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RESEARCH ARTICLE / ARAŞTIRMA MAKALESİ

Spherical Fuzzy Version of EDAS and An Application

Küresel Bulanık EDAS ve Bir Uygulama

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Abstract
Several fuzzy set concepts have been developed after the first invention of fuzzy sets in the 1960s to demonstrate the uncertainty and vagueness in human preferences. Spherical fuzzy sets (SFS) as a recent one of these developments support this aim by giving a comprehensive preference domain to decision-makers. The distinctive feature of SFS is its rule saying that the squared sum of membership, non-membership, and hesitancy degrees should be within the interval of [0,1] while each element is independently assigned within the same interval. With this study, EDAS (Evaluation Based on Distance from Average Solution), one of the younger but stronger multiple attribute decision-making tools is modified for spherical fuzzy environment. Entropy-based objective attribute weighting is also integrated with this novel version of EDAS to avoid the undesired potential effects of subjective weighting such as longer data collection time. The novel version proposed is applied in an example of a product design selection problem for additive manufacturing. The proposed method is compared with neutrosophic sets and Pyhtagorean fuzzy extensions of EDAS. Also, a sensitivity analysis is conducted in order to check its robustness against the changes in attribute weights.
Keywords: EDAS, spherical fuzzy sets, entropy measure, linguistic assessment.

Öz
İnsan tercihlerindeki belirsizliği ve kesinsizliği temsil etmek amacıyla 1960'larda bulanık kümeler kavramı ortaya atılmış ve daha sonraki süreçte bir çok farklı bulanık küme tanımı geliştirilmiştir. Bu gelişmelerden oldukça yeni bir tanesi olan küresel bulanık kümeler (KBK), karar vericilere kapsamlı bir tercih alanı vererek bu amacı desteklemektedir. KBK'in ayırt edici özelliği, üyelik, üye-olmama ve tereddüt derecelerinin kareler toplamının [0,1] aralığında olması ve her bir unsurun aynı aralık içinde bağımsız olarak tanımlanabilmesidir. Bu çalışma ile, oldukça yeni ancak yazında sıkça kendine yer bulan bir çok ölçütlü karar verme yöntemi olan EDAS (Evaluation Based on Distance from Average Solution – Ortalama Çözüme Olan Uzaklığa Dayalı Değerlendirme), küresel bulanık ortam için uyarlanmaktadır. Entropi tabanlı nesnel ölçüt ağırlıklandırma, daha uzun veri toplama süresi gibi öznel ağırlıklandırma yöntemlerinin dezavantaj yaratan potansiyel etkilerinden kaçınmak için EDAS'ın bu yeni sürümüyle entegre edilmiştir. Önerilen yeni versiyon, eklemeli imalat (*additive manufacturing*) için bir ürün tasarımı seçme problemi örneğinde uygulanmıştır. Önerilen yöntem neutosophic küme ve Pisagor bulanık küme kullanılan EDAS versiyonları ile karşılaştırılmış, ayrıca ölçüt ağırlıklarındaki değişim karşısında yöntemin gürbüzlüğünün test edilmesi amacıyla bir de duyarlılık analizi yapılmıştır.
Anahtar kelimeler: EDAS, küresel bulanık kümeler, entropi ölçüsü, dilsel değerlendirme.

I. INTRODUCTION

Decision-makers usually have to state their opinions or judgments when they are consulted about a decision problem. Linguistic terms can support them by providing different sorts of fuzzy sets. Fuzzy set concept was developed by Zadeh (1965) and symbolizes the judgments in the decision makers' subconscious. In classical fuzzy sets, there is only one element called membership degree (μ_A) and it can take any value in the range of [0,1]. Membership degree can represent the optimism or agreement level of a judgment.

During the last 6 decades, many researchers from various scientific fields developed different representation styles to ease the symbolization of the vagueness. Some important developments are summarized below.

Atanassov (1986) described the intuitionistic fuzzy sets and introduce a new element into fuzzy sets, called non-membership function. Non-membership degree (v_A) adds flexibility into the representation of judgments because it states the pessimistic view or disagreement level of a judgment. After specifying this new parameter, Atanassov (1986) also presented a new measure regarding decision makers' hesitancy level so that intuitionistic fuzzy sets can handle three dimensions of judgments (membership, non-membership, and hesitancy). But here the hesitancy is related to the other two degrees and decision-maker does not have the opportunity to appoint any independent hesitancy degree: $\pi_A = 1 - \mu_A\,(x) - v_A\,(x)$.

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Yager (2013)'s Pythagorean fuzzy sets concept is developed for supporting decision-makers about the boundaries of fuzzy sets or intuitionistic fuzzy sets. This system defines the membership (μ_A) and non-membership (v_A) degrees in the manner of holding a rule saying that the squared sum of these two degrees should be between 0 and 1. Therefore, the hesitancy degree depends again on these two predetermined ones: $\pi_A = \sqrt{(1 - \mu_A^2(x) - \upsilon_A^2(x))}$. As seen from the equation, the hesitancy is derived from the others.

q-rung orthopair fuzzy sets concept has emerged from Pythagorean fuzzy set concept and it can be accepted as its generalization. Yager (2017) identified this new understanding by changing the squaring rule with taking q^{th} power. So, the dependent hesitancy degree can be computed by the equation of $\pi_A = \sqrt[q]{(1 - \mu_A^q(x) - \upsilon_A^q(x)}$ where $q \geq 1$.

Using the degree of hesitancy/indeterminacy as an independent component, Smarandache (1999) initiated the neutrosophic set theory. Neutrosophic set is defined where each element of the universe has specific nonstandard membership (μ_A), non-membership (v_A), and hesitancy (π_A) degrees which satisfy the conditions that each degree should be within $]0^-, 1^+[$ and their sum should be within $]0^-, 3^+[$. Wang et al (2010) developed single-valued neutrosophic sets and Wang et al (2005) defined interval valued neutrosophic sets as instances with the aim of making this nonstandard notation more understandable and operationalizable. In this concept, there is an opportunity about assigning independent values for each of three elements in the set but there are different kinds of criticism about it, especially in terms of their mathematical complexity.

Intuitionistic, Pythagorean, q-rung orthopair fuzzy sets and neutrosophic sets are accepted as three-dimensional fuzzy sets because they can represent the uncertainty more comprehensively by involving three fuzziness degrees. In order to deal with the drawbacks explained above, Kutlu Gündoğdu and Kahraman (2019a) invented spherical fuzzy sets (SFS) as a synthesis of Pythagorean fuzzy sets and neutrosophic sets. SFS consists of three elements that can take independently any value between 0 and 1, namely membership, non-membership, and hesitancy degrees. The distinctive feature of SFS is that the squared sum of these three degrees should be within the interval of [0,1]. Suppose a decision-maker saying that "I partially agree (0.80) and partially disagree (0.25) with your idea and I need to stay hesitant a little bit (0.10)". The sum of these values is greater than 1 (0.80+0.25+0.10=1.15) while the squared sum is lower than 1 (0.7125). This judgment can only be represented by SFS. So, it is seen that this novel form of fuzziness provides a more extensive preference domain to the decision-maker.

Among many multiple attribute decision-making methods, EDAS (Evaluation Based on Distance from Average Solution) was developed by Keshavarz Ghorabaee et al (2015) to search the distances between each alternative and average solution. It is a method from distance-based method family such as TOPSIS and VIKOR. They consider the distances between each alternative and positive/negative ideal alternatives and then reach a decision accordingly: the best alternative should relatively be as distant as possible from the negative ideal and as close as possible to the positive ideal. EDAS eliminates the step of determining the positive/negative ideal solutions which can be complex that expected in terms of some situations since it measures the distance between each alternative and average solution. In the literature, authors chose to use various ideal alternative determination processes. For instance, Lima Junior et al (2014) considered constant ideal points such as triangular fuzzy numbers: (1,1,1) for positive ideal solution and (0,0,0) for negative ideal solution while Kutlu Gündoğdu and Kahraman (2019a) defuzzified the decision matrix for selecting the maximum values for the positive ideal and the minimum values for the negative ideal solutions. Biswas et al (2016) developed their own methodology for this process. EDAS does not require these types of discussions because it eliminates this step as a contribution to the literature. In this manner, EDAS makes the compromise solution method more stable in ranking alternatives (Feng et al, 2018; Darko and Liang, 2020).

