type-2 trapezoidal fuzzy numbers and the center line value of the EWMA control chart are also shown in Table 1.

Observations	Linguistic Terms	Corresponded trapezoidal type-2 fuzzy number
1	Approx. 34.5	<[(33, 34, 35, 36)(0.9, 0.9)], [(33.5, 34, 35, 35.5)(0.7, 0.7)]>
2	Approx. 34.2	<[(32.7, 33.7, 34.7, 35.7)(0.9, 0.9)], [(33.2, 33.7, 34.7, 35.2)(0.7, 0.7)]>
3	Approx. 31.6	<[(30.1, 31.1, 32.1, 33.1)(0.9, 0.9)], [(30.6, 31.1, 32.1, 32.6)(0.7, 0.7)]>
4	Approx. 31.5	<[(30, 31, 32, 33)(0.9, 0.9)], [(30.5, 31, 32, 32.5)(0.7, 0.7)]>
5	Approx. 35.0	<[(33.5, 34.5, 35.5, 36.5)(0.9, 0.9)], [(34, 34.5, 35.5, 36)(0.7, 0.7)]>
6	Approx. 34.1	<[(32.6, 33.6, 34.6, 35.6)(0.9, 0.9)], [(33.1, 33.6, 34.6, 35.1)(0.7, 0.7)]>
7	Approx. 32.6	<[(31.1, 32.1, 33.1, 34.1)(0.9, 0.9)], [(31.6, 32.1, 33.1, 33.6)(0.7, 0.7)]>
8	Approx. 33.8	<[(32.3, 33.3, 34.3, 35.3)(0.9, 0.9)], [(32.8, 33.3, 34.3, 34.8)(0.7, 0.7)]>
9	Approx. 34.8	<[(33.3, 34.3, 35.3, 36.3)(0.9, 0.9)], [(33.8, 34.3, 35.3, 35.8)(0.7, 0.7)]>
10	Approx. 33.6	<[(32.1, 33.1, 34.1, 35.1)(0.9, 0.9)], [(32.6, 33.1, 34.1, 34.6)(0.7, 0.7)]>
11	Approx. 31.9	<[(30.4, 31.4, 32.4, 33.4)(0.9, 0.9)], [(30.9, 31.4, 32.4, 32.9)(0.7, 0.7)]>
12	Approx. 38.6	<[(37.1, 38.1, 39.1, 40.1)(0.9, 0.9)], [(37.6, 38.1, 39.1, 39.6)(0.7, 0.7)]>
13	Approx. 35.4	<[(33.9, 34.9, 35.9, 36.9)(0.9, 0.9)], [(34.4, 34.9, 35.9, 36.4)(0.7, 0.7)]>
14	Approx. 34.0	<[(32.5, 33.5, 34.5, 35.5)(0.9, 0.9)], [(33, 33.5, 34.5, 35)(0.7, 0.7)]>
15	Approx. 37.1	<[(35.6, 36.6, 37.6, 38.6)(0.9, 0.9)], [(36.1, 36.6, 37.6, 38.1)(0.7, 0.7)]>
16	Approx. 34.9	<[(33.4, 34.4, 35.4, 36.4)(0.9, 0.9)], [(33.9, 34.4, 35.4, 35.9)(0.7, 0.7)]>
17	Approx. 33.5	<[(32, 33, 34, 35)(0.9, 0.9)], [(32.5, 33, 34, 34.5)(0.7, 0.7)]>
18	Approx. 31.7	<[(30.2, 31.2, 32.2, 33.2)(0.9, 0.9)], [(30.7, 31.2, 32.2, 32.7)(0.7, 0.7)]>
19	Approx. 34.0	<[(32.5, 33.5, 34.5, 35.5)(0.9, 0.9)], [(33, 33.5, 34.5, 35)(0.7, 0.7)]>
20	Approx. 35.1	<[(33.6, 34.6, 35.6, 36.6)(0.9, 0.9)], [(34.1, 34.6, 35.6, 36.1)(0.7, 0.7)]>
21	Approx. 33.7	<[(32.2, 33.2, 34.2, 35.2)(0.9, 0.9)], [(32.7, 33.2, 34.2, 34.7)(0.7, 0.7)]>
22	Approx. 32.8	<[(31.3, 32.3, 33.3, 34.3)(0.9, 0.9)], [(31.8, 32.3, 33.3, 33.8)(0.7, 0.7)]>
23	Approx. 33.5	<[(32, 33, 34, 35)(0.9, 0.9)], [(32.5, 33, 34, 34.5)(0.7, 0.7)]>
24	Approx. 34.2	<[(32.7, 33.7, 34.7, 35.7)(0.9, 0.9)], [(33.2, 33.7, 34.7, 35.2)(0.7, 0.7)]>
$\widetilde{\widetilde{CL}}$		<[(32.5, 33.5, 34.5, 35.5)(0.9, 0.9)], [(33, 33.5, 34.5, 35)(0.7, 0.7)]>

