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An Application: A Model with Sequence Dependent Setup Times for Parallel Machines for the Die House Station in a White Goods Manufacturing Company

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ABSTRACT

Excessive buffer inventory may disrupt production and constitute one of the main problems. One of the ways of coping with the inventory problems in the mass production lines is to achieve and implement a detailed production schedule. In this study, a company with a mass production line in the die house station of a white goods sector is in consideration. A mixed-integer programming model with sequence-dependent setup times has been developed to solve the excessive work in process problems for the dye house station. The developed model has been applied to the company to test the model by using real data and the problem has been solved by using the General Algebraic Modeling System (GAMS) CPLEX 24,1 solver. The optimal solution is obtained in 10 hours and 3 minutes. In the solution, the total earliness and tardiness time for 30 jobs is 4299 minutes.

Bir Uygulama: Bir Beyaz Eşya Üreten Firmadaki İstasyonda Paralel Makineler için Sıra Bağımlı Hazırlık Zamanları ile bir Model

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ABSTRACT

Gereğinden fazla miktarda ara stok, üretimi kesintiye uğratabilmekte ve temel sorunlardan birini oluşturmaktadır. Kütle üretim yapılan hatlarda envanter sorunları ile başa çıkmanın yollarından biri, ayrıntılı bir üretim programı yapmak ve uygulamaktır. Bu çalışmada beyaz eşya sektörü firmasındaki boyahane istasyonu için seri üretim hattı olan bir firma ele alınmıştır. Boyahane istasyonu için ara stok problemlerindeki aşırı iş yükünü çözmek için sıraya bağlı kurulum sürelerine sahip bir karma tamsayı programlama modeli geliştirilmiştir. Geliştirilen model, gerçek veriler kullanarak test etmek suretiyle söz konusu firmaya uygulanmış ve problem Genel Cebirsel Modelleme Sistemi (GAMS) CPLEX 24,1 çözücü kullanılarak çözülmüştür. Optimal çözüm 10 saat 3 dakikada elde edilmektedir. Elde edilen çözümde 30 iş için toplam erken ve gecikmiş bitirme zamanı 4299 dakikadır.

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1. Introduction

The main objective of a production system is to meet the market demand in time. To realize this aim, it is necessary to create production plans considering production constraints including the inventory, capacity of machines, maintenance plans, workers' productivity and continually update production plans. Scheduling is the problem of optimal assignment of the system resources to apply a combination of jobs over some time duration. The optimal resource allocation problem leads to the optimal modeling of the scheduling problem [1]. The production schedule is the management and designation of resources, events, and procedures to create goods and services. It sets down which resources should be used at what phase of manufacturing. Considering the forecasts, a production schedule should be made to make the company not to have any resource shortage within the production procedures. Effective scheduling enables certain activities to be done in less time using fewer resources. One of the difficult problems is the Job Shop Scheduling Problem (JSSP) is a difficult problem in which a set of n jobs to be carried out on m machines while every job *i* includes of n_i procedures that should be fulfilled by using the machines with the satisfaction of some priority constraints. JSSP tries to obtain a suitable sequence, which optimizes the sequence of operations, to be processed with the related machines according to the determined performance measurements.

The Flexible Job Shop Scheduling Problem (FJSP) is an extended version of the well-known JSSP in which the jobs are assigned to a suitable machine set. This makes FJSP more difficult to solve due to both routings of the jobs and operations schedule. Thus, FJSP is also considered NP-hard [2].

Parallel machine models are widely used in many sectors such as casting, textile, food, printing press sectors. There are many types of parallel machine models. Some of these models are identical machines in parallel, machines in parallel with different speeds, and unrelated machines in parallel. There are "*m*" identical machines in parallel in the identical machines in the parallel model. A job requires a single operation and may be processed on any one of the "*m*" machines. On the other hand, there are "*m*" different machines in parallel in the unrelated parallel machines model. Each machine has a different speed.

The white goods manufacturing company that we apply the model conforms to the flexible job shop model for its whole production. The focal points of our study are the dye house station, which uses identical parallel machines, and the transfer to the press station, which uses an unrelated parallel machine in the company. This study develops a mixed-integer mathematical model for scheduling the jobs in this company for parallel machines with sequence-dependent setup times, solve the problem with real data for the so-called white goods company as an application, and analyze the results.