This study aims to extend this new (it was developed just five years ago) and easily implementable (an extensive literature has been formed during a short period of time because its technical aspects are not so complex) methodology under spherical fuzzy environment because the crisp or current fuzzy versions of EDAS are not capable of considering independent hesitancy of decision-makers. The literature of EDAS is summarized in Section 2.

Another important issue that is handled in this study is the objective weighting of attributes. Subjective approaches such as pairwise comparison logic of AHP (Analytic Hierarchy Process) or direct importance allocation are based on the decision makers' expertise on the topic but objective methods conversely do not consider these individualistic preferences and just look at the performances which were collected with respect to each attribute. To minimize the risk arisen from the self-seeking/manipulative decision-makers or too long data collection period, many objective weighting methods are developed in the literature. In some real-life cases, such as credit rating of the firms by the creditors or performance assessment of an occupational health and safety policy application, the subjective weights based on expert opinions are not desired today.

For making the decision process more comprehensive and competent, this paper integrates entropy-based objective attribute weighting and EDAS methodology under spherical fuzzy environment. Its capability of operating decision makers' hesitancies as independent inputs, this novel version of EDAS, and objective weighting with spherical fuzzy numbers (SFN) can be accepted as the contributions of this study to the decision-making literature. The rest is organized as follows. While section 2 shares the results of the extensive literature review on three-dimensional fuzzy versions of EDAS, section 3 includes the preliminaries of SFSs. In Section 4, the spherical fuzzy version of EDAS (SF-EDAS) is detailed step-by-step. In order to show its application possibility, an illustrative example about product design for additive manufacturing is demonstrated in Section 5. Section 6 concludes the study with the findings and the further research potential.

II. LITERATURE REVIEW ON THREE-DIMENSIONAL FUZZY VERSIONS OF EDAS

Traditional fuzzy extension of EDAS is a research area getting importance day-by-day in the literature. EDAS which was developed by Keshavarz Ghorabaee et al (2015) found its very first application in multi-criteria inventory classification. There are many crisp versions of EDAS in the literature, e.g., Özmen (2020) proposed an SMAA-EDAS integration for evaluating OECD countries in terms of broadband infrastructures and structural services of the telecommunication sector; Özbek (2019) used EDAS method for ranking the cities of Turkey in terms of the livability criteria; Ulutaş (2018) integrated entropy-based objective attribute weighting and EDAS method in order to analyze the performances of logistics companies. There are also traditional fuzzy extensions of EDAS in the literature, e.g., Aldalou and Perçin (2020) utilized entropy for weighting attributes of financial performance evaluation problem of food and drink index and fuzzy EDAS for obtaining the best results, and then CRITIC-based scenario analysis was conducted for checking the robustness of the solution; Kas Bayrakdaroğlu and Kundakcı (2019) used a fuzzified version of EDAS while selecting the best R&D project; Mukul et al (2019) obtained the attribute weights via fuzzy AHP and the alternatives were evaluated through fuzzy EDAS in a strategic analysis of intelligent transportation systems; Erkayman et al (2018) modified fuzzy DEMATEL and integrated it with fuzzy EDAS for choosing the best ERP deployment project.

In this paper, we focus on three-dimensional (3D) fuzzy versions of EDAS rather than the traditional fuzzy versions of crisp ones. Table 1 depicts the results of the extensive literature survey on the related area. In Table 1, versions of 3D fuzzy sets which are considered by papers, the integrated methods with the reason for this integration, and the application area of the study are summarized in columns.

According to our extensive literature review on the field of 3D fuzzy extension attempts of EDAS, there are 15 studies: 4 papers using intuitionistic fuzzy versions, 8 papers using neutrosophic sets, 1 study using Pythagorean, and 2 ones *q*-rung orthopair fuzzy set versions. Different MADM methods are utilized for comparison purposes, i.e., 7 studies compared their EDAS version's results with fuzzified TOPSIS' results, and 3 papers used VIKOR for the same purpose. CODAS, WASPAS, ARAS, and GRA are other methods that were exploited for comparison results. In terms of attribute weighting, CRITIC, SWARA, entropy-based, divergence-based, closeness to ideal solution-based methods were used in an objective or subjective manner or a mixture of them. As seen from the literature, most of the studies have handled the weighting problem via various objective methods by considering their positive sides against subjective weighting. All these 15 papers have used their methods in solving real-life or numerical/experimental applications.

From the literature review, it is seen that there is a very important gap about a recent kind of fuzzy sets representing decision-makers' preferences in a more comprehensive way. Spherical fuzzy version of EDAS has not been developed yet. As mentioned in Chapter 1, intuitionistic and Pythagorean fuzzy sets or *q*-rung orthopair sets can handle hesitancy of the decision-makers in a limited manner because they do not consider an independent hesitancy degree. In neutrosophic sets, there is an independent hesitancy consideration, but different complexities are arising from the rule saying that the sum of membership, non-membership, and hesitancy degrees should be between 0 and 3. In order to handle these issues, spherical fuzzy sets are developed, and their operations are proposed clearly. This study's main contribution to the literature is that it develops a spherical fuzzy extension of EDAS for coping with the issue of hesitancy consideration which is hidden in the judgments of decision-makers. Another important contribution is that a novel entropy that has been proposed recently in the literature finds an application area in this study. This novel entropy is used for gathering the objective weights of attributes that are considered in the application section.

Table 1. The literature on three-dimensional fuzzy versions of EDAS

Paper	*Version of fuzziness*	*Integrated Methods*	*Application*
Kahraman et al (2017)	Interval-valued intuitionistic FS	-	Solid waste disposal site selection
Schitea et al (2019)	Intuitionistic FS	Closeness coefficient-based attribute weighting; WASPAS, COPRAS, and TOPSIS for comparison	Hydrogen mobility roll-up site selection in Romania
Mishra et al (2020)	Intuitionistic FS	Divergence measure-based attribute weighting; TOPSIS and VIKOR for comparison	Assessment of health-care waste disposal technology
Liang (2020)	Intuitionistic FS	CRITIC for attribute weighting; VIKOR and GRA for comparison	Evaluating green building energy-saving design project
Peng and Liu (2017)	Single-valued neutrosophic sets	Grey system theory for attribute weighting	Selection of a software development project to invest
Karaşan and Kahraman (2018a)	Interval-valued neutrosophic sets	TOPSIS for comparison	Prioritization of United Nations national sustainable development goals
Karaşan and Kahraman (2018b)	Interval-valued neutrosophic sets	TOPSIS for comparison	Selecting the best fertilizer supplier for the plantation area
Li et al (2019)	Linguistic neutrosophic numbers	Closeness based attribute weighting	Selecting a suitable property management company in Zhengzhou
Wang et al (2019)	Linguistic neutrosophic numbers	Different aggregation operators for comparison	Selecting the best construction project
Han and Wei (2020)	Multivalued neutrosophic sets	TODIM and TOPSIS for comparison	Choosing the most appropriate investment project
Supciller and Toprak (2020)	Single-valued neutrosophic sets	SWARA for attribute weighting; TOPSIS, VIKOR, and MULTIMOORA for comparison	Selection of wind turbines
Xu et al (2020)	Single-valued complex neutrosophic sets	-	Green supplier selection
Mohagheghi and Mousavi (2020)	Interval-valued Pythagorean FS	Entropy-based attribute weighting; COADAP, ARAS, and D-WASPAS for comparison	Sustainable Project Portfolio Problem
Li et al (2020)	q-rung Orthopair FS	Different aggregation operators for comparison	Selection of household refrigerator
Darko and Liang (2020)	q-rung Orthopair FS	Best-Worst Method for attribute weighting; TOPSIS, GRA, and COPRAS for comparison	Mobile payment platform selection

III. SPHERICAL FUZZY SETS

The concept of spherical fuzzy sets (SFS) is proposed by Kutlu Gündoğdu and Kahraman (2019a) as a generalization and synthesis of Pythagorean fuzzy sets and neutrosophic sets. It is aimed to let decision-makers express their hesitancy independently in their linguistic judgments on an alternative or attribute (Kutlu Gündoğdu, 2020). By using SFS, decision-makers can mention their hesitancy degree just like membership and non-membership degrees, independently. SFS copes with the mostly undesired features of neutrosophic sets (sum of three degrees is larger than 1) and Pythagorean fuzzy sets (it disregards independent hesitancy) (Kutlu Gündoğdu and Kahraman, 2019b).

