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# Sığ Tüneller ve Yapıların Etkileşimi

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**ÖZET**

Tünellerden çıkarılan zemin nedeniyle, yüzeyde çökmeler ve yanal hareketlere neden olur. Mevcut yapıların altından geçecek tünellerde, yapılar üzerinde meydana gelecek etkilerin hesaplanmasının önemli olduğu görülmektedir. Bu çalışmanın amacı, sığ tünel ve yapının bu sorunu inceleyerek etkileşimi araştırmak ve uygun çözümler önermektir. Tünel-yapı etkileşimindeki en büyük sorun, zemin yüzeyinde oturmanın neden olduğu problemdir. Bu nedenle bu çalışmada yeryüzüne yakın tüneller nedeniyle yüzey deplasmanları elde edilmiş ve bu deplasmanlardan dolayı üst yapıda oluşacak kuvvetler incelenecektir. Bunun için indirekt sınır eleman yöntemi kullanılarak elde edilen sonuçlar sonlu elemanlar ile elde edilen sonuçlarla karşılaştırılacaktır. Ayrıca literatürden elde edilen değerlerle karşılaştırılarak optimum çözümler önerilecektir.

# Interaction Between Shallow Tunnels and Structures

**Research Article**

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**ABSTRACT**

The soil lost due to the tunnels built under the ground causes surface collapses and lateral movements. Therefore, it is important to calculate the effects of the tunnels on the existing structures above the ground. The purpose of this study is to investigate the interaction between shallow tunnels and structures and how they affect each other and to suggest suitable solutions. The biggest problem in tunnel-structure interaction is the one caused by sitting on the ground surface. For this reason, in this study, the surface displacements due to the tunnels close to the ground are obtained, and the forces to occur in the upper structure due to these displacements will be examined. To this end, the results obtained using the indirect boundary element method will be compared with those obtained using the finite element method. In addition, optimum solutions will be proposed by comparison with the values obtained from the literature.

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## 1. Introduction

Tunnelling through soils results in ground loss, causing surface settlements and transverse movements. Where a tunnel drive passes below an existing structure, it is important to estimate its effects on the structure. However, the free ground deformations should not simply be imposed upon a structure, because the structure contributes to stiffening of the ground. Computational soil-structure interaction analysis is required to obtain detailed stress–deformation response. In this study, the indirect boundary element method will be used to estimate the ground movements due to a tunnel in free ground, and these results will be compared with the results of the finite element method.

In some studies in the literature, the superstructure has been considered as a beam element or a shell element [1], [2]. Generally, the finite element method has been used in the literature. The tunnel pile interaction problem has been studied by [3] using FEM and FDM methods. In many studies, the damage to the upper structure due to the effect of settling on the ground surface while opening the tunnel was examined.

Quantitative and accurate prediction of soil-structure interaction will be of significant importance in terms of many aspects including construction safety, ground settlements and potential damage to existing structures and facilities, and operation of the subway systems. Due to the complex nature of the soil structure interaction problems, one may have to resort to a numerical approach for analyzing them. It is well known that the boundary element method is a powerful tool for the analysis of soil–structure interaction in geomechanics and has been applied to tunnel excavations by taking into account the construction process, the different soil layers, complex geometries, various loading conditions, and soil–structure interfaces.

In this study, the indirect boundary element approach was used to solve the geotechnical problems with interfaces. The results show a good agreement with finite element solutions. The results presented in this study show that the proposed numerical procedure can be used to effectively estimate the deformation and stresses on the soils and the structures.

Some examples of utilizing the theoretical solution were given for design analysis. Where a tunnel drive passes below an existing structure, it is important to estimate its effects on the structure. However, the free ground deformations should not simply be imposed upon a structure, because the structure contributes to stiffening of the ground. A computational soil-structure interaction analysis is required to obtain a detailed stress–deformation response. This analysis is used to assess the interaction between the ground and the structures sitting on tunnels with different depths. A survey allowed the estimation of free ground movements. In the soil-structure interaction problems, the behavior of the structure and the perimeter of the ground are interconnected, and the solution requires that both the structure and the ground be properly analyzed. Many soil-structure interaction problems are analyzed using numerical methods.