In this study, a mixed-integer mathematical model for scheduling the jobs in this company for parallel machines with sequence-dependent setup times is developed. Then, the problem with real data for the so-called white goods company is solved and the results are analyzed, some suggestions are made for further studies. In the first of this study, studies related to scheduling are explained. In the second part, the mathematical model is explained and the model is applied by using real data. In the last part of the study, the results of the study, and the suggestions for future studies are presented.

1.1. *Literature Review*

Scheduling with multiple criteria is one of the most attractive topics for researchers. When the relevant studies on a single and parallel machine in literature are searched, studies on parallel machines are increasing in recent years. Considering the literature, the scheduling problems on parallel machines are generally solved by using heuristic algorithms. However, some examples exist in which mathematical algorithms have been used in the solution of the problems.

Baez et al. [3] developed a scheduling problem to minimize total completion time for identical parallel machines including dependent setup times. They presented two time-dependent formulations for the problem that performed much better than a classical formulation based on a formulation for the TSP.

Fanjul-Peyro et al. [4] modeled two integer linear programming problems to minimize the makespan. As the models presented are incapable of solving medium-sized instances to optimality, they proposed three metaheuristic strategies for both models. The algorithms proposed are tested over an extensive computational experience. Results show that the metaheuristic strategies significantly outperform the mathematical models. Mundim and Queiroz [5] studied on a heuristic method including a variable neighborhood search and a mixed-integer programming model in order to solve the identical parallel machine scheduling problem including sequence-dependent setup time. They tried to minimize two objectives, which are the makespan and the flow time.

Gedik et al. [6] focused on non-preemptive unrelated parallel machine scheduling problems (PMSP) with job sequence and machine-dependent setup times. They tried to minimize the makespan. Their study provided novel constraint programming (CP) model with two customized branching strategies that utilize CP's global constraints, interval decision variables, and domain filtering algorithms.

Soper and Strusevich [7] also tried to minimize the makespan. They studied the schedules with a single preemption on uniform parallel machines.

Yepes-Borrero et al. [8] demonstrate a bi-objective scheduling problem on parallel machines by considering job and machine sequence-based setup times and the required resources throughout setups.

Kramer et al. [9] develop the scheduling problem of parallel machines by considering family-based setup times and minimization of the weighted total completion time.

Chen et al. [10] consider the scheduling problem by renewable speed-up resources. Considering the identical machines, the resources, and the jobs, the objective of the study is minimizing the makespan.

Özpeynirci et al. [11] present an approach, which includes mixed-integer programming and integrates the problems of scheduling and the assignment of a tool, considering the parallel machine environments.

Wan et al. [12] study the minimization problem of the maximum total time of completion per machine on identical and parallel machines.

Jiang et al. [13] develop a problem, which considers preemptive scheduling and parallel machines by a single server. It is demonstrated in the study that the suggested algorithm can generate optimal schedules.

Ham and Cakici [14] studied an FJSP with parallel batch processing machines (PBM). They developed a mixed integer programming (MIP) model and a constraint programming (CP) model to reduce computational time. They got the following results: the proposed MIP model improves the computational time significantly compared to the original model in the literature; also, the valid inequalities decrease the solution time, and CP is the best of all three MIP models.

Özgüven et al. [15] addressed F-JSSP with the process plan, routing flexibility with sequence-independent set-up times (SDST), and sequence-dependent set-up times (SIST). They tried to minimize the makespan balance the workloads of the machines by using a mixed-integer goal programming model (MIGP).

Ham [16] introduced a mathematical model for FJSP with PBM with the objective of minimizing the maximum computational time.

Moradi et al. [17] analyze the problem of flexible job shop by preventive maintenance operations considering the multi-objective optimization methods.

Özgüven et al. [18] consider two NP-hard optimization problems. The first one is the flexible job shop scheduling problems, which include sequencing and routing sub-problems. The second one is the flexible job shop scheduling problems by process plan flexibility.

Fattahi and Fallahi [19] consider the problem of the flexible job shop dynamic scheduling. The numerical results prove the efficiency of the suggested algorithm for the problem.

Fattahi et al. [20] discuss the scheduling problem of the flexible job shop by a new approach considering overlapping in tasks. The numerical results in the study demonstrate that the suggested approach can improve the makespan.