Definition 1. An SFS $\tilde{A}_S$ of the universe of discourse *X* is given by

$$\tilde{A}_S = \{< x, \mu_{\tilde{A}_S}(x), \upsilon_{\tilde{A}_S}(x), \pi_{\tilde{A}_S}(x) | x \in X >\} \quad (1)$$

where $\mu_{\tilde{A}_S}(x) : X \rightarrow [0,1]$, $\upsilon_{\tilde{A}_S}(x) : X \rightarrow [0,1]$, $\pi_{\tilde{A}_S}(x) : X \rightarrow [0,1]$ and

$$0 \leq \mu^2_{\tilde{A}_S}(x) + \upsilon^2_{\tilde{A}_S}(x) + \pi^2_{\tilde{A}_S}(x) \leq 1 \ , \ \forall x \in X \quad (2)$$

For each *x*, the $\mu_{\tilde{A}_S}(x), \upsilon_{\tilde{A}_S}(x)$, and $\pi_{\tilde{A}_S}(x)$ are the membership degree, non-membership degree, and hesitancy degree of *x* to $\tilde{A}_S$, respectively.

Definition 2. Let X_1 and X_2 be two universes and let $\tilde{A}_S = (\mu_{\tilde{A}_S}, \upsilon_{\tilde{A}_S}, \pi_{\tilde{A}_S})$ and $\tilde{B}_S = (\mu_{\tilde{B}_S}, \upsilon_{\tilde{B}_S}, \pi_{\tilde{B}_S})$ be two SFSs from the universes of discourse X_1 and X_2. Basic operators are defined as given below.

Addition

$$\tilde{A}_S \oplus \tilde{B}_S = \{(\mu^2_{\tilde{A}_S} + \mu^2_{\tilde{B}_S} - \mu^2_{\tilde{A}_S}\mu^2_{\tilde{B}_S})^{\frac{1}{2}}, \upsilon_{\tilde{A}_S}\upsilon_{\tilde{B}_S}, ((1-\mu^2_{\tilde{B}_S})\pi^2_{\tilde{A}_S} + (1-\mu^2_{\tilde{A}_S})\pi^2_{\tilde{B}_S} - \pi^2_{\tilde{A}_S}\pi^2_{\tilde{B}_S})^{\frac{1}{2}}\} \quad (3)$$

Multiplication

$$\tilde{A}_S \otimes \tilde{B}_S = \{\mu_{\tilde{A}_S}\mu_{\tilde{B}_S}, (\upsilon^2_{\tilde{A}_S} + \upsilon^2_{\tilde{B}_S} - \upsilon^2_{\tilde{A}_S}\upsilon^2_{\tilde{B}_S})^{\frac{1}{2}}, ((1-\upsilon^2_{\tilde{B}_S})\pi^2_{\tilde{A}_S} + (1-\upsilon^2_{\tilde{A}_S})\pi^2_{\tilde{B}_S} - \pi^2_{\tilde{A}_S}\pi^2_{\tilde{B}_S})^{\frac{1}{2}}\} \quad (4)$$

Multiplication by a scalar (λ>0)

$$\lambda * \tilde{A}_S = \{\left(1-(1-\mu^2_{\tilde{A}_S})^{\lambda}\right)^{\frac{1}{2}}, \upsilon^{\lambda}_{\tilde{A}_S}, ((1-\mu^2_{\tilde{A}_S})^{\lambda} - (1-\mu^2_{\tilde{A}_S} - \pi^2_{\tilde{A}_S})^{\lambda})^{\frac{1}{2}}\} \quad (5)$$

Definition 3. For aggregation purposes, spherical weighted arithmetic mean (SWAM) is defined as follows. The weights are given as w_j $(j = 1, \ldots, n)$ where $w_j \in [0,1]$ and $\sum_{j=1}^{n} w_j = 1$.

$$SWAM_w(\tilde{A}_{S1}, \tilde{A}_{S2}, \ldots, \tilde{A}_{Sn}) = w_1\tilde{A}_{S1} + w_2\tilde{A}_{S2} + \cdots + w_n\tilde{A}_{Sn}$$

$$= \{\left[1 - \prod_{i=1}^{n}\left(1-\mu^2_{\tilde{A}_{Si}}\right)^{w_i}\right]^{\frac{1}{2}}, \prod_{i=1}^{n} \upsilon^{w_i}_{\tilde{A}_{Si}}, [\prod_{i=1}^{n}\left(1-\mu^2_{\tilde{A}_{Si}}\right)^{w_i} - \prod_{i=1}^{n}\left(1-\mu^2_{\tilde{A}_{Si}} - \pi^2_{\tilde{A}_{Si}}\right)^{w_i}]^{\frac{1}{2}}\} \quad (6)$$

Definition 4. Score function (that can be used for defuzzification operation) and Accuracy function that can be used for ranking SFSs are defined by

$$sc(\tilde{A}_S) = (\mu_{\tilde{A}_S} - \pi_{\tilde{A}_S})^2 - (\upsilon_{\tilde{A}_S} - \pi_{\tilde{A}_S})^2 \quad (7)$$

$$ac(\tilde{A}_S) = \mu^2_{\tilde{A}_S} + \upsilon^2_{\tilde{A}_S} + \pi^2_{\tilde{A}_S} \quad (8)$$

$\tilde{A}_S$ can be preferred to $\tilde{B}_S$ if and only if *(i)* $sc(\tilde{A}_S) > sc(\tilde{B}_S)$ or *(ii)* $sc(\tilde{A}_S) = sc(\tilde{B}_S)$ and $ac(\tilde{A}_S) > ac(\tilde{B}_S)$.

Definition 5. Normalized spherical distance between $\tilde{A}_S = (\mu_{\tilde{A}_S}, \upsilon_{\tilde{A}_S}, \pi_{\tilde{A}_S})$ and $\tilde{B}_S = (\mu_{\tilde{B}_S}, \upsilon_{\tilde{B}_S}, \pi_{\tilde{B}_S})$ can be found via

$$dis_n(\tilde{A}_S, \tilde{B}_S) = \frac{2}{n\pi}\sum_{i=1}^{n} \arccos(\mu_{\tilde{A}_S}\mu_{\tilde{B}_S} + \upsilon_{\tilde{A}_S}\upsilon_{\tilde{B}_S} + \pi_{\tilde{A}_S}\pi_{\tilde{B}_S}) \quad (9)$$

Decision analyst dealing with the decision problem has to choose a proper weighting approach for representing the importance of decision attributes since the aggregated performance values are obtained by using these weights. In literature, we have two general approaches for this purpose. Subjective weighting methods consider the decision-makers' judgments and opinions regarding the aforementioned importance while objective weighting approaches ignore these personal preferences and directly operationalize the performance values that are obtained with respect to each attribute (Koksalmis and Kabak, 2019). In literature, there are some drawbacks cited for subjective weighting, e.g., some risks can arise from self-seeking behaviors of decision-makers or long periods may be needed while collecting data from the decision-makers. To overcome these issues, one can apply objective weighting understanding (Aydoğdu and Gül, 2020).

The basic motivation of using an objective attribute weighting method in the study is to minimize the experts' possible negative influence on the decision and minimize the risk of manipulation. In the literature, there are some objective methods such as maximizing deviation method (Xiong and Cheng, 2018; Peng et al, 2018; Ju et al, 2021), variation coefficient method (Tian et al, 2017), CRITIC (Piasecki and Kostyrko, 2020), entropy-based approaches (Jin et al, 2019; Barukab et al, 2019; Aydoğdu and Gül, 2020), etc. Except for entropy, the mentioned methods are based on statistical measures such as variance and standard deviation and they also utilize mathematical programming models in calculating the unknown attribute weights. The study preferred to use the entropy tool in attribute weighting because it is easier than other methods, especially in SFS-based MADM. Also, few articles are proposing objective attribute weighting in this field and novel approaches are required for this purpose. Developing new mathematical modeling and statistics-based models is out of the study's scope. Future studies should consider this issue.