It is known that it is important to calculate the effects of the tunnels to pass under the existing structures. Since ground and structure show different properties in terms of material, the indirect boundary element method equations should be examined in the two-material region [4, 5, 6].

## 2. Indirect Boundary Elements Method

The numerical methods are generally used in solving engineering problems. In the boundary element method (BEM), discretization is done in the boundaries. In the finite element method (FEM), discretization is done in the region. The boundary element method is used in two different ways: direct or indirect method [7, 8, 9]. In the indirect method, first of all, the fictive values at the boundary are calculated. Then, using these fictitious values, the stresses or displacements in the boundary or region are calculated.

If the fictitious values calculated at the boundary are stresses, this is called “fictitious stress method” (FSM). If the values at the boundary are displacement discontinuities, this method is called “displacement discontinuity method” (DDM). These displacement discontinuities can also be called “fictitious cracks.”

In this study, we propose a two-dimensional indirect boundary element method for analyzing the soil-structure interactions. In recent years, it has been observed that the soil-structure interaction problems have been solved by the DDM. Because the soil related problems can be modeled more easily using DDM than other methods.

In the numerical application, the interaction between the tunnel and wall under the effect of the internal pressure inside the tunnel and the effect of the horizontal and vertical loads on the wall has been investigated. The stress and displacements on the ground surface were calculated, and the results were compared with those in the literature.

The problem with an elastic body is that there are stresses or displacements under the influence of the external load or body force acting on the object. For this case, the equations of equilibrium in the elastic body may be written as follows;

$$\sigma_{ij,j} + \beta_i = 0 \tag{1}$$

It is usually assumed that the body forces $\beta_i$ are known; so, the solution we seek from the fifteen equations listed here is for the six stresses $\sigma_{ij}$, the six strains $\varepsilon_{ij}$, and the three displacements $u_i$. In combination with the equilibrium equations, these equations comprise a system for the solution of the stress components, but it is not an especially easy system to solve.

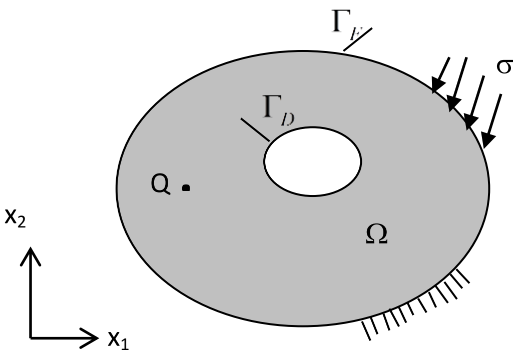


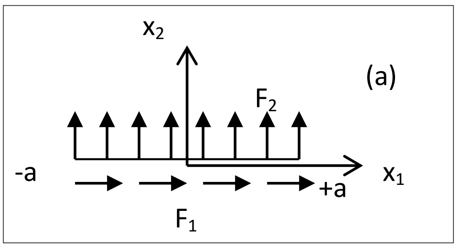

**Figure 1.** Hollow area in the plane

In the boundary element method, there is a need for a solution called “fundamental solution” which corresponds to the homogeneous solution of the differential equation of the problem (Figure 1). This can be the Kelvin solution for three-dimensional objects, the Kelvin or Mindlin solution for half-space problems. In two dimensional problems, Flament solution, Kelvin solution or Melan solution may be written as follows;

$$u_i(Q) = U_{ij}(P,Q)F_j(P) \tag{2}$$

$$\sigma_{ij}(Q) = S_{ijk}(P,Q)F_k(P) \tag{3}$$

where singular force Fj(P) is the force at the point P. The functions $U_{ij}$(P, Q) and $S_{ijk}$(P, Q) are displacements and stresses, respectively, in the xi direction due to a unit force in the $x_j$ direction.
In this study, the indirect boundary element method, the fictitious stress method, and the displacement discontinuity methods were used (Figure 2). In these methods, the material will be considered as a homogeneous, isotropic, and linear elastic material. In the indirect boundary element method, the influence functions are obtained precisely in closed form. The advantages of the integration solved in closed form over the numerical integration lie in that their solutions for influence functions are precise. Since the stiffness singularity is very low in the fictitious stress method, it is appropriate to model the outer boundary of the body using the fictional stress influence functions.