Table 1 Type-2 trapezoidal fuzzy numbers corresponding to the observations

The upper control limit (UCL) and lower control limit (LCL) values, calculated by using Eqs. 12-14, are shown in Table 2. In the calculation of these limit values, it is taken as $\lambda = 0.45$ and $L = 5$.

i	$\widetilde{\widetilde{LCL}}$	$\widetilde{\widetilde{UCL}}$
1	<[(28.79, 29.79, 30.79, 31.79)(0.9, 0.9)], [(29.29, 29.79, 30.79, 31.29)(0.7, 0.7)]>	<[(36.22, 37.22, 38.22, 39.22)(0.9, 0.9)], [(36.72, 37.22, 38.22, 38.72)(0.7, 0.7)]>
2	<[(28.27, 29.27, 30.27, 31.27)(0.9, 0.9)], [(28.77, 29.27, 30.27, 30.77)(0.7, 0.7)]>	<[(36.74, 37.74, 38.74, 39.74)(0.9, 0.9)], [(37.24, 37.74, 38.74, 39.24)(0.7, 0.7)]>
3	<[(28.12, 29.12, 30.12, 31.12)(0.9, 0.9)], [(28.62, 29.12, 30.12, 30.62)(0.7, 0.7)]>	<[(36.89, 37.89, 38.89, 39.89)(0.9, 0.9)], [(37.39, 37.89, 38.89, 39.39)(0.7, 0.7)]>
4	<[(28.08, 29.08, 30.08, 31.08)(0.9, 0.9)], [(28.58, 29.08, 30.08, 30.58)(0.7, 0.7)]>	<[(36.93, 37.93, 38.93, 39.93)(0.9, 0.9)], [(37.43, 37.93, 38.93, 39.43)(0.7, 0.7)]>
5	<[(28.06, 29.06, 30.06, 31.06)(0.9, 0.9)], [(28.56, 29.06, 30.06, 30.56)(0.7, 0.7)]>	<[(36.94, 37.94, 38.94, 39.94)(0.9, 0.9)], [(37.44, 37.94, 38.94, 39.44)(0.7, 0.7)]>
6	<[(28.06, 29.06, 30.06, 31.06)(0.9, 0.9)], [(28.56, 29.06, 30.06, 30.56)(0.7, 0.7)]>	<[(36.95, 37.95, 38.95, 39.95)(0.9, 0.9)], [(37.45, 37.95, 38.95, 39.45)(0.7, 0.7)]>
7	<[(28.06, 29.06, 30.06, 31.06)(0.9, 0.9)], [(28.56, 29.06, 30.06, 30.56)(0.7, 0.7)]>	<[(36.95, 37.95, 38.95, 39.95)(0.9, 0.9)], [(37.45, 37.95, 38.95, 39.45)(0.7, 0.7)]>
8	<[(28.06, 29.06, 30.06, 31.06)(0.9, 0.9)], [(28.56, 29.06, 30.06, 30.56)(0.7, 0.7)]>	<[(36.95, 37.95, 38.95, 39.95)(0.9, 0.9)], [(37.45, 37.95, 38.95, 39.45)(0.7, 0.7)]>
9	<[(28.06, 29.06, 30.06, 31.06)(0.9, 0.9)], [(28.56, 29.06, 30.06, 30.56)(0.7, 0.7)]>	<[(36.95, 37.95, 38.95, 39.95)(0.9, 0.9)], [(37.45, 37.95, 38.95, 39.45)(0.7, 0.7)]>
10	<[(28.06, 29.06, 30.06, 31.06)(0.9, 0.9)], [(28.56, 29.06, 30.06, 30.56)(0.7, 0.7)]>	<[(36.95, 37.95, 38.95, 39.95)(0.9, 0.9)], [(37.45, 37.95, 38.95, 39.45)(0.7, 0.7)]>
11	<[(28.06, 29.06, 30.06, 31.06)(0.9, 0.9)], [(28.56, 29.06, 30.06, 30.56)(0.7, 0.7)]>	<[(36.95, 37.95, 38.95, 39.95)(0.9, 0.9)], [(37.45, 37.95, 38.95, 39.45)(0.7, 0.7)]>
12	<[(28.06, 29.06, 30.06, 31.06)(0.9, 0.9)], [(28.56, 29.06, 30.06, 30.56)(0.7, 0.7)]>	<[(36.95, 37.95, 38.95, 39.95)(0.9, 0.9)], [(37.45, 37.95, 38.95, 39.45)(0.7, 0.7)]>