As mentioned above, entropy measure is a good alternative way while objectively weighting attributes. It is a mathematical tool that is used for measuring fuzziness degree and a significant quantitative measure of uncertain information. Entropy-based weighting is based on one idea: an attribute has higher importance if a greater dispersion in evaluations of alternatives has occurred. According to this definition, the dispersion of the data in an attribute can be a measurement of its importance. In decision-making applications, after collecting the scores of the alternatives with respect to the attributes, these two data are processed together. In entropy-based weighting, the entropy value of each attribute is calculated and then they are normalized to get the weights. Subjective expert judgments are used here as representations of the alternative scores, not as the attribute weights. In most of the fuzzy-based decision-making applications, the authors prefer to collect additional subjective data about the importance set of attributes but in some cases, this additional operation can be improper. Thus, the expert's judgments are solely utilized for the purpose of alternative scoring. In this manner, the subjectivity is not totally removed, but limited. Aydoğdu and Gül (2020) developed an entropy measure for SFS as given below.

Definition 6. Let $\tilde{A}_S$ be an SFS on X. The mapping

$$E(\tilde{A}_S) = \frac{1}{n}\sum_{i=1}^{n}\left(1-\frac{4}{5}\left[\left|\mu_{\tilde{A}_S}^2(x_i)-v_{\tilde{A}_S}^2(x_i)\right|+\left|\pi_{\tilde{A}_S}^2(x_i)-0.25\right|\right]\right) \quad (10)$$

is an entropy measure for SFS.

IV. SPHERICAL FUZZY VERSION OF EDAS (SF-EDAS)

EDAS is one of the distance-based multiple attribute decision-making methods such as TOPSIS and VIKOR. These methods measure the distances between each alternative and positive/negative ideal alternatives and then consider these measures as a decision criterion. Under these circumstances, decision-makers should determine one positive and one negative ideal solution. EDAS can eliminate this additional step since it measures the distance between alternative and average solutions. So, decision-makers do not have to obtain positive/negative ideal solutions and just need to compute the average performance scores of each attribute. EDAS utilizes two measures: positive distance from average (*PDA*) and negative distance from average (*NDA*). Naturally, the decision-makers desire higher positive distance and lower negative distance.

In this study, we have extended EDAS multiple attribute decision-making method under spherical fuzzy environment as a contribution to the literature. The novel concept of SFS can supply higher independence opportunities to the decision-makers since SFS allows them to state an independent hesitancy degree. In SFS, the geometric spherical surface gives a broader preference declaration opportunity to the decision-makers. The traditional EDAS and the existing 3D fuzzy versions of ARAS which are shown in Table 1, do not or rarely take this independent hesitancy element under consideration. To deal with this problem, SFS extension of EDAS is introduced in this study for the first time in the literature.

1.1 Step 1: Obtaining and Aggregating Decision Makers' Judgments

Decision-makers (e=1,…,k) are asked to express their judgments with linguistic terms stated in Table 2 about alternatives' (i=1,…,m) performances with respect to attributes (j=1,…,n). So, after collecting data from decision-makers, there will be k decision matrices $(X^1, X^2, \dots, X^k)$. The linguistic terms have correspondences as membership, non-membership, and hesitancy degrees. If a decision-maker is fully informed or experienced with SFSs, he/she can express their own SF number instead of the given linguistic term. This is one of the benefits arisen from using an SFS extended EDAS.

The aggregated judgments are computed via SWAM operator (Equation 11) and the resulting aggregated matrix is obtained as given in Equation (12). In this step, the decision-makers can be weighted according to their expertise (ω_e). $\langle \mu_{ij}^e, v_{ij}^e, \pi_{ij}^e \rangle$ depicts the linguistic evaluation of e^{th} decision-maker and $\langle \mu_{ij}, v_{ij}, \pi_{ij} \rangle$ represents the aggregated performance evaluation. In this application, decision-makers select the most appropriate linguistic terms from Table 2 so that there are no outlier values in these evaluations as seen in the table and also, there are no proportional relations among them. For these reasons, we propose the usage of SWAM operator in the present SF-EDAS approach rather than the usage of SWGM (spherical weighted geometric mean) operator. Besides, Sharaf (2021) compared SWAM and SWGM operators in an application with SF-VIKOR and showed that SWAM operator give more stable and robust implementation results. If required, a decision analyst can use other operators like SWGM.

Table 2. Linguistic terms and corresponding SF numbers (Kutlu Gündoğdu and Kahraman, 2020a)

Linguistic Term	*Abb.*	μ	v	π
Absolutely More Satisfactory	AMS	0.9	0.1	0.1
Very High Satisfactory	VHS	0.8	0.2	0.2
High Satisfactory	HS	0.7	0.3	0.3
Slightly More Satisfactory	SMS	0.6	0.4	0.4
Medium Satisfactory	MS	0.5	0.5	0.5
Slightly Low Satisfactory	SLS	0.4	0.6	0.4
Low Satisfactory	LS	0.3	0.7	0.3
Very Low Satisfactory	VLS	0.2	0.8	0.2
Absolutely Low Satisfactory	AL	0.1	0.9	0.1

$$\begin{aligned} X^{agg} &= SWAM_{\omega}(X^1, X^2, \ldots, X^k) \\ &= \omega_1 X^1 + \omega_2 X^2 + \cdots + \omega_k X^k = \langle \mu_{ij}, \upsilon_{ij}, \pi_{ij} \rangle \\ &= \{\left[1 - \prod_{e=1}^{k}\left(1 - (\mu_{ij}^e)^2\right)^{\omega_e}\right]^{\frac{1}{2}}, \prod_{e=1}^{k}(\upsilon_{ij}^e)^{\omega_e}, \\ &[\prod_{e=1}^{k}\left(1 - (\mu_{ij}^e)^2\right)^{\omega_e} - \prod_{i=1}^{k}\left(1 - (\mu_{ij}^e)^2 - (\pi_{ij}^e)^2\right)^{\omega_e}]^{\frac{1}{2}}\} \end{aligned} \quad (11)$$

$$X^{agg} = \begin{bmatrix} \langle \mu_{11}, \upsilon_{11}, \pi_{11} \rangle & \cdots & \langle \mu_{1n}, \upsilon_{1n}, \pi_{1n} \rangle \\ \vdots & \ddots & \vdots \\ \langle \mu_{m1}, \upsilon_{m1}, \pi_{m1} \rangle & \cdots & \langle \mu_{mn}, \upsilon_{mn}, \pi_{mn} \rangle \end{bmatrix} \quad (12)$$

Step 2: Calculation of Weights

In order to operate the model, the decision analyst needs to find the attribute weights since all the attributes can have different importance for the problem at hand. Objective weighting is chosen in this study for minimizing the potential manipulative effect of misbehaving decision-makers. If required, the decision analyst is free to use any subjective method such as SWARA or AHP. In entropy-based objective weighting, there two basic operations. Firstly, the entropy of each attribute should be calculated via Equation (10). For this purpose, alternatives' aggregated performance scores are considered. Then, objective weights of attributes are computed via Equation (13). *m* is here the number of attributes.

$$w_j = \frac{(1 - En_j)}{\sum_{i=1}^{m}(1 - En_j)} \quad (13)$$

Step 3: Finding Average Solutions

The basic distinctive feature of EDAS is the consideration of average scores rather than positive or negative ideals. In this step, the SF average scores of each attribute will be obtained. For this purpose, all the aggregated performance scores depicted in columns of X^{agg} matrix are averaged as given in Equation (14). For the addition operation, Equation (3) is used. Then, multiplication by a scalar ($\lambda = 1/m$) operator which is defined in Equation (5) is executed.