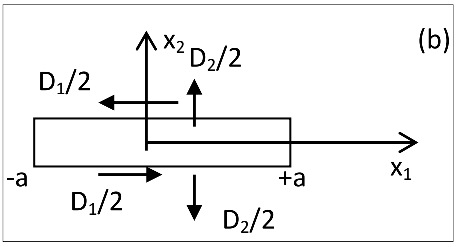




**Figure 2.** Constant elements: (a) FSM; (b) DDM

*2.1. Fictitious Stress Method (FSM)*

In the fictitious stress method, the solution of the problem is obtained by applying fictitious forces to the boundaries of the region. Fictitious forces are calculated by solving the set of linear equations obtained by superposition of boundary integrals. Then, using these fictitious forces, the stresses and displacements in the region and boundaries are calculated. Accordingly, as shown in (Figure 2a), using the equations (2) and (3), the displacement and stresses at the point of Q are calculated with the integration within the range (-a, a). The displacement and stress equations may be written as follows;

$$u_i(Q) = \int_{\Gamma} U_{ij}(P\,;Q)F_j(P)d\Gamma(P) \tag{4}$$

$$\sigma_{ij}(Q) = \int_{\Gamma} S_{kij}(P\,;Q)F_k(P)d\Gamma(P) \tag{5}$$

*2.2. Displacement Discontinuity Method (DDM)*

The solution method in the displacement discontinuity method is exactly the same as the indirect boundary element method. The difference lies in that two opposing surfaces are considered instead of a surface (in any element). Accordingly, surfaces can be considered as an element. When moving from one side of the line to the other, the displacements undergo a change which is stated to be constant in $D_i = (D_1, D_2)$. $D_i$ refers to displacement discontinuity.

The displacement discontinuity in the thickness D in opposite directions in the infinite plane is considered (see Figure 2b). The displacement discontinuity in the infinitesimal element is as given in Eqn. (6) below:

$$D_i = u_i(x_1, 0_-) - u_i(x_1, 0_+) \tag{6}$$

As seen in the equation, it is defined as the displacement difference between the two sides of the part (Figure 2-b). Influence functions of displacement discontinuity are obtained using the singular force fundamental solution in infinite plane. For this, the individual forces are applied in an element mutually and the displacements in that element are calculated.

If we consider a point on the infinite plane, when singular forces in opposite directions are applied to an element the thickness of which is Δ in an infinite zone, the displacement functions can be calculated as follows,

$$u_i(Q) = -\frac{\partial U_{ij}}{\partial x_2} F_j \Delta \tag{7}$$

The derivatives of fundamental solutions in the direction of forces are calculated. Here FjΔ can be named “dipole stress.”

Accordingly, as shown in Figure 2b, using the equations (2), (3), and (7), the displacement and stresses at the point of Q are calculated with integration of the constant displacement discontinuities within the range (-a, a) in any element on the boundary. It can be obtained in the form.

$$u_i(Q) = \int_{\Gamma} U_{ij}^{d}(P\,;Q) D_j(P) d\Gamma(P) \tag{8}$$

$$\sigma_{ij}(Q) = \int_{\Gamma} S_{kij}^{d}(P\,;Q) D_k(P) d\Gamma(P) \tag{9}$$

## 3. Sub-Regional BEM for 2D Contact Bodies

In this section, the indirect boundary element equations in the two-material region will be examined. For this purpose, as seen in Figure 3, considering an object consisting of two sub-regions, the material was accepted as homogeneous, isotropic, and linear elastic in both regions. In general, in addition to the boundary element equations written at the boundary, boundary element equations are obtained by using continuity conditions at the interface.

Sub-region method was used for two different regions. For this, the geometric conformity condition ($u_2$-$u_3$=0) and the equilibrium equation ($t_2$ +$t_3$=0) were used at the boundaries where the two regions contact each other (Figure 3). The equations of the whole system are obtained using these conditions.

The outer boundary of a body is divided into m elements, while at the contact boundary it is divided into 2m elements. Therefore, the number of independent equations is written according to these elements. The equations on the outer boundary of the object and the equations on the contact surface are combined to obtain the equation set for the whole system. After this equation set is solved, the displacement and stress values at the boundaries and the interior region are calculated.