13	<[(28.06, 29.06, 30.06, 31.06)(0.9, 0.9)], [(28.56, 29.06, 30.06, 30.56)(0.7, 0.7)]>	<[(36.95, 37.95, 38.95, 39.95)(0.9, 0.9)], [(37.45, 37.95, 38.95, 39.45)(0.7, 0.7)]>
14	<[(28.06, 29.06, 30.06, 31.06)(0.9, 0.9)], [(28.56, 29.06, 30.06, 30.56)(0.7, 0.7)]>	<[(36.95, 37.95, 38.95, 39.95)(0.9, 0.9)], [(37.45, 37.95, 38.95, 39.45)(0.7, 0.7)]>
15	<[(28.06, 29.06, 30.06, 31.06)(0.9, 0.9)], [(28.56, 29.06, 30.06, 30.56)(0.7, 0.7)]>	<[(36.95, 37.95, 38.95, 39.95)(0.9, 0.9)], [(37.45, 37.95, 38.95, 39.45)(0.7, 0.7)]>
16	<[(28.06, 29.06, 30.06, 31.06)(0.9, 0.9)], [(28.56, 29.06, 30.06, 30.56)(0.7, 0.7)]>	<[(36.95, 37.95, 38.95, 39.95)(0.9, 0.9)], [(37.45, 37.95, 38.95, 39.45)(0.7, 0.7)]>
17	<[(28.06, 29.06, 30.06, 31.06)(0.9, 0.9)], [(28.56, 29.06, 30.06, 30.56)(0.7, 0.7)]>	<[(36.95, 37.95, 38.95, 39.95)(0.9, 0.9)], [(37.45, 37.95, 38.95, 39.45)(0.7, 0.7)]>
18	<[(28.06, 29.06, 30.06, 31.06)(0.9, 0.9)], [(28.56, 29.06, 30.06, 30.56)(0.7, 0.7)]>	<[(36.95, 37.95, 38.95, 39.95)(0.9, 0.9)], [(37.45, 37.95, 38.95, 39.45)(0.7, 0.7)]>
19	<[(28.06, 29.06, 30.06, 31.06)(0.9, 0.9)], [(28.56, 29.06, 30.06, 30.56)(0.7, 0.7)]>	<[(36.95, 37.95, 38.95, 39.95)(0.9, 0.9)], [(37.45, 37.95, 38.95, 39.45)(0.7, 0.7)]>
20	<[(28.06, 29.06, 30.06, 31.06)(0.9, 0.9)], [(28.56, 29.06, 30.06, 30.56)(0.7, 0.7)]>	<[(36.95, 37.95, 38.95, 39.95)(0.9, 0.9)], [(37.45, 37.95, 38.95, 39.45)(0.7, 0.7)]>
21	<[(28.06, 29.06, 30.06, 31.06)(0.9, 0.9)], [(28.56, 29.06, 30.06, 30.56)(0.7, 0.7)]>	<[(36.95, 37.95, 38.95, 39.95)(0.9, 0.9)], [(37.45, 37.95, 38.95, 39.45)(0.7, 0.7)]>
22	<[(28.06, 29.06, 30.06, 31.06)(0.9, 0.9)], [(28.56, 29.06, 30.06, 30.56)(0.7, 0.7)]>	<[(36.95, 37.95, 38.95, 39.95)(0.9, 0.9)], [(37.45, 37.95, 38.95, 39.45)(0.7, 0.7)]>
23	<[(28.06, 29.06, 30.06, 31.06)(0.9, 0.9)], [(28.56, 29.06, 30.06, 30.56)(0.7, 0.7)]>	<[(36.95, 37.95, 38.95, 39.95)(0.9, 0.9)], [(37.45, 37.95, 38.95, 39.45)(0.7, 0.7)]>
24	<[(28.06, 29.06, 30.06, 31.06)(0.9, 0.9)], [(28.56, 29.06, 30.06, 30.56)(0.7, 0.7)]>	<[(36.95, 37.95, 38.95, 39.95)(0.9, 0.9)], [(37.45, 37.95, 38.95, 39.45)(0.7, 0.7)]>

Table 2 UCL and LCL values calculated according to type-2 fuzzy EWMA formula

The CL, LCL and UCL values calculated in Tables 1-2 and Z_i values calculated according to the formula $Z_1 = \lambda x_1 + (1 - \lambda)Z_0$ are shown as type-2 trapezoidal fuzzy numbers in Figure 2. In Table 2 and Figure 2, it can be observed that the control limits approach their stable values as the observation value grows. It is shown that the observation values are within the control limits in this process.