$$\widetilde{AV}_j = \frac{1}{m} * \sum_{i=1}^{m} \langle \mu_{ij}, \upsilon_{ij}, \pi_{ij} \rangle = \langle \mu_{ij}^{AV}, \upsilon_{ij}^{AV}, \pi_{ij}^{AV} \rangle \quad (14)$$

Step 4: Calculation of Positive and Negative Distances from Average Solution

In a classical decision-making problem, the attributes can be either beneficial or non-beneficial ones but the current problem has just beneficial attributes because the performance scores are linguistic terms. A more positive statement means a higher value in SFS-based linguistic terms (please see Table 2). The basic difference is not about attribute types but the elements of SFS. A membership degree has a positive meaning and a higher membership degree is always desired. Conversely, non-membership and hesitancy degrees have negative meanings and lower values are naturally desired. Also, all these three degrees are independently determined, and they can be operated independently as well. So, while measuring the distances from average solutions, Equation (15) and Equation (16) are used for positive distance ($\widetilde{PDA}_{ij}$) and negative distance ($\widetilde{NDA}_{ij}$) from average, respectively.

$$\widetilde{PDA}_{ij} = \begin{cases} \frac{\max(0, (\mu_{ij} - \mu_{ij}^{AV}))}{sc(\widetilde{AV}_j)} & \textit{if membership degree is considered,} \\ \frac{\max(0, (\upsilon_{ij}^{AV} - \upsilon_{ij}))}{sc(\widetilde{AV}_j)} & \textit{if non} - \textit{membership degree is considered,} \\ \frac{\max(0, (\pi_{ij}^{AV} - \pi_{ij}))}{sc(\widetilde{AV}_j)} & \textit{if hesitancy degree is considered,} \end{cases} \quad (15)$$

$$\widetilde{NDA}_{ij} = \begin{cases} \frac{\max(0, (\mu_{ij}^{AV} - \mu_{ij}))}{sc(\widetilde{AV}_j)} & \textit{if membership degree is considered,} \\ \frac{\max(0, (\upsilon_{ij} - \upsilon_{ij}^{AV}))}{sc(\widetilde{AV}_j)} & \textit{if non} - \textit{membership degree is considered,} \\ \frac{\max(0, (\pi_{ij} - \pi_{ij}^{AV}))}{sc(\widetilde{AV}_j)} & \textit{if hesitancy degree is considered,} \end{cases} \quad (16)$$

In literature, there is no proposition for division operation of SFS so that we had to consider the score function value of the average solution ($sc(\widetilde{AV}_j)$) rather than the average solution itself ($\widetilde{AV}_j$). The development of division operation on SFS is out of our study's scope since it requires more complex mathematics. This situation can be accepted as a limitation of our proposition. The score value of the average solution will be a scalar number so that we can use Equation (5) as a representation of division as multiplication by a scalar.

Step 5: Weighting of PDA and NDA Values

In this step, the objective attribute weights are aggregated with *PDA* and *NDA* values. The multiplication by a scalar operator (Equation 5) is used and then, Equation (3) showing the addition operation is utilized. Weighted PDA and NDA values are defined in Equation (17) and Equation (18).

$$\widetilde{SP}_i = \sum_{j=1}^{m} w_j * \widetilde{PDA}_{ij} \quad (17)$$

$$\widetilde{SN}_i = \sum_{j=1}^{m} w_j * \widetilde{NDA}_{ij} \quad (18)$$

Step 6: Normalization Process and Selection of the Best Alternative according to Appraisal Score

To aggregate positive and negative weighted values, they all require to be normalized. Firstly, $\widetilde{SP}_i$ and $\widetilde{SN}_i$ SF numbers should be defuzzified via score function (Equation 7) and then, Equation (19) and Equation (20) are operated for finding normalized values.

$$NSP_i = \frac{sc(\widetilde{SP}_i)}{\max_i sc(\widetilde{SP}_i)} \quad (19)$$

$$NSN_i = 1 - \frac{sc(\widetilde{SN}_i)}{\max_i sc(\widetilde{SN}_i)} \quad (20)$$

As aggregation of normalized weighted positive and negative distances, the appraisal score of each alternative (AS_i) is calculated via Equation (21). All the alternatives can be ranked according to AS_i.

$$AS_i = \frac{NSP_i + NSN_i}{2} \quad (21)$$

V. AN APPLICATION ON PRODUCT DESIGN SELECTION PROBLEM

In this study, we have aimed to develop a fuzzified version of EDAS under a SF environment. We have also tried to keep the computations spherical fuzzy until the very end of the steps. The proposed SF-EDAS is here applied in an illustrative product design selection problem for additive manufacturing (Kutlu Gündoğdu and Kahraman, 2020b). In this example, 7 attributes are considered: speed of movement (C_1), temperature control range (C_2), machine overall cost (C_3), safety standards (C_4), rate of failure occurrence (C_5), modular design (C_6), and axes motion accuracy (C_7). Three experts with the same significance level were asked to assess 5 alternative designs: A, B, C, D, and E.

In Step 1, three decision-makers (e=1,2,3) share their judgments involving the evaluations of alternatives (i=1,…,5) with respect to attributes (j=1,…,7) according to the linguistic terms given in Table 2. The personal decision matrices (X^1, X^2, X^3) are obtained as given in Table 3. Then, they are aggregated by performing SWAM operator given in Equation (11). Kutlu Gündoğdu and Kahraman (2020b) stated that the expert group is composed of experienced engineers and customers in additive manufacturing. They used an expert weight set of (0.3, 0.4, 0.3) but we assume $\omega_j = 1/3$ as the weight of each decision-maker in this study because we use an existing data set and do not have enough information about the expertise differences among them. In fact, the original weights are very close to each other so that we use equal expert weights in this hypothetical application of the proposed SF-EDAS. Table 4 gives the aggregated SF decision matrix (X^{agg}).

In Step 2, the entropy-based objective weights are obtained. Firstly, the entropies are computed by operating Equation (10). As a depiction of calculations, the entropy of C_1 is given below. Other entropies are En_2=0.588, En_3=0.624, En_4=0.824, En_5=0.564, En_6=0.544, and En_7=0.459. The weights of attributes are determined via Equation (13): w_1=0.180, w_2=0.141, w_3=0.129, w_4=0.060, w_5=0.149, w_6=0.156, and w_7=0.185.

Step 3 finds the average values by operating Equation (14). The last row of Table 4 includes the spherical fuzzy average solutions of each attribute and corresponding score function values, respectively. In Step 4, the positive and negative spherical distances from average solution of each alternative are measured degree by degree. The appropriate equation (in Equation 15 and Equation 16) is used for each case representing membership, non-membership, and hesitancy degrees. The $\widetilde{PDA}_{ij}$ and $\widetilde{NDA}_{ij}$ values are given in Table 5. In Step 5, the weighted positive and negative distances are calculated via Equation (17) and Equation (18) and they are depicted in Table 6. The summation of these values is given in Table 7. Step 6 is the last step of the methodology and includes the defuzzification, normalization (Equation 19 and Equation 20), and ranking the alternatives according to their AS_i (Equation 21).

At the end of the procedure, the ranks of the alternatives are obtained: $E \succ B \succ D \succ A \succ C$. So, the best one is the product design alternative E. It can be concluded that E is relatively the most distant alternative to the average solution.

Table 3. Evaluations of the experts

		C_1	C_2	C_3	C_4	C_5	C_6	C_7
	E1	HS	MS	AMS	LS	AMS	MS	AMS
A	E2	VHS	VHS	SLS	HS	SLS	VHS	SLS
	E3	HS	AMS	SLS	VLS	SLS	AMS	SLS
	E1	AMS	HS	HS	HS	HS	HS	HS
B	E2	MS	HS	AMS	LS	AMS	HS	AMS
	E3	VHS	HS	VHS	SLS	VHS	HS	VHS
	E1	HS	MS	AMS	LS	AMS	MS	AMS
C	E2	VHS	VHS	SLS	HS	SLS	VHS	SLS
	E3	HS	AMS	SLS	VLS	SLS	AMS	SLS
	E1	VHS	SMS	HS	VLS	AMS	SMS	HS
D	E2	SMS	VLS	LS	HS	SMS	VLS	VHS
	E3	HS	SMS	SLS	SMS	SMS	SMS	HS
	E1	LS	MS	LS	HS	SLS	MS	AMS
E	E2	VHS	VHS	HS	SMS	MS	VHS	MS
	E3	VHS	SLS	VLS	LS	MS	SLS	VHS

VI. COMPARISONS

In order to check the validity of the proposed entropy-based SF-EDAS tool, we have performed two comparisons. As indicated before, SFS concept is the generalization of Pythagorean fuzzy sets and neutrosophic sets. In the first comparison, for showing the novel extension's validity, the same case is solved by neutrosophic set version of EDAS which is developed by Supciller and Toprak (2020), and Pythagorean fuzzy extension of EDAS which is proposed by Mohagheghi and Mousavi (2020). The interested reader can examine the methods in the papers which are specified. Appropriate linguistic term conversions are operated, and the relevant steps are performed respectively. The results are depicted in Table 8.