Saidi-Mehrabad and Fattahi [21] present a tabu search algorithm to solve the scheduling problem of flexible job shop to minimize the makespan. The experimental results of the study demonstrate that the proposed approach can generate optimal solutions in short computational times.

Low et al. [22] use a global criterion approach, which is a technique for multi-objective decision making, to solve the scheduling problems of the flexible manufacturing system. In the study, hybrid heuristics, which include tabu search and simulated annealing, are suggested to solve the considered problem. The adaptability and feasibility of the suggested approach are analyzed by the numerical experiments in the study.

Gomes et al. [23] suggest a novel integer linear programming model for flexible job shop scheduling. The suggested model considers intermediate buffers, parallel machines, and set-up effects. The numerical experiments show that the suggested model in the study gives optimal results with acceptable solution times.

Seyyedi et al. [24] addressed a multi-objective FSSP that minimizes maximum completion time, maximum machine workload, total machine workload, and earliness/tardiness penalty while satisfying various constraints. They proposed an overall mathematical model that contains the related constraints and assumptions for the problem. They showed that the developed model can be implemented in an acceptable time with better efficiency as well as in real-life problems which has a flexible job shop system.

Scheduling with multiple criteria is one of the most attractive topics for researchers. When the relevant studies on the single and parallel machines in literature are searched, studies on parallel machines are increasing in recent years. Some of the studies are given in Table 1 below. Considering the literature, the scheduling problem on parallel machines is generally solved by heuristic algorithms. Some examples in which the mathematical algorithms are used in the solution of the problems are given in Table 2 below.

Table 1. Studies related to parallel machine scheduling in the literature

Reference	Problem Addressed	Objectives
Baez et al. [3]	Time-dependent model to minimize the total completion time for a parallel machine scheduling problem	Minimize the total completion time
Fanjul-Peyro et al. [4]	Unrelated parallel machines with an additional resource are considered	Minimize the makespan
Mundim and Queiroz [5]	The bi-objective identical parallel machine scheduling problem	Minimize the makespan and the flow time
Gedik et al. [6]	Non-preemptive unrelated parallel machine scheduling problem (PMSP) with job sequence and machine-dependent setup times	Minimize the makespan
Soper and Strusevich [7]	Schedules with a single preemption on uniform parallel machines	Minimize the make-span
Yepes-Borrero et al. [8]	The bi-objective parallel machine scheduling problem	Minimize the makespan and the number of additional resources
Kramer et al. [9]	Parallel machine scheduling problem with family dependent setup times and total weighted completion time minimization	Minimize the total weighted completion time
Chen et al. [10]	Scheduling with renewable speed-up resources.	Minimize the makespan
Özpeynirci et al. [11]	Parallel machine scheduling with tool loading	Minimize the makespan
Wan et al. [12]	Minimizing the maximum total completion time per machine on *m* parallel and identical machines	Minimize the maximum total completion time
Jiang et al. [13]	A preemptive scheduling problem on two parallel machines with a single server	Minimize the makespan

Table 2. Studies related to mathematical models for FJSSP

Reference	Math Models	Problem Addressed	Objectives
Ham and Cakici [14]	MIP, CP	FJSP	Reducing computational time
Özgüven et al. [15]	MIGP	F-JSSP with process plan and routing flexibility with SDST &SIST	Minimize the Makespan Balance the workloads of the machines
Ham [16]	MIP	FJSP	Minimize Cmax
Demir and İşleyen [2]	MIP	FJSP	Minimize the make-span
Moradi et al. [17]	Bi-criterion MILP	F-JSSP with preventive maintenance task	Make-span & minimization of system unavailability
Özgüven et al. [18]	MILP	F-JSSP	Minimize the Make-span
Fattahi and Fallahi [19]	Dynamic scheduling	F-JSSP	The balance between efficiency and stability of the schedules
Fattahi et al. [20]	MILP	F-JSSP with overlapping in operations	Minimize the make-span
Saidi-Mehrabad and Fattahi [21]	MILP	F-JSSP with SDST	Minimize the make-span
Low et al. [22]	MILP	F-JSSP with SIST	Minimize the mean flow time, the mean job tardiness, the mean machine idle time
Gomes et al. [23]	Two MILPs	F-JSSP with and without recirculation	Minimize the costs related to just in time due dates, in-process inventories and orders not fully completed
Seyyedi et al. [24]	MILP	F-JSSP	Minimizes maximum completion time, machine workload, and earliness tardiness penalty

