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AN UPPER BOUND ON THE DEGREE OF CHOICE REGULARITIES ASSOCIATED WITH SARP-LIKE REQUIREMENTS*

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Abstract

Yildiz (2016a) proposes a novel tool, namely *choice regularities*, to classify choice theories according to the syntactic structure of the underlying choice axioms. We focus on the regularity of the SARP-like acyclicity requirements. Axiomatic characterizations of many choice theories contain conditions formulated in the vein of the *strong axiom of revealed preference* (SARP), such as Masatlioglu et al. (2012)and Yildiz (2016b), in which a binary relation –based on the observed choice behavior– is formulated and its acyclicity is required. We focus on the regularity of the SARP-like acyclicity requirements. For those binary relations that can be represented in a specific form, we find an upper bound on the degree of the free regularities that impose the binary relation to be acyclic.

Keywords: *Rational choice, Bounded rationality, Choice axioms, Mathematical logic, Propositional calculus, Descriptive complexity.*

SARP BENZERİ KISITLARA İLİŞKİN SEÇİM DÜZENLİLİKLERİ DERECELERİ ÜZERİNE BİR ÜST SINIR

Öz

Yildiz (2016a) seçim kuramlarının altında yatan aksiyomların sentaks yapısına bağlı olarak tasnifi amacıyla yeni bir ölçüt olarak *seçim düzenlilikleri* kavramını öne sürmüştür. Burada SARP benzeri seçim aksiyomlarının düzenlilikleri üzerinde yoğunlaşıyoruz. Masatlioglu et al. (2016) ve Yildiz (2016b) gibi pek çok seçim kuramının karakterizasyonu *güçlü açıklanmış tercihler aksiyomu* (SARP) benzeri koşullar içermektedir. Bu koşullar gözlemlenen seçim davranışından çıkarsanan bir ikili karşılaştırma ilişkisinin çevrimsiz olmasını gerektirmektedir. Biz SARP benzeri çevrimsizlik koşullarına odaklanıyoruz. Belli bir formda ifade edilebilen ikili ilişkiler için, bu ilişkilerin çevrimsiz olmasını garanti eden serbest düzenlilikler için bir üst sınır buluyoruz.

Anahtar Kelimeler: *Rasyonel seçim, Kısmi rasyonellik, Seçim aksiyomları, Matematiksel mantık, Önermeler kalkülüsü, Tanımsal karmaşıklık.*

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* This study is used to be contained in an old and unpublished manuscript of the author titled "Choice Regularities: Relative identification of choice theories", which is available online through the web pages of some universities in which the author has presented this old version of the paper in departmental seminars. The author also orally presented the content of this paper with the title "Choice Regularities and SARP-like Requirements" in the 4th International Management and Social Sciences Conference (UYSAD 2020) and the abstract appeared in the abstract book of this conference, Yildiz (2020).

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1. INTRODUCTION

In the classical revealed preference framework, the observable content of a decision maker's choices is summarized by a *choice function*, which singles out an alternative from each *choice set*.[1] In this framework, a *choice theory* is a collection of choice functions. For example, the rational choice theory is the collection of choice functions that can be represented as if maximizing a single preference relation. Similarly, a choice axiom proposes a requirement to be satisfied by choice functions. That is, a choice axiom is associated with the choice theory consisting of the choice functions that satisfy the axiom. We use this association between choice axioms and choice theories to explore the structure of choice axioms.

Several studies analyze the *computational complexity* of decision-making, which measures the time or effort needed for the empirical refutation of a choice theory. Depending on the proposed measure, some of these studies favor boundedly rational choice theories, while others find the rational choice theory computationally more tractable. In contrast, Yildiz (2016a) introduces a *descriptive measure* for analyzing the structure of choice axioms which paves the way for the axiomatic classification and comparison of choice theories.[2]

Axiomatic characterizations of many choice theories contain conditions formulated in the vein of the *strong axiom of revealed preference* (SARP) introduced by Samuelson (1938), such as Masatlioglu et al. (2012) and Yildiz (2016b), in which a binary relation –based on the observed choice behavior– is formulated and its acyclicity is required. We focus on the regularity of the SARP-like acyclicity requirements. For those binary relations that can be represented in a specific form, we find an upper bound on the degree of the free regularities that impose the binary relation to be acyclic.