The other two method's basic drawback is their early defuzzification perspective. After collecting the data from the experts and aggregating them, they preferred to defuzzify the decision matrix and proceed to the remaining steps as the classical way of EDAS suggests. For clarifying the importance of keeping the whole process spherical fuzzy until the very end of the method, we also performed a modified version of the proposed SF-EDAS. For this purpose, the aggregated decision matrix is defuzzified in the early stages and the classical EDAS is proceeded as the other extensions implement.

Table 8 shows that the proposed SF-EDAS gave significantly different results than the others in terms of alternative rankings. Early defuzzification implementing methods, namely neutrosophic EDAS (NS-EDAS), Pythagorean fuzzy EDAS (PF-EDAS), and SF-EDAS (Early-Def) found the same rankings. This interesting finding emphasizes the importance of keeping the whole process spherical fuzzy until the very end.

In the second comparison, a sensitivity analysis is conducted to introduce the robustness of the method against the attribute weight changes. We use the same methodology by considering different weight sets. In the first trial, we assume that all the attribute weights are equal (w_j = 1/7). In the second trial, the entropy measure proposed by Barukab et al (2019) which is given in Eq. (22) is used in calculating the weights. The resulting weight set is found as (0.158, 0.143, 0.136, 0.105, 0.146, 0.149, 0.162). When we compare this new set with the first weight set which is based on Eq. (10), (0.180, 0.141, 0.129, 0.060, 0.149, 0.156, 0.185), it is seen that the attribute weights changed in magnitude but their priority ranking does not.

$$E(\tilde{A}_S) = \frac{1}{n}\sum_{i=1}^{n}[\{1 - |\mu_{\tilde{A}_S}^2(x_i) - \pi_{\tilde{A}_S}^2(x_i)|\}\frac{2-\mu_{\tilde{A}_S}^2(x_i)-v_{\tilde{A}_S}^2(x_i)-\pi_{\tilde{A}_S}^2(x_i)}{2}] \quad (22)$$

Table 9 shows the results of the sensitivity analysis. From the table, it is seen that there are no significant differences between the comparisons. The leading alternative (E) is keeping its ranking while the pairs of (B,D) and (A,C) takes the order pairs of (2,3) and (4,5), respectively. Therefore, the method's robustness against the changes in attribute weights is shown in this manner. Future studies can retest it via applying the method in solving larger and more complex decision problems.

Table 4. Aggregated SF decision matrix (X^{agg}) and average solutions

	C₁			*C₂*			*C₃*			*C₄*			*C₅*			*C₆*			*C₇*		
A	0.739	0.262	0.266	0.793	0.215	0.255	0.699	0.330	0.274	0.486	0.552	0.286	0.699	0.330	0.274	0.793	0.215	0.255	0.699	0.330	0.274
B	0.793	0.215	0.255	0.700	0.300	0.300	0.821	0.182	0.194	0.519	0.501	0.338	0.821	0.182	0.194	0.700	0.300	0.300	0.821	0.182	0.194
C	0.739	0.262	0.266	0.793	0.215	0.255	0.699	0.330	0.274	0.500	0.528	0.307	0.699	0.330	0.274	0.793	0.215	0.255	0.699	0.330	0.274
D	0.714	0.288	0.298	0.517	0.504	0.369	0.519	0.501	0.338	0.566	0.458	0.330	0.757	0.252	0.284	0.629	0.400	0.290	0.739	0.262	0.266
E	0.714	0.304	0.229	0.625	0.391	0.364	0.486	0.552	0.286	0.577	0.438	0.344	0.470	0.531	0.474	0.625	0.391	0.364	0.793	0.215	0.255
$\widetilde{AV}_j$	0.742	0.265	0.264	0.707	0.307	0.305	0.675	0.353	0.271	0.532	0.494	0.323	0.714	0.305	0.297	0.720	0.294	0.292	0.756	0.257	0.252
$sc(\widetilde{AV}_j)$		0.228			0.161			0.157			0.015			0.174			0.183			0.254	

Table 5. $\widetilde{PDA}_{ij}$ and $\widetilde{NDA}_{ij}$ values

$\widetilde{PDA}_{ij}$	C_1			C_2			C_3			C_4			C_5			C_6			C_7		
A	0.000	0.000	0.000	0.654	0.000	0.122	0.376	0.000	0.000	0.000	0.000	0.300	0.000	0.000	0.055	0.582	0.000	0.085	0.000	0.000	0.000
B	0.453	0.000	0.019	0.000	0.000	0.013	0.796	0.000	0.182	0.000	0.000	0.000	0.692	0.000	0.238	0.000	0.000	0.000	0.480	0.000	0.114
C	0.000	0.000	0.000	0.654	0.000	0.122	0.376	0.000	0.000	0.000	0.000	0.136	0.000	0.000	0.055	0.582	0.000	0.085	0.000	0.000	0.000
D	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.955	0.000	0.000	0.475	0.000	0.030	0.000	0.000	0.005	0.000	0.000	0.000
E	0.000	0.000	0.073	0.000	0.000	0.000	0.000	0.000	0.000	0.979	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.370	0.000	0.000

$\widetilde{NDA}_{ij}$	*C₁*			*C₂*			*C₃*			*C₄*			*C₅*			*C₆*			*C₇*		
A	0.113	0.000	0.003	0.000	0.000	0.000	0.000	0.000	0.009	0.980	0.000	0.000	0.288	0.000	0.000	0.000	0.000	0.000	0.455	0.000	0.044
B	0.000	0.000	0.000	0.201	0.000	0.000	0.000	0.000	0.000	0.765	0.000	0.119	0.000	0.000	0.000	0.322	0.000	0.019	0.000	0.000	0.000
C	0.113	0.000	0.003	0.000	0.000	0.000	0.000	0.000	0.009	0.946	0.000	0.000	0.288	0.000	0.000	0.000	0.000	0.000	0.455	0.000	0.044
D	0.339	0.000	0.070	0.854	0.000	0.144	0.813	0.000	0.158	0.000	0.000	0.056	0.000	0.000	0.000	0.638	0.000	0.000	0.256	0.000	0.027
E	0.342	0.000	0.000	0.642	0.000	0.143	0.859	0.000	0.036	0.000	0.000	0.171	0.894	0.000	0.354	0.649	0.000	0.164	0.000	0.000	0.006

Table 6. $\widetilde{SP}_{ij}$ and $\widetilde{SN}_{ij}$ values

$\widetilde{SP}_{ij}$		C_1			C_2			C_3			C_4			C_5			C_6			C_7	
A	0.000	0.009	0.000	0.275	0.124	0.059	0.139	0.045	0.000	0.000	0.000	0.075	0.000	0.000	0.021	0.250	0.114	0.040	0.000	0.000	0.000
B	0.201	0.093	0.009	0.000	0.014	0.005	0.052	0.235	0.103	0.000	0.000	0.000	0.304	0.166	0.124	0.000	0.000	0.000	0.218	0.152	0.055
C	0.000	0.009	0.000	0.032	0.124	0.059	0.008	0.045	0.000	0.000	0.000	0.033	0.000	0.000	0.021	0.250	0.114	0.040	0.000	0.000	0.000
D	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.368	0.000	0.000	0.193	0.080	0.013	0.000	0.000	0.002	0.000	0.000	0.000
E	0.000	0.000	0.031	0.000	0.000	0.000	0.000	0.000	0.000	0.417	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.164	0.098	0.000