Table 3. List of abbreviations of Table 2

Abbreviation	Meaning of abbreviation
BFI	Best feasible integer
BIV	Number of integer variables
FJSSP	The flexible job-shop scheduling problem
MILP	Mixed-integer linear programming
MINLP	Mixed-integer non-linear programming
SIST	Sequence-independent set-up times
SDST	Sequence-dependent set-up times

There are many studies in the literature aiming to reduce the total completion time and total delay time. However, to the best of our knowledge, there is no mathematical programming study in the literature to minimize the total earliness and tardiness times. In this study, a mathematical programming model is developed to minimize the total earliness and tardiness times in order to reduce the amount of inventory around the station.

In our study, there are two identical machines, eight types of colors, and different changeover times between changing the colors. The products can be produced at the same time as the previous one on each machine. Therefore, the products in the order are can be produced after a 0,2 minutes setup time without waiting for the completion of previous product production. Moreover, a classification of the product by its color is needed to minimize the time spent on changeovers.

2. Application for Identical Parallel Machines

This process includes a conveyor that is 429-meter-long and has 429 hangers. The conveyor's cycle time is 53 minutes. The number of semi-manufactured parts can be hanged on hangers and it is changeable according to the types of product parts. In addition to this, the dye house has 2 cabinets and in the current situation, 8 different colors can be used for dying, such as black, white, silver, claret red, arc744, arc 764, stone coal, Manhattan grey, and cataphoretic. If the color should be changed, change over time must be considered. Changeover times generally take 15 minutes, except for black to white paint color changes. The black color can affect the other colors and changeover time takes 25 minutes.

2.1. *Mixed Integer Mathematical Model*

A mixed-integer mathematical model with sequence-dependent setup times has been developed based on the purpose to solve the excessive work in process inventory problem for the so-called company.

2.1.1. *Problem Definition*

There is an N number of jobs that come to the station at the time t=0. All the products can be processed by both machines. Each job is processed only once by one of these identical machines. Each job has the same precedencies. Pj and dj represent the processing time and the due date for the job j. Cj represents the completion time for job j. In the model, there are $8n3+10n2+10n-2$ constraints (n: the total number of jobs). We simplify our complicated models by using limited jobs that need to schedule in the dye house.

2.1.2. *Notations*

<u>Indices</u>

i:	index for machines where $1\leq i \leq m$
j and q:	index for jobs where $1\leq j \leq n$
K:	Index for job's color where $0\leq k \leq 7$
b:	Index for order where $1\leq b \leq$ n

* The colors of products are white, stone coal, Arc744, Arc764, claret red, black, Manhattan grey, cataphoretic. To make it easy, we use k = {0, 1, 2, 3, 4, 5, 6, 7} for the colors respectively.

<u>Sets</u>

N:	the set of jobs {1, 2, 3, 4, 5, 6, 7, 8, 9, 10}
M:	the set of machines {1, 2}
K:	the set of job's colors
O:	the set of order {1, 2, 3, 4, 5, 6, 7, 8, 9, 10}

<u>Parameters</u>

n:	Total number of jobs
m:	Total number of machines
d_j:	Due date for j^{th} job
P:	Processing time for jobs = 53 minutes
TT:	Total available machine time = 430 minutes
M:	A big positive number
h_{jq}:	The required setup time between j and q (0,2 if colors of j and q job are identical, 15 if none of the colors is black, 25 if one of the colors is black, and M if j=q)
R_{jk}:	1, if the j^{th} job's color is k, 0 otherwise.

<u>Decision Variables</u>

X_{ijb}:	Binary variable (1 if the job j in b^{th} order on the machine i^{th}, 0 if any job on any machine)
Y_{ijqb}:	Binary variable (1 if the job j is in $(b\text{-}1)^{th}$ order on the i^{th} machine and q^{th} job is in b^{th} order on the machine i^{th}, 0 otherwise
T_{ijb}:	The tardiness for j^{th} job in b^{th} order on the i^{th} machine
E_{ijb}:	The earliness for j^{th} job in b^{th} order on the i^{th} machine
C_{ijb}:	The completion time for j^{th} job in b^{th} order on the i^{th} machine