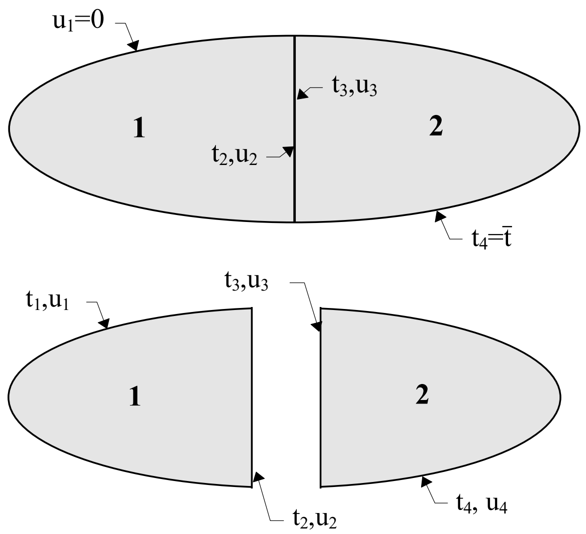



**Figure 3.** Two elastic bodies and boundary conditions

$U_{ij}$ and $T_{ij}$ are the displacements and tractions of nodes in boundary, respectively. We first formulate the coupling condition for both subdomains separately,

$$\begin{bmatrix} T_{11} & T_{12} & 0 \\ T_{21} & T_{22} & 0 \\ 0 & T_{33} & T_{34} \\ 0 & T_{43} & T_{44} \end{bmatrix} \begin{Bmatrix} u_1 \\ u_2 \\ u_4 \end{Bmatrix} = \begin{bmatrix} U_{11} & U_{12} & 0 \\ U_{21} & U_{22} & 0 \\ 0 & -U_{33} & U_{34} \\ 0 & -U_{43} & U_{44} \end{bmatrix} \begin{Bmatrix} t_1 \\ t_2 \\ t_4 \end{Bmatrix} \tag{10}$$

Note that unlike in finite element methods, the single blocks do not overlap but just slide beneath each other. This means that the number of equations does not diminish. Of all six vectors $u_i$ and $t_i$, only two are determined by boundary conditions (e.g. $u_1$ and $t_4$); so, exactly four unknown vectors must be determined by solving four matrix equations. Given a compound domain as in Figure 3, the system of equations reads

$$\begin{bmatrix} T_{12} & -U_{12} & 0 & -U_{11} \\ T_{22} & -U_{22} & 0 & -U_{21} \\ T_{33} & U_{33} & T_{34} & 0 \\ T_{43} & U_{43} & T_{44} & 0 \end{bmatrix} \begin{Bmatrix} u_2 \\ t_2 \\ u_4 \\ t_1 \end{Bmatrix} = \begin{bmatrix} 0 \\ 0 \\ U_{34} \\ U_{44} \end{bmatrix} \{t_4\} \quad (11)$$

## 4. Numerical Calculation Method

The boundary integral equations (12) and (13) are used in the numerical calculation method. Here, due to the displacement discontinuity Di(P) at the point P, the fundamental solutions at the position Q are in the form of $U_{ij}^{d}(P,Q)$ and $S_{ijk}^{d}(P,Q)$, respectively. The displacements and stresses at any point Q, due to $D_i$(P), can be determined by

$$u_i(Q) = \int_{\Gamma} U_{ij}^{d}(P\,;Q) D_j(P) d\Gamma(P) \quad (12)$$

$$\sigma_{ij}(Q) = \int_{\Gamma} S_{kij}^{d}(P\,;Q) D_k(P) d\Gamma(P) \quad (13)$$

where Γ represents the boundary of the surface.

The borders of the region were divided into constant elements of length 2a. The stresses and displacements in the element j due to the loading at the midpoint of the element i are calculated. These calculations are made by integrating the influence functions.