$\widetilde{SN}_{ij}$		C_1			C_2			C_3			C_4			C_5			C_6			C_7	
A	0.048	0.000	0.001	0.000	0.000	0.000	0.000	0.000	0.003	0.420	0.000	0.000	0.113	0.042	0.000	0.000	0.000	0.000	0.205	0.149	0.021
B	0.000	0.000	0.000	0.076	0.000	0.000	0.000	0.000	0.000	0.227	0.000	0.045	0.000	0.000	0.000	0.130	0.014	0.008	0.000	0.000	0.000
C	0.001	0.000	0.001	0.000	0.000	0.000	0.000	0.000	0.003	0.356	0.000	0.000	0.113	0.042	0.000	0.000	0.000	0.000	0.205	0.149	0.021
D	0.012	0.511	0.031	0.410	0.241	0.096	0.361	0.209	0.092	0.000	0.000	0.014	0.000	0.000	0.000	0.280	0.148	0.000	0.112	0.022	0.012
E	0.012	0.558	0.000	0.269	0.114	0.068	0.398	0.265	0.023	0.000	0.000	0.042	0.462	0.278	0.327	0.286	0.138	0.082	0.000	0.000	0.003

Table 7. Defuzzification, normalization, and AS_i values for ranking alternatives

		$\widetilde{SP}_i$		$sc(\widetilde{SP}_i)$	NSP_i		$\widetilde{SN}_i$		$sc(\widetilde{SN}_i)$	NSN_i	AS_i	$Rank$
A	0.388	0.000	0.099	0.074	0.429	0.473	0.000	0.019	0.205	0.000	0.215	4
B	0.416	0.000	0.159	0.040	0.235	0.270	0.000	0.045	0.049	0.762	0.498	2
C	0.252	0.000	0.079	0.024	0.137	0.417	0.000	0.020	0.158	0.232	0.184	5
D	0.410	0.000	0.012	0.158	0.921	0.584	0.000	0.121	0.200	0.025	0.473	3
E	0.443	0.000	0.028	0.172	1.000	0.660	0.000	0.289	0.054	0.739	0.870	1

Table 8. Comparison of alternative rankings

	AS	*SF-EDAS*	*AS*	*SF-EDAS (Early-Def)*	*AS*	*NS-EDAS*	*AS*	*PF-EDAS*
A	0.215	4	0.393	3	0.652	2 , 3	0.662	2 , 3
B	0.498	2	0.957	1	0.993	1	0.998	1
C	0.184	5	0.513	2	0.652	2 , 3	0.662	2 , 3
D	0.473	3	0.294	4	0.279	4	0.169	4
E	0.870	1	0.235	5	0.082	5	0.078	5

Table 9. Results of sensitivity analysis

	AS	*SF-EDAS (Eq. 10)*	*AS*	*SF-EDAS (Equal)*	*AS*	*SF-EDAS (Eq. 22)*
A	0.215	4	0.072	5	0.110	5
B	0.498	2	0.424	3	0.445	3
C	0.184	5	0.142	4	0.156	4
D	0.473	3	0.671	2	0.612	2
E	0.870	1	0.922	1	0.907	1

VII. CONCLUSION

This study proposes a novel spherical fuzzy version of EDAS with the integration of entropy-based objective weighting to solve the problem of consideration of hesitancy which is possibly hidden in the decision maker's preference and judgments. The main contributions can be explained as given below.

- EDAS is one of the literature's younger but stronger methods in recent years. It is extended into a spherical fuzzy environment in this study. The novel method's contribution stems from its representation strength of hesitancy.
- Entropy-based objective attribute weighting is utilized with the integration of SF-EDAS, but the methodology is generalizable with both objective and subjective approaches.
- Most of the computation required by EDAS is adapted by the mathematics of SFS and the entire calculation stage is tried to be kept spherical fuzzy until the end of SF-EDAS. So, the early defuzzification problem is not valid for this version.

Besides its contributions, the study also requires some improvement. Firstly, the requirement involving the utilization of a predetermined and constant linguistic term set (Table 2) should be revised. A future study can research the decision makers' direct allocations of membership, non-membership, and hesitancy degrees so that the preferences and opinions would be modeled more practically. Also, the measurement of consensus in a decision team should be investigated under spherical fuzzy environment. From a technical viewpoint, one last limitation of the EDAS method can be its consideration of the average solution. In case of having outlier expert evaluations, the method's behavior should be extensively researched because the EDAS solution of a decision problem may potentially be affected by these outlier evaluations.

REFERENCES

[1] Aldalou, E., Perçin, S. (2020). Financial Performance Evaluation of Food and Drink Index Using Fuzzy MCDM Approach. *Int. J. Econ. Innov.*, 6(1), 1-19.

[2] Atanassov, K.T. (1986). Intuitionistic fuzzy sets. *Fuzzy Set. Syst.*, 20, 87-96.

[3] Aydoğdu, A., Gül, S. (2020). A novel entropy proposition for spherical fuzzy sets and its application in multiple attribute decision-making. *Int. J. Intell. Syst.*, 35(9), 1354-1374.

[4] Barukab, O., Abdullah, S., Ashraf, S., Arif, M., Khan, S.A. (2019). A New Approach to Fuzzy TOPSIS Method Based on Entropy Measure under Spherical Fuzzy Information. *Entropy*, 21, 1231.

[5] Biswas, P., Pramanik, S., Giri, B.C. (2016). TOPSIS method for multi-attribute group decision-making under single-valued neutrosophic environment. *Neural Comput. & Applic.*, 27, 727-737.

[6] Darko, A.P., Liang, D. (2020). Some q-rung orthopair fuzzy Hamacher aggregation operators and their application to multiple attribute group decision making with modified EDAS method. *Eng. Appl. Artif. Intel.*, 87, 103259.

[7] Erkayman, B., Khorshidi, M., Usanmaz, B. (2018). An integrated fuzzy approach for ERP deployment strategy selection under conflicting criteria. *Atatürk Üniv. İİB Dergisi*, 32(3), 807-823.

[8] Feng, X., Wei, C., Liu, Q. (2018). EDAS Method for Extended Hesitant Fuzzy Linguistic Multicriteria Decision Making. *Int. J. Fuzzy Syst.*, 20(8), 2470-2483.

[9] Han, L., Wei, C. (2020). An Extended EDAS Method for Multicriteria Decision-Making Based on Multivalued Neutrosophic Sets. *Complexity*, 7578507, 9 pages.

[10] Jin, Y., Ashraf, S., Abdullah, S. (2019). Spherical Fuzzy Logarithmic Aggregation Operator Based on Entropy and Their Application in Decision Support Systems. *Entropy*, 21, 628.

[11] Ju, Y., Liang, Y., Luo, C., Dong, P., Gonzalez, E.D.R.S., Wang, A. (2021). T-spherical fuzzy TODIM method for multi-criteria group decision-making problem with incomplete weight information. *Soft Comput*, 25, 2981-3001.

[12] Kahraman, C., Keshavarz Ghorabaee, M., Zavadskas, E.K., Cevik Onar, S., Yazdani, M., Oztaysi, B. (2017). Intuitionistic fuzzy EDAS method: an application to solid waste disposal site selection. *J. Environ. Eng. Landsc.*, 25(01), 1-12.

[13] Karaşan, A., Kahraman, C. (2018a). A novel interval-valued neutrosophic EDAS method: prioritization of the United Nations national sustainable development goals. *Soft Comput.*, 22, 4891-4906.

[14] Karaşan, A., Kahraman, C. (2018b). Interval-Valued Neutrosophic Extension of EDAS Method. In: Kacprzyk J, Szmidt E, Zadrozny S, Atanassov KT, Krawczak M (eds) Advances in Fuzzy Logic and Technology 2017, Proceedings of EUSFLAT'17, September 11-15, Warsaw, Poland & IWIFSGN'17, September 13-15, Warsaw, Poland. Advances in Intelligent Systems and Computing, Switzerland: Springer 642, 343-357.