Our algorithm;

$$\text{Min } z = \sum_{b=1}^{n} \sum_{i=2}^{m} \sum_{j=1}^{n} (E_{ijb} + T_{ijb}) \tag{1}$$

Subject to;

$\sum_{b=1}^{n} \sum_{i=1}^{m} X_{ijb} = 1$	$\forall i \in N$	(2)
$\sum_{j=1}^{n} X_{ijb} \leq 1$	$\forall i \in M, \forall b \in O$	(3)
$\sum_{k=0}^{7} R_{jk} = 1$	$\forall i \in N$	(4)
$\sum_{j=1}^{n} x_{iqb} \leq \sum_{j=1}^{n} x_{ij(b-1)}$	$\forall i \in M, \forall b \in O\ b > 1$	(5)
$C_{ijb} + M(1 - x_{ijb}) \geq P$	$\forall i \in M, \forall j \in N\ \ \forall b \in O\ b = 1$	(6)
$C_{ijb} - M(1 - x_{ijb}) \leq P$	$\forall i \in M, \forall j \in N\ \ \forall b \in O\ b = 1$	(7)
$Y_{ijqb} \geq 1 - (2 - x_{iqb} - x_{ij(b-1)}) * M$	$\forall i \in M, \forall j \in N, \forall q \in N\ \forall b \in O$	(8)
$C_{iqb} + M(2 - x_{iqb} - x_{ij(b-1)}) \geq C_{ij,b-1} + h_{jq} * Y_{ijqb}$	$\forall i \in M, \forall j \in N, \forall q \in N\ \forall b \in O\ b > 1$	(9)
$C_{iqb} - M(2 - x_{iqb} - x_{ij(b-1)}) \leq C_{ij,b-1} + h_{pq} * Y_{ijqb}$	$\forall i \in M, \forall j \in N, \forall q \in N\ \forall b \in O\ b > 1$	(10)
$x_{ijb} * C_{ijb} \leq TT$	$\forall i \in M \forall j \in N\ \forall q \in O$	(11)
$T_{ijb} \geq C_{ijb} - d_j$	$\forall i \in M \forall j \in N\ \forall q \in O$	(12)
$E_{ijb} \geq d_j - C_{ijb}$	$\forall i \in M \forall j \in N\ \forall q \in O$	(13)
$X_{ijb} = \{0, 1\}$	$\forall i \in M \forall j \in N\ \forall q \in O$	(14)
$Y_{ijqb} = \{0, 1\}$	$\forall i \in M \forall j \in N\ \forall q \in O$	(15)
$C_{ijb}, T_{ijb}, E_{ijb} \geq 0$	$\forall i \in M \forall j \in N\ \forall q \in O$	(16)
$T_{ijb} \geq 0$	$\forall i \in M \forall j \in N\ \forall q \in O$	(17)
$E_{ijb} \geq 0$	$\forall i \in M \forall j \in N\ \forall q \in O$	(18)

The objective function, equation (1) minimizes total tardiness, and earliness time. Constraints (2) force that each job can be performed by only a machine and at order. Constraints (3) ensure that just one job at most can be assigned to an order of a machine. Constraint (4) guarantees that each job has only one color. Constraint (5) ensures that if there is a job in b^{th} order, there also is a job in the *(b-1)*th order for each machine and each order. Constraints (6) and (7) calculate the completion time of jobs assigned to the first order of each machine. Constraint (8) allows the formation of preparation time during the transition from job *j* to job *q*, if the job *j* is in the *(b-1)*th order, job *q* is in the b^{th} order and if they are the same machine. Constraint (9) and (10) determine the total completion time for each job in each order (except first order) on each machine. Constraints (11) guarantee that total completion time does not exceed the total available time for each job in each order on each machine. Constraints (12) determine tardiness duration for each job in each order on each machine. Constraints (13) determine earliness duration for each job in each order on each machine. Constraints (14) and (15) define the X_{ijb} and Y_{ijqb} as binary variables. Constraints (16), (17), and 18 are non-negativity constraints.

2.2. *Mathematical Model*

In the company, 4,000 products can be produced in a shift and 8,000 products can be produced daily. These quantities may increase to 12,000 products, depending on seasonal effects. The dye-house station can control its production speed and capacity according to fluctuations in demand. However, many constraints should be taken into consideration. In the proposed model, the number of constraints increases according to the number of jobs. Related real data are shown in Table 4.