2.CHOICE REGULARITIES AND SARP-LIKE REQUIREMENTS

Let A be a fixed nonempty finite alternative set with n alternatives. Let Ω denote the collection of all subsets of A with at least two alternatives. Let $\mathcal{N}$ stand for the pairs of nested choice sets, that is $\mathcal{N} = \{(S_i, S_j) | S_i, S_j \in \Omega \text{ such that } S_i \subset S_j\}$. A choice function is a mapping $c: \Omega \to A$ such that for each $S \in \Omega$, $c(S) \in S$.

A choice theory τ is a collection of choice functions. We consider two choice procedures with possibly different formulations as equivalent if these procedures are observationally indistinguishable in the revealed preference framework. That is, two choice procedures rationalize the same set of choice functions. Similarly, we associate a choice axiom with the choice functions that satisfy the axiom and use this association to explore the structure of a choice axiom.

We introduce a few basic concepts from Mathematical Logic to define our regularity notions as *formulas* in a properly defined *language* of propositional calculus.[3] Here we consider a specific language $\mathcal{L}$ with the following alphabet:

1. Variables: "alternatives" $(a_1, \ldots, a_n)$, "choice sets" $(S_1, \ldots S_m)$, "nested pairs of choice sets" $((S_1, S_2), \ldots, (S_k, S_l))$.

2. Logical connectives: conjunction ($\wedge$), implication ($\Rightarrow$).

3. Quantifiers: the universal quantifier, "for each" ($\forall$).

[1] A choice set is a subset of the grand alternative set with at least two alternatives. We read $a = c(S)$ as alternative a is chosen from the choice set S.

[2] A branch of *computational complexity theory* and of *finite model theory*, namely, *descriptive complexity theory* investigates the relationship between the computational complexity and logic.

[3] For more on Mathematical Logic, one can consult Crossley (2006).

4. Atomic statement (predicate): “a is chosen from S" ($a = c(S)$).

For a given language, such as $\mathcal{L}$, a **formula** in this language is constructed from the atomic statements by using the logical connectives. Let q be a formula in a given language. A variable v is **universal** in q if "$\forall v$" appears in q. Otherwise, it is said to be **free**. A formula is a **sentence** if it contains no free variables. For the given language $\mathcal{L}$ that is specified above, for each choice function c, **a first-order free regularity** (**1-*f*reg**) is a formula of the form:

$$\text{If } a_1 = c(S_1), \text{then } a_2 = c(S_2)$$

for some $a_1, a_2 \in A$ and $S_1, S_2 \in \Omega$. Similarly, a $\boldsymbol{k^{th}}$**-order free regularity** (**k-*f*reg**) is a formula of the form:

$$\text{If } a_1 = c(S_1) \text{ and } a_2 = c(S_2) \cdots \text{and } a_k = c(S_k), \text{then } a_{k+1} = c(S_{k+1}).$$

for some $a_1, \dots, a_{k+1} \in A$ and $S_1, \dots S_{k+1} \in \Omega$.

A choice theory τ **satisfies a set of free regularities** Q if each $c \in \tau$ satisfies each regularity $q \in Q$. Now, we are ready to provide the main definitions used to classify the choice theories based on the structure of the underlying choice axioms.