[15] Kas Bayrakdaroğlu, F., Kundakcı, N. (2019). Bulanık EDAS Yöntemi ile Ar-Ge Projesi Seçimi. *Uluslararası İkt. ve İd. İnc. Dergisi*, 24, 151-170.

[16] Keshavarz Ghorabaee, M., Zavadskas, E.K., Olfat, L., Turskis, Z. (2015). Multi-Criteria Inventory Classification Using a New Method of Evaluation Based on Distance from Average Solution (EDAS). *Informatica*, 26(3), 435-451.

[17] Koksalmis, E., Kabak, Ö. (2019). Deriving decision makers' weights in group decision making: An overview of objective methods. *Inform. Fusion*, 49, 146-160.

[18] Kutlu Gündoğdu, F. (2020). Principals of Spherical Fuzzy Sets. In: Kahraman C, Cebi S, Cevik Onar S, Oztaysi B, Tolga A, Sari I (eds) Intelligent and Fuzzy Techniques in Big Data Analytics and Decision Making (INFUS'19), July 23-25, İstanbul, Turkey. Advances in Intelligent Systems and Computing, Switzerland: Springer 1029, 15-23.

[19] Kutlu Gündoğdu, F., Kahraman, C. (2019a). Spherical fuzzy sets and spherical fuzzy TOPSIS method. *J. Intell. Fuzzy Syst.*, 36(1), 337-352.

[20] Kutlu Gündoğdu, F., Kahraman, C. (2019b). A novel fuzzy TOPSIS method using emerging interval-valued spherical fuzzy sets. *Eng. Appl. Artif. Intel.*, 85, 307-323.

[21] Kutlu Gündoğdu, F., Kahraman, C. (2020a). A novel spherical fuzzy analytic hierarchy process and its renewable energy application. *Soft Comput.*, 24, 4607-4621.

[22] Kutlu Gündoğdu, F., Kahraman, C. (2020b). A novel spherical fuzzy QFD method and its application to the linear delta robot technology development. *Eng. Appl. Artif. Intel.*, 87, 103348.

[23] Li, Y.Y., Wang, J.Q., Wang, T.L. (2019). A Linguistic Neutrosophic Multi-criteria Group Decision-Making Approach with EDAS Method. *Arab. J. Sci. Eng.*, 44, 2737-2749.

[24] Li, Z., Wei, G., Wang, R., Wu, J., Wei, C., Wei, Y. (2020). EDAS method for multiple attribute group decision making under q-rung orthopair fuzzy environment. *Technol. Econ. Dev. Eco.*, 26(1), 86-102.

[25] Liang, Y. (2020). An EDAS Method for Multiple Attribute Group Decision-Making under Intuitionistic Fuzzy Environment and Its Application for Evaluating Green Building Energy-Saving Design Projects. *Symmetry*, 12, 484.

[26] Lima Junior, F.R., Osiro, L., Carpinetti, L.C.R.. (2014). A comparison between Fuzzy AHP and Fuzzy TOPSIS methods to supplier selection. *Appl. Soft Comput.*, 21, 194-209.

[27] Mishra, A.R.., Mardani, A., Rani, P., Zavadskas, E.K. (2020). A novel EDAS approach on intuitionistic fuzzy set for assessment of health-care waste disposal technology using new parametric divergence measures. *J. Clean. Prod.*, 272, 122807.

[28] Mohagheghi, V., Mousavi, S.M. (2019). D-WASPAS: Addressing Social Cognition in Uncertain Decision-Making with an Application to a Sustainable Project Portfolio Problem. *Cogn. Comput.*, 12, 619-641.

[29] Mukul, E., Büyüközkan, G., Güler, M. (2019). Strategic analysis of intelligent transportation systems. *Beykoz Akad. Dergisi*, Özel Sayı, 148-158.

[30] Özbek, A. (2019). Türkiye'deki illerin EDAS ve WASPAS yöntemleri ile taşınabilirlik kriterlerine göre siralanmasi. *Kırıkkale Üniv. SB Dergisi*, 9(1), 177-200.

[31] Özmen, M. (2020). OECD ülkelerinin telekomünikasyon sektörü açisindan SMAA-EDAS yöntemi ile değerlendirilmesi. *NOHU J. Eng. Sci.*, 9(1), 224-237.

[32] Peng, H.G., Zhang, H.Y., Wang, J.Q. (2018). Probability multi-valued neutrosophic sets and its application in multi-criteria group decision-making problems. *Neural Comput. & Applic.*, 30, 563-583.

[33] Peng, X., Liu, C. (2017). Algorithms for neutrosophic soft decision making based on EDAS, new similarity measure and level soft set. *J. Intell. Fuzzy Syst.*, 32, 955-968.

[34] Piasecki, M., Kostyrko, K. (2020). Development of Weighting Scheme for Indoor Air Quality Model Using a Multi-Attribute Decision Making Method. *Energies*, 13, 3120.

[35] Schitea, D., Deveci, M., Iordache, M., Bilgili, K., Akyurt, İ.Z., Iordache, I. (2019). Hydrogen mobility roll-up site selection using intuitionistic fuzzy sets based WASPAS, COPRAS and EDAS. *Int. J. Hydrogen Energ.*, 44, 8585-8600.

[36] Sharaf, I.M. (2021). Spherical Fuzzy VIKOR with SWAM and SWGM Operators for MCDM. In: Kahraman C, Kutlu Gündoğdu F (eds) *Decision Making with Spherical Fuzzy Sets*. Studies in Fuzziness and Soft Computing 392, Switzerland: Springer, 217-240.

[37] Smarandache, F. (1999). A unifying field in logics neutrosophy: neutrosophic probability, set and logic, Rehoboth: American Research Press.

[38] Supciller, A.A., Toprak, F. (2020). Selection of wind turbines with multi-criteria decision making techniques involving neutrosophic numbers: A case from Turkey. *Energy*, 207, 118237.

[39] Tian, Z.P., Wang, J., Wang, J.Q., Zhang, H.Y. (2017). An improved MULTIMOORA approach for multi-criteria decision-making based on interdependent inputs of simplified neutrosophic linguistic information. *Neural Comput. & Applic.*, 28 (Suppl 1), S585-S597.

[40] Ulutaş, A. (2018). Entropi tabanli EDAS yöntemi ile lojistik firmalarinin performans analizi. *Uluslararası İkt. ve İd. İnc. Dergisi*, 23, 53-66.

[41] Wang, H., Smarandache, F., Zhang, Y.Q., Sunderraman, R. (2005). Interval Neutrosophic Sets and Logic: Theory and Applications in Computing, Hexis, USA.

[42] Wang, H., Smarandache, F., Zhang, Y.Q., Sunderraman, R. (2010). Single valued neutrosophic sets. In: Smarandache F (ed) Multispace & Multistructure. Neutrosophic Transdisciplinarity Vol. IV. North-European Scientific Publishers, Hanko, Finland, 410-413.

[43] Wang, P., Wang, J., Wei, G. (2019). EDAS method for multiple criteria group decision making under 2-tuple linguistic neutrosophic environment. *J. Intell. Fuzzy Syst.*, 37(2), 1597-1608.

[44] Xiong, W., Cheng, J. (2018). A Novel Method for Determining the Attribute Weights in the Multiple Attribute Decision-Making with Neutrosophic Information through Maximizing the Generalized Single-Valued Neutrosophic Deviation. *Information*, 9, 137.

[45] Xu, D., Cui, X., Xian, H. (2020). An Extended EDAS Method with a Single-Valued Complex Neutrosophic Set and Its Application in Green Supplier Selection. *Mathematics*, 8, 282.

[46] Yager, R.R. (2013). Pythagorean fuzzy subsets. *Proceedings of Joint IFSA World Congress and NAFIPS Annual Meeting*, June 24-28, 2013, Edmonton, Canada, 57-61.

[47] Yager, R.R. (2017). Generalized Orthopair Fuzzy Sets. *IEEE Trans. Fuzzy Syst.*, 25(5), 1222-1230.

[48] Zadeh, L.A. (1965). Fuzzy sets. *Inform. Control*, 8, 338-353.