In the calculations made using DDM, the boundary of the object is divided into N displacement discontinuities. These unknown DDMs were obtained by gathering N elements at the same point to meet the boundary conditions. If stresses and displacements are prescribed on the $i^{th}$ element, then the $i^{th}$ equations of the system are as follows,

$$u_s^i = \sum_{j=1}^{N} B_{ss}^{ij} \phi_s^j + \sum_{j=1}^{N} B_{sn}^{ij} \phi_n^j \quad (14)$$

$$u_n^i = \sum_{j=1}^{N} B_{ns}^{ij}\, \varphi_s^j + \sum_{j=1}^{N} B_{nn}^{ij}\, \varphi_n^j \quad (15)$$

$$\sigma_s^i = \sum_{j=1}^{N} A_{ss}^{ij} \phi_s^j + \sum_{j=1}^{N} A_{sn}^{ij} \phi_n^j \quad (16)$$

$$\sigma_n^i = \sum_{j=1}^{N} A_{ns}^{ij} \phi_s^j + \sum_{j=1}^{N} A_{nn}^{ij} \phi_n^j \quad (17)$$

where $\sigma_s$ and $\sigma_n$ are the tangential and normal stresses, respectively. $\boldsymbol{A_{ss}^{ij}}$ and $\boldsymbol{B_{ss}^{ij}}$ etc. are the stress and displacement influence coeffcients, respectively, which relates the stresses (or displacements) of the node i to a unit displacement discontinuity at the node j.

$D_s$ and $D_n$ are the shear and normal relative displacements, respectively, between the faces of a crack as shown in Figure 2b. Here, in the fictitious stress method, the influence functions are the results of the integration of the Eqn. (4) and (5). In the displacement discontinuity method, they are the results of the integration of the Eqn. (8) and (9).

As shown in Figure 1, consider a closed space in infinite plane. Here, the boundaries are divided into 2a long linear elements, and stress and displacements at the middle point of the element i will be calculated due to the loading in the element j. The fictitious values at the boundary are obtained by solving of equations (14), (15), (16), and (17) under the given boundary conditions. Then, using these fictitious values, stresses and displacements in the region and boundaries are calculated. The fictitious values here are the displacement discontinuities for the displacement discontinuity method ($\varphi = D_i$), and the fictitious stresses for the fictitious stress method fictitious strain method ($\varphi = F_i$).

## 5. Numerical Applications

In these examples, the results of the interactive computations for soil-masonry wall were shown in Figure 4. The free ground settlements are shown for comparison. Both the displacements and the interactive inward movements were very small, particularly within the 8m either side of the center-line; however, their effects on the stresses should not be overlooked.

Parameters of example
r (radius of tunnel) =4m
c (axle depth of tunnel ) =10m
h (height of wall) =5m
L (length of wall) =16m
$E_w$ (elasticity of wall) =10000MPa
$v_w$ (Poisson’s ratio of wall) = 0,2
$E_s$ (elasticity of soil) =50MPa
$v_s$ (Poisson’s ratio of soil) = 0,49
w: wall, s: soil

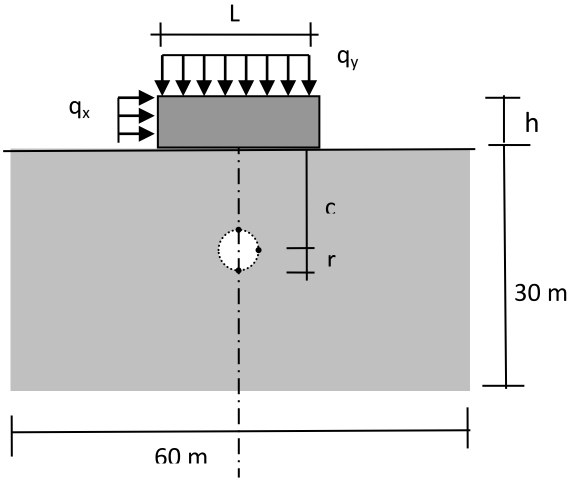



**Figure 4.** Tunnel and superstructure in various loading situations

*5.1. Internal Pressure Effect in Tunnel (Example 1)*

In this example, the interaction between a tunnel opened on the clay floor and a wall on the ground surface will be examined. The tunnel is under a radial pressure of 200 kPa. In the boundary element solution (DDM), the total number of elements is 131 and the number of elements is increased in the region where the radial load is affected by the tunnel and in the area where the wall and the floor interact. Using the symmetry condition, a solution was made with 1/2 of the problem. The results obtained from the solution reached using ANSYS [10] were compared with the results in Selby [6] and the displacement graph on the ground surface is given in the Figures 5 and 6.