Definition: *Let τ be a choice theory. τ is* $\boldsymbol{k^{th}}$***-order free regular (k-fregular)*** *if a collection of k-fregs Q^f identifies τ, i.e. a choice function c satisfies each $q^f \in Q^f$ if and only if $c \in \tau$.*

3. SARP-LIKE REQUIREMENTS

Characterizations of many choice theories contain axioms formulated in the vein of the *strong axiom of revealed preference* (SARP) introduced by Samuelson (1938). To introduce SARP, for a given choice function c and each distinct $x, y \in A$, x is *revealed to be preferred* to y, denoted by xRy, if there is a choice set S such that $y \in S$ and $x = c(S)$. SARP requires R to be acyclic. In this section, we focus on the free regularities that represent SARP-like conditions. For those binary relations that can be represented in a specific form, we find an upper bound on the degree of free regularities that impose the binary relation to be acyclic. Next, we introduce this specific form that a binary relation should have.[4]

Definition: *Let R be a binary relation on the alternative set A, R is representable as a* ***disjunction of conjunctions of atomic formulas of order*** $\boldsymbol{m}$ *(DCAF-m), if for each $x, y \in A$, there is a family of collections of alternative-choice set pairs $\{\{(a_i^j, S_i^j)\}_{i=1}^{m_j}\}_{j\in J}$ such that*

$$xRy \text{ if and only if } \bigvee_{j\in J} \bigwedge_{i=1}^{m_j} [a_i^j = c(S_i^j)],$$

where for each $j \in J$, $m_j \leq m$.

Note that the revealed preference relation R is representable as DCAF-1 since for each distinct $x, y \in A$, xRy if and only if $\vee_{\{S\in\Omega: y\in S\}} [x = c(S)]$. In the following examples, we show that the binary relations used to characterize choice with limited attention and list-rational choice are DCAF-2.

[4] This requirement is similar to a preference relation being *definable* in the words of Rubinstein (1998). A preference relation is said to be definable in a language of propositional calculus if there is a formula that is satisfied precisely for those pairs which satisfy the relation. Under some minor assumptions, he shows that lexicographic preferences are the only definable preferences. Rubinstein (1998) formulates atomic statements by using the given primitive rankings that justify an alternative is better than another, whereas our atomic statements describe what is chosen from each choice set.

Example 1 (Choice with Limited Attention) Masatlioglu et al. (2012) *show that a choice function c belongs to the theory of choice with limited attention if and only if the following binary relation P is acyclic.*

$$xPy \text{ if there exists } S \in \Omega \text{such that } c(S) = x \text{ and } x \neq c(T \backslash y).$$

Now, define $\mathcal{B} = \{(S,z) \in \Omega \times A: z \in S \backslash \{x,y\}\}$. Note that P is representable as DCAF-2, since xPy if and only if $\vee_{\{(S,z) \in \mathcal{B}\}} [x = c(S) \wedge z = c(S \backslash y)]$.

Example 2 (List Rational Choice) Yildiz (2013) *shows that a choice function c is list rational if and only if c satisfies weak path independence and the following binary relation F is acyclic.*

$$xFy \text{ if there exists } S \in \Omega \text{ with } y \in S \text{ and } c(S) = x \text{ such that } x \neq c(T \setminus y) \text{ or } y = c(x,y).$$

Note that F is representable as DCAF-2 since xFy if and only if $\vee_{\{(S,z) \in \mathcal{B}\}} [x = c(S) \wedge z = c(S \setminus y)] \vee_{\{S \in \Omega: y \in S\}} [x = c(S) \wedge y = c(x,y)]$.

Proposition 1 *Let A be an alternative set with n elements.*

i. Any choice theory is $(2^n - n - 2)$-*f*regular.

ii. If a choice theory τ is characterized by the acyclicity of a binary relation representable as DCAF-m, then τ is nm-*f*regular.