Table 4. Relation of number of jobs and model constraints

Number of jobs	Number of constraints
1	26
2	122
4	710
5	1,298

Table 4 (continuation). Relation of number of jobs and model constraints

Number of jobs	Number of constraints
6	2,146
8	4,814
10	9,098
15	29,398
25	131,498
30	225,298
40	528,398
50	1,025,498

After the model is built, the model has been validated. It was implemented by solving problems with different sizes of samples by using GAMS CPLEX solver to test the validity of the model. After the analysis of the solutions, it has been seen that when the sample size increases, solution gaps increase. The required times for the GAMS CPLEX 24.1 solver to solve the model are shown in Table 5.

Table 5. The relation of elapsed time for obtained outputs in GAMS and number of jobs when the number of iteration and resources are not defined

Number of jobs	Elapsed time for calculation	Relative Gaps
1	0:06	0
2	0:07	0
3	0:08	0
4	0:07	0,068
5	0:09	0,076
6	0:08	0,099
7	0:26	0,099
8	3:19	0,1
9	16:48	0,2302
10	16:5	0,39
20	16:47	0,2968
30	16:47	0,3023

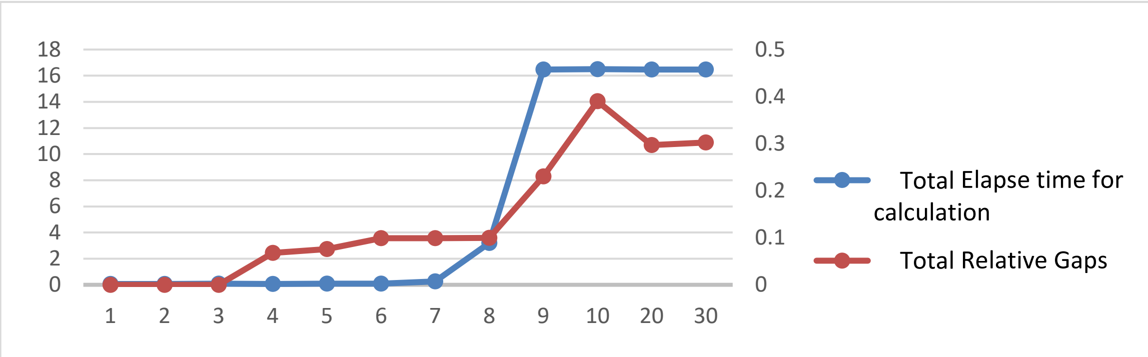

Figure 1. The graph for the solution performance

The results of the samples is shown in Table 5 and Figure 1. When the quantity of products produced in the company is compared with the sample including 50 jobs, to obtain the optimal solution will be hard to get. As a result, using a mathematical model in a company with a high production rate may not be suitable. However, the model can be used in a company with a low Production rate and give optimal solutions to create an efficient production schedule. To obtain a reasonable solution for high numbers of jobs a heuristic algorithm should be developed.

2.2.1. *Application Results with Real Data*

The sample includes 30 numbers of jobs and the jobs' features are shown in Table 6.

Table 6. The due dates and product's colors

J	d_j	Color (*k*)	*J*	d_j	Color
1	450	White (0)	16	65	Black (5)
2	450	Black (5)	17	200	Stone coal (1)
3	450	Arc744 (2)	18	100	Arc744 (2)
4	90	Arc744 (2)	19	200	White (0)
5	110	Stone coal (1)	20	150	Arc764 (3)
6	65	White (0)	21	250	Arc744 (2)
7	75	Black (5)	22	150	Arc744 (2)
8	100	Stone coal (1)	23	100	Stone coal (1)
9	80	Arc744 (2)	24	200	White (0)
10	75	White (0)	25	210	Arc764 (3)
11	95	Cataphoretic (7)	26	250	White (0)
12	60	White (0)	27	230	Arc744 (2)
13	75	Arc744 (2)	28	260	Claret red (4)
14	100	Cataphoretic (7)	29	270	Manhattan Grey (6)
15	75	Black (5)	30	260	Manhattan Grey (6)

The company paints the jobs according to the First in First Out rule. Based on this rule, the jobs assigned to the first machine are 1-3-4-7-9-11-14-15-16-18-20-23-25-26-28 respectively, and the jobs assigned to the second machine are 2-5-6- 8-10-12-13-17-19-21-22-24-27-29-30 respectively. As a result of this assignment, the total earliness and tardiness time is 10,195 minutes.