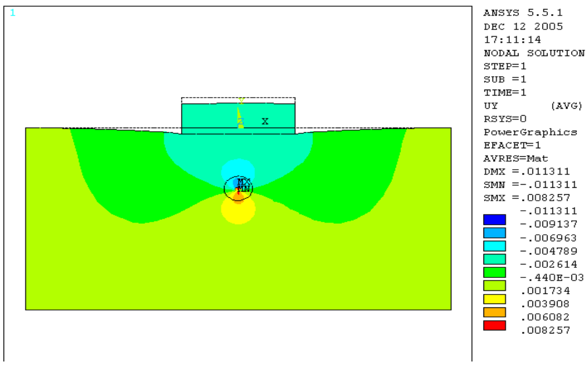



**Figure 5.** Vertical displacements on the ground surface using the FEM method under the influence of internal pressure

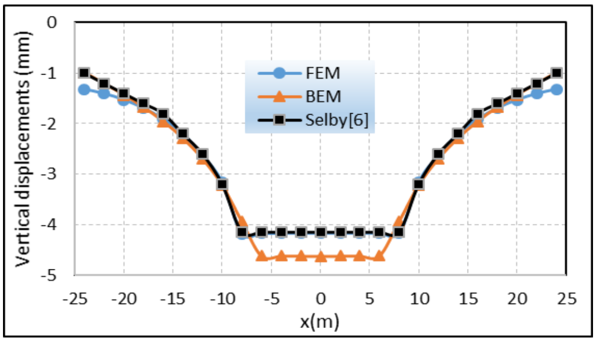



**Figure 6.** Vertical displacements on the ground surface under the influence of internal pressure

*5.2. Only the Vertical Distributed Load Effect on the Structure (Example 2)*

There is a distributed load of 200 kPa on the structure. The results obtained from the solution reached using ANSYS were compared and the displacements were given below. In addition, the calculations were carried out by BEM under the same loading, taking r = 4 m for the changing values of c / r (L = 16 m constant) and likewise taking c = 10 m, this time for the changing values of L / r (L = 16 m constant). The graphics for the displacements and stresses solved by BEM are given in the Figures 7, 8, 9, and 10.