Proof. i. Let τ be any choice theory. First, fix any distinct $a, b \in A$, and let $\Omega \backslash \{a,b\} = \{S_1, \ldots, S_m\}$, where $m = 2^n - n - 2$. Next, consider any choice function $c' \not\in \tau$. Assume w.l.o.g. that $a = c'(\{a,b\})$. Now, formulate the $2^n - n - 2$-regularity $q_{c'}$ that requires: $\wedge_{i=1}^{m} [c'(S_i) = c(S_i)] \Rightarrow b = c(\{a,b\})$. Finally, consider the collection of these regs $\{q_{c'}\}_{c' \not\in \tau}$. We argue that $\{q_{c'}\}_{c' \not\in \tau}$ identifies τ. To see this, first note that for each $c' \not\in \tau$, by construction, c' fails to satisfy $q_{c'}$. Next, we argue that for each $c \in \tau$ and $c' \not\in \tau$, c satisfies $q_{c'}$. Suppose not, then it follows that there exists $c' \not\in \tau$ such that c satisfies the precedent part of the requirement $q_{c'}$, but not the consequent part. By the construction of $q_{c'}$, this means that for each $S \in \Omega$, $c(S) = c'(S)$. Hence we obtain a contradiction.

ii. Let τ be a choice theory characterized by the acyclicity of a binary relation R, which is representable as DCAF-k. We argue that acyclicity of R can be imposed via a set of regularities. To see this, first note that each cycle in any binary relation can be decomposed into non-intersecting cycles. Next, note that the length of each non-intersecting cycle in R can be at most n. Now, consider a cycle $x_1 R x_2 R \cdots x_k R x_1$ with length $k \leq n$. We argue that we can rule out such a cycle of length k via a set of $2k$-*f*regs.

Since R is representable as DCAF-k, for each $j < k$, $x_j R x_{j+1}$; for $j = k$, $x_j R x_1$ means that $\wedge_{i=1}^{m_j} a_i^j = c(S_i^j)$ for a collection of alternative-choice set pairs $\{(a_i^j, S_i^j)\}_{i=1}^{m_j}$ such that $m_j \leq k$. Now, for the given cycle consider the following $2k$-*f*reg:

$$\bigwedge_{j=1}^{k} \bigwedge_{i=1}^{m_j} a_i^j = c(S_i^j) \Rightarrow x = c(S_{m_k}^k)$$

for some $x \in S_{m_k}^k \backslash \{a_{m_k}^k\}$. Let Q be the collection of these $2k$-*f*regs written for each possible cycle of length k. Note that for each choice function c, the relation R, induced from c, does not have a cycle of length k if and only if c satisfies each $2k$-*f*reg in Q. Since the longest cycle of R has a length of n, it follows that τ is nk-regular. and for each $j \in J$, $m_j \leq m$.

We can interpret the number $2^n - n - 2$ as the softbound on the order of the free regularities that identify a choice theory. This makes the characterization of rational choice particularly appealing since it shows that the theory is 1-regular as opposed to being $(2^n - n - 2)$-regular. Similarly, we might measure the "normative appeal" or "ease of identification" obtained out of axiomatic characterizations in terms of the gain in the order of regularities. For example, consider choice with limited attention and list-rational choice. It directly follows from Proposition 1 their axiomatic characterizations that these choice theories are $2n$-regular.

4. CONCLUDING REMARKS

Here, we assess the 'niceness' of choice axioms in terms of choice regularities. This is not the only criteria to evaluate a choice axiom. Choice axioms are relevant since they provide insights into the underlying structure of the analyzed choice behavior. From a theoretical point of view, different axiomatic characterizations might facilitate the comparison between similar choice theories; from a more empirical point of view, they might facilitate identification problems in different ways. In this vein, although SARP can not be represented as a first or second regularity, it is the relevant choice axiom to verify if a consumer is choosing from different budget sets as if maximizing a utility function. We believe that similar relative identification exercises for different choice theories would contribute to our understanding of the behavioral differences among different boundedly rational choice theories. We hope that researchers would find choice regularities useful in the identification of the simple behavioral properties of a choice theory or to check the robustness of their axiomatic results.

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