The outputs given in Table 7 are obtained when we solved the case study by using GAMS CPLEX solver.

Table 7. Results for the application

X_{ijb}		C_{ijb}		X_{ijb}		C_{ijb}	
X_{1119}=	1	C_{1119}=	373	X_{12711}=	1	C_{12711}=	233
X_{1217}=	1	C_{1217}=	333	X_{12815}=	1	C_{12815}=	294
X_{1318}=	1	C_{1318}=	358	X_{12914}=	1	C_{12914}=	278
X_{144}=	1	C_{144}=	108	X_{13016}=	1	C_{13016}=	308
X_{1113}=	1	C_{1113}=	93	X_{255}=	1	C_{255}=	103,4
X_{1132}=	1	C_{1132}=	78	X_{261}=	1	C_{261}=	53
X_{1145}=	1	C_{1145}=	123	X_{273}=	1	C_{273}=	78.2
X_{1156}=	1	C_{1156}=	148	X_{284}=	1	C_{284}=	103,2
X_{1161}=	1	C_{1161}=	53	X_{298}=	1	C_{298}=	118,8
X_{1179}=	1	C_{1179}=	203	X_{2109}=	1	C_{2109}=	133,8
X_{1198}=	1	C_{1198}=	188	X_{2122}=	1	C_{2122}=	53,2
X_{1207}=	1	C_{1207}=	173	X_{2187}=	1	C_{2187}=	118,6
X_{12113}=	1	C_{12113}=	263	X_{22210}=	1	C_{22210}=	148,8
X_{12510}=	1	C_{12510}=	218	X_{2236}=	1	C_{2236}=	103,6
X_{12612}=	1	C_{12612}=	148	X_{22411}=	1	C_{22411}=	163,8

The objective function value, which gives the total earliness and tardiness of the application, is 4,299.0 minutes. The jobs that are assigned to machine 1 respectively are: 16-13-11-4-14-15-20-19-17-25-27-26-21-29-28-30-2-3-1 and to machine 2 respectively are: 6-12-7-8-5-23-18-9-10-22-24. The solver gives us the optimal outputs by considering due dates and constraints. The elapsed time to solve the model is 10 hours 3 minutes.

By using the proposed model, the total earliness and tardiness time is improved 42% regarding the total earliness and tardiness time obtained by using the FIFO rule that the company uses.

3. Conclusion

The study aims to minimize work in process inventories around the dye house stations. To accomplish this aim, a mathematical production scheduling model that can work according to the changeable demands for the external body production unit is developed.

In the current situation, the production scheduling of the dye house station is done based on a human's experience. A mathematical model has been developed to obtain an optimal solution. An identical parallel machine scheduling model is developed and an optimal solution has been found by solving the model by using GAMS CPLEX 24.1. However, the solution is obtained in a long-time period. Increasing the number of jobs raises the total time required. If the company practices high-demand mass production such as 4,000 products in a shift, the mean period may not satisfy the company especially in case the company is in the need of frequent revisions on the schedule.

The best solution can be obtained through mathematical programming. However, as the number of jobs in the mathematical model increases, the time to reach a solution increases. With the mathematical model developed, the time to obtain solutions is slow compared to the dynamic structure of the companies that make mass production. For this reason, it will be useful to use heuristic methods that give the best solution in a short time to solve the scheduling problems where the quantity of product to be produced is high. It will be useful to create heuristic and metaheuristic algorithms using the parameters of the model for future studies. Also, metaheuristic methods can be used for large-scale problems.

Statement of Conflict of Interest
Authors have declared no conflict of interest.

Author's Contributions
The contribution of the authors is equal

References

[1] Ahmad F., Khan SA. Module-based architecture for a periodic job-shop scheduling problem, Computers & Mathematics with Applications 2012, 64(1): 1-10.

[2] Demir Y., Işleyen SK. Evaluation of mathematical models for flexible job-shop

scheduling problems. Applied Mathematical Modelling 2013; 37(3): 977-988.