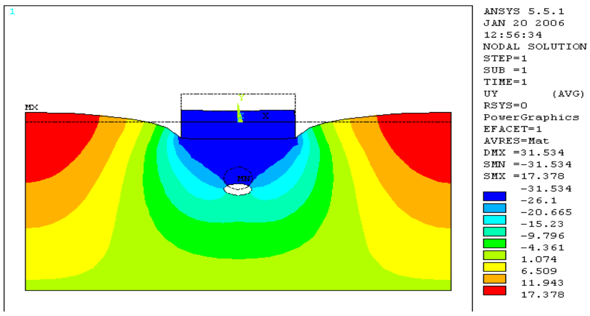



**Figure 7.** Vertical displacements on the ground surface using the FEM method under the vertical loading

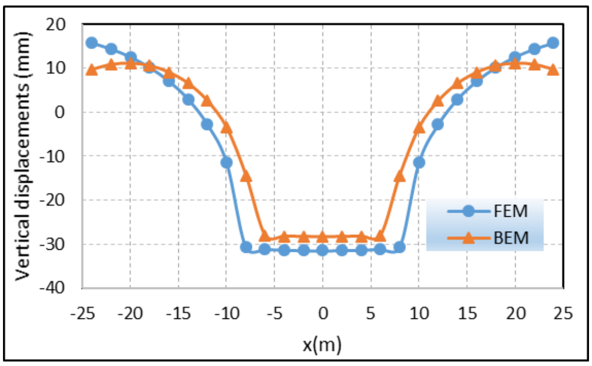



**Figure 8.** Vertical displacements on the ground surface under the vertical loading

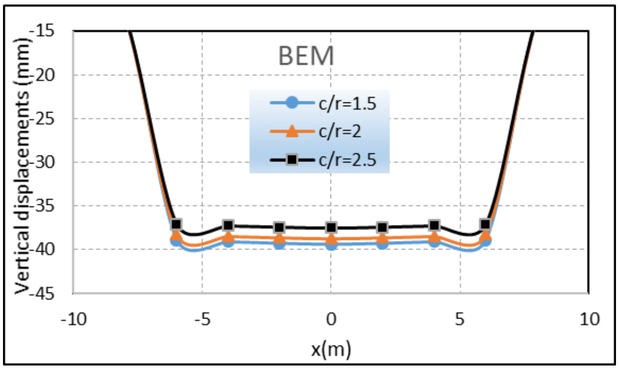



**Figure 9.** Vertical displacements according to c/r, on the ground surface under the vertical loading

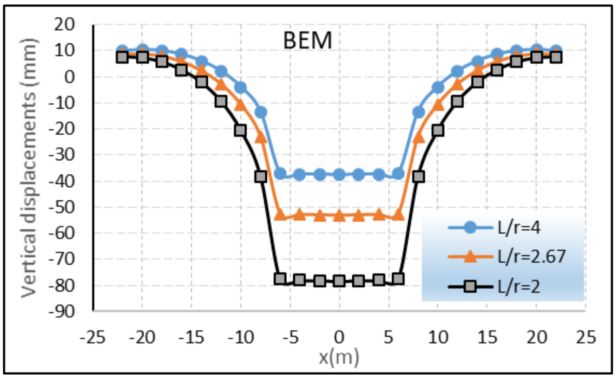



**Figure 10.** Vertical displacements according to L/r, on the ground surface under the vertical loading

*5.3. Only the Horizontal Distributed Load Effect on the Structure (Example 3)*

The horizontal distributed load of 200 kPa is affected by the face of the structure. In the boundary element solution (DDM), the total number of elements is 158, the number of elements is increased in the regions where the tunnel is located, where the load is affected and the structure and the ground interact. The ANSYS and the results obtained from the solution were compared with the displacement and stress graphs below. In addition, the calculations were carried out by BEM under the same loading, taking r = 4 m for the changing values of c / r (L = 16 m constant) and likewise taking c = 10 m, this time for the changing values of L / r (L = 16 m constant). The graphics of resolved displacements and stresses are given below.