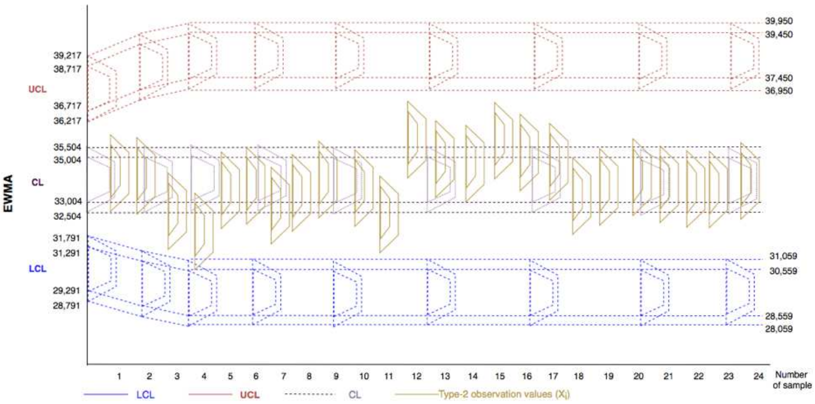

Fig. 2: Type-2 EWMA control chart with trapezoidal fuzzy numbers

EWMA control chart with type-2 trapezoidal fuzzy numbers shown in Figure 2 is defuzzified, and the control chart given in Figure 3 is obtained. When the control chart with the defuzzified values is examined, it is seen that there is no value outside the control limits and observations fluctuate randomly around center line. As a result, it can be said that the process is under control according to Figure 3 as well.

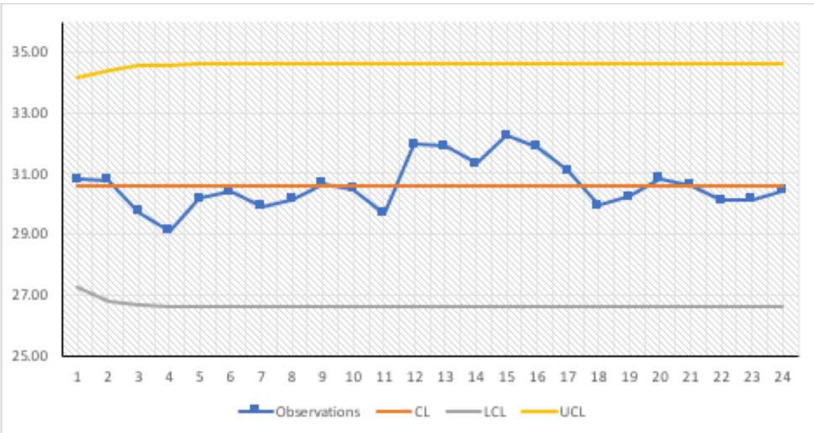

Fig. 3: EWMA control chart obtained after the defuzzification

5 Conclusion

Comprehending and improving quality play an important role in attracting customers and staying competitive in the market. Since adopting quality as an essential part of business strategies brings significant benefits, many approaches such as quality control charts have been developed to improve the quality of products and services by monitoring process' stability. The effective utilization of control charts is quite important to ensure stability by reducing variations in processes. EWMA and CUSUM control charts are preferred over SCCs for detecting small process shifts. The superiority of type-2 fuzzy sets is its ability to effectively model uncertainty even when the membership functions cannot be represented with crisp values. Therefore, in this study, type-2 fuzzy sets are integrated into EWMA and CUSUM control charts which enable preventive actions to be taken and consequently eliminate the unusual sources of variations. Furthermore, illustrative example is provided to show the applicability of the proposed technique. The results indicate that the proposed statistical quality control tool produces reliable outputs.

For future research, other types of fuzzy sets such as intuitionistic fuzzy sets, Pythagorean fuzzy sets and hesitant fuzzy sets can be used for the comparative purposes since they are able to represent uncertainty in a different manner. Process capability analysis can be implemented as further study.

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