[3] Baez S., Angel-Bello F., Alvarez A. Time-dependent formulations for minimizing total completion time in a parallel machine scheduling problem with dependent setup times, IFAC-PapersOnLine 2016; 49(12): 857-862.

[4] Fanjul-Peyro L., Perea F., Ruiz R. Models and metaheuristics for the unrelated parallel machine scheduling problem with additional resources, European Journal of Operational Research 2017; 260(2): 482-493.

[5] Mundim LR., Queiroz TAD. Using a variable neighborhood search to solve a bi-objective identical parallel machine scheduling problem, Electronic Notes in Discrete Mathematics 2018; 66: 127–134.

[6] Gedik R., Kalathia D., Egilmez G., Kirac E. A constraint programming approach for solving unrelated parallel machine scheduling problem, Computers & Industrial Engineering 2018; 121: 139-149.

[7] Soper AJ., Strusevich VA. Schedules with a single preemption on uniform parallel machines, Discrete Applied Mathematics 2018; 261: 332-343.

[8] Yepes-Borrero JC., Perea F., Ruiz R., Villa F. Bi-objective parallel machine scheduling with additional resources during setups, European Journal of Operational Research 2020 (In Press).

[9] Kramer A., Iori M., Lacomme P. Mathematical formulations for scheduling jobs on identical parallel machines with family setup times and total weighted completion time minimization, European Journal of Operational Research 2019; 289(3): 825-840.

[10] Chen L., Ye D., Zhang G. Parallel machine scheduling with speed-up resources, European Journal of Operational Research 2018; 268(1): 101-112.

[11] Özpeynirci S., Gökgür B., Hnich B. Parallel machine scheduling with tool loading, Applied Mathematical Modelling 2016, 40(9-10): 5660-5671.

[12] Wan L., Ding Z., Li Y., Chen Q., Tan Z. Scheduling to minimize the maximum total completion time per machine, European Journal of Operational Research 2015, 242(1): 45-50.

[13] Jiang Y., Dong J., Ji M. Preemptive scheduling on two parallel machines with a single server, Computers & Industrial Engineering 2013; 66(2): 514-518.

[14] Ham AM., Cakici E. Flexible job shop scheduling problem with parallel batch processing machines: MIP and CP approaches, Computers & Industrial Engineering 2016; 102: 160-165.

[15] Özgüven C., Yavuz Y., Özbakır L. Mixed integer goal programming models for the flexible job-shop scheduling problems with separable and non-separable sequence dependent setup times, Applied Mathematical Modelling 2012; 36(2): 846-858.

[16] Ham A. Flexible job shop scheduling problem for parallel batch processing machine with compatible job families, Applied Mathematical Modelling 2017, 45: 551-562.

[17] Moradi E., Ghomi SF., Zandieh M. Bi-objective optimization research on integrated fixed time interval preventive maintenance and production for scheduling flexible job-shop problem, Expert Systems with Applications 2011, 38(6): 7169-7178.

[18] Özgüven C., Özbakır L., Yavuz Y. Mathematical models for job-shop scheduling problems with routing and process plan flexibility, Applied Mathematical Modelling 2010; 34(6): 1539-1548.

[19] Fattahi P., Fallahi A. Dynamic scheduling in flexible job shop systems by considering simultaneously efficiency and stability, CIRP Journal of Manufacturing Science and Technology 2010, 2(2): 114-123.

[20] Fattahi P., Jolai F., Arkat J. Flexible job shop scheduling with overlapping in operations, Applied Mathematical Modelling 2009, 33(7): 3076-3087.

[21] Saidi-Mehrabad M., Fattahi P. Flexible job shop scheduling with tabu search algorithms, International Journal of Advanced Manufacturing Technology 2007; 32(5-6): 563-570.

[22] Low C., Yukling Y., Tai-His W. Modelling and heuristics of FMS scheduling with multiple objectives, Computers & Operations Research 2006, 33(3): 674-694.

[23] Gomes MC., Barbosa-Povoa AP., Novais AQ. Optimal scheduling for flexible job shop operation, International Journal of Production Research 2005; 43(11): 2323-2353.

[24] Seyyedi MH., Saghih AMF., Pooya A., Azimi ZN. Designing a multi-objective mathematical model for flexible job shop scheduling problem with the earliness/tardiness penalty, The Journal of Social Sciences Research 2019; 5(10): 1375-1382.