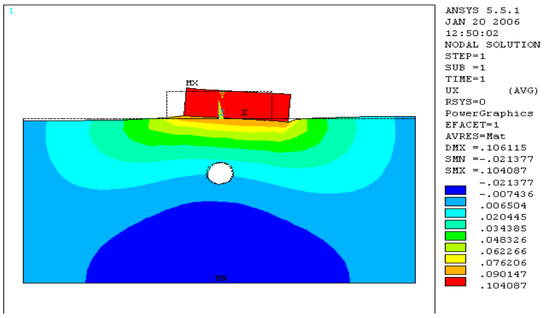



**Figure 11.** Horizontal displacements on the ground surface using the FEM method under the horizontal loading

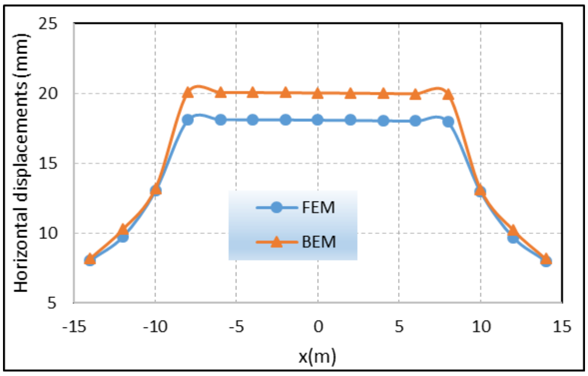



**Figure 12.** Horizontal displacements on the ground surface, under the horizontal loading

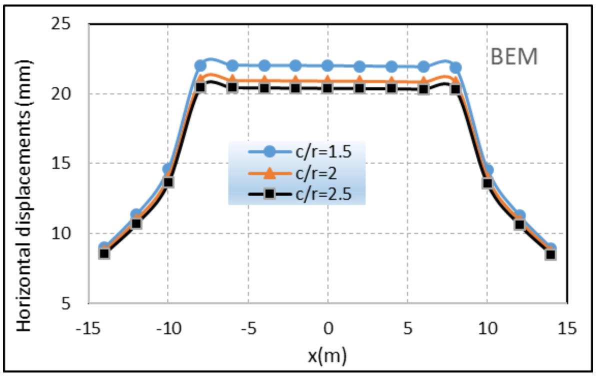



**Figure 13.** Horizontal displacements according to c/r, on the ground surface under the horizontal loading

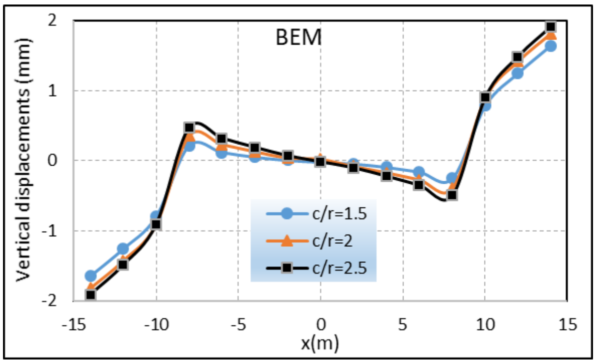



**Figure 14.** Vertical displacements according to c/r, on the ground surface under the horizontal loading

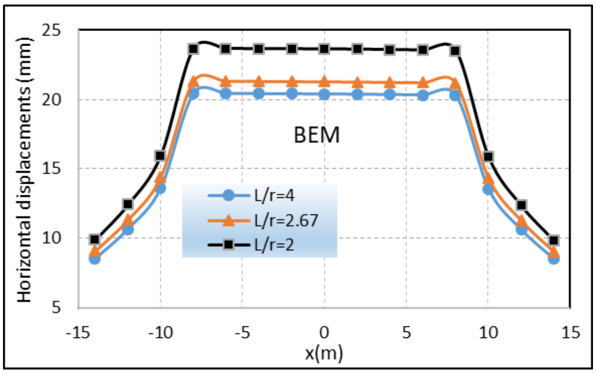



**Figure 15.** Horizontal displacements according to L/r, on the ground surface under the horizontal loading

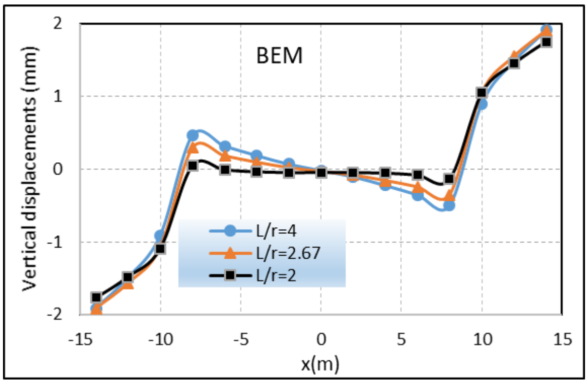



**Figure 16.** Vertical displacements according to L/r, on the ground surface under the horizontal loading

## 6. Results

In this study, two dimensional plane strain problems were investigated using the fictitious stress method (FSM) and the displacement discontinuity method (DDM). In both methods, boundary element equations were created by integrating the Kelvin basic solution using fixed elements. In the Example 1, the problem of interaction between the wall on the ground surface and the tunnel under the effect of radial internal pressure was solved using DDM.

The numerical values in this example were taken from Selby [6] and it was observed that the displacements on the ground surface changed very little along the wall, and after the wall was terminated, the displacements rapidly decreased and changed. When the results obtained from ANSYS and those reached by Selby [6] were compared, the vertical displacements were found to be close enough.

In the Example 2, the problem of the interaction of the tunnel with the structure with a vertical spreading load was solved using DDM. It was observed that the displacements changed very little throughout the structure and decreased rapidly with the termination of the structure, while the stresses remained small throughout the structure, and the main stress occurred at the point where the structure left the ground.

In the Example 3, the problem of the interaction of the tunnel with the structure with a horizontally distributed load on one side was solved using DDM. It was observed that the displacements changed very little throughout the structure and decreased rapidly with the termination of the structure, while the stresses remained small throughout the structure, and the main stress occurred at the point where the structure left the ground.

Based on the results reached in the Examples 2 and 3, it was observed that the growth of the diameter of a tunnel with a fixed axle distance was more effective than the tunnel with a fixed diameter approaching the